
Data Structures and Program Design in C++

NAVIGATING THE DISK

For information on using the Acrobat toolbar and other Acrobat commands, consult the Help document within Acrobat. See especially the section “Navigating Pages.”

Material displayed in green enables jumps to other locations in the book, to transparency masters, and to run sample demonstration programs. These come in three varieties:

- ➔ The green menu boxes in the left margin of each page perform jumps to frequently used parts of the book:
- ➔ Green material in the text itself will jump to the place indicated. After taking such a jump, you may return by selecting the << icon (go back) in the Acrobat toolbar.
- ➔ The transparency-projector icon (🖨️) brings up a transparency master on the current topic. Return by selecting the << icon (go back) in the Acrobat toolbar.
- ➔ The Windows (🖥️) icon in the left margin select and run a demonstration program, which will operate only on the Windows platform.

This CD contains a folder `textprog` that contains the source code for all programs and program segments appearing in the book. These files cannot be compiled directly, but they can be copied and used for writing other programs.

HINTS FOR PAGE NAVIGATION

- ➔ Each chapter (or other major section) of the book is in a separate pdf file, so you may start Acrobat directly on a desired chapter.
- ➔ To find a particular section in the current chapter, hit the **Home** key, or select |< in the Acrobat toolbar or 🏠 in the green menu bar, which will jump to the first page of the chapter where there is a table of contents for the chapter.
- ➔ After jumping to a new location in the book, you can easily return to your previous location by selecting << (go back) in the Acrobat toolbar.
- ➔ To find a particular topic, select the index icon (📖) in the left margin.
- ➔ To find a particular word in the current chapter, use the binoculars icon in the Acrobat toolbar.
- ➔ The **PgDown** and **Enter** (or **Return**) keys advance one *screenful*, whereas >, ↓, →, and ▶ advance one *page*. Of these, only ▶ will move from the last page of one chapter to the first page of the next chapter.
- ➔ To move backwards, **PgUp** and **Shift+Enter** move up one *screenful*, whereas <, ↑, ←, and ◀ move back one *page*. Of these, only ◀ will move from the first page of one chapter to the last page of the previous chapter.

Data Structures and Program Design in C++

Robert L. Kruse
Alexander J. Ryba

CD-ROM prepared by
Paul A. Mailhot



Prentice Hall
Upper Saddle River, New Jersey 07458

Library of Congress Cataloging-in-Publication Data

KRUSE, ROBERT L.
Data structures and program design in C++ / Robert L. Kruse,
Alexander J. Ryba.

p. cm.
Includes bibliographical references and index.

ISBN 0-13-087697-6

1. C++ (Computer program language) 2. Data Structures
(Computer Science) I. Ryba, Alexander J. II. Title.

QA76.73.C153K79 1998
005.13'3—dc21

98-35979
CIP

Publisher: *Alan Apt*
Editor in Chief: *Marcia Horton*
Acquisitions Editor: *Laura Steele*
Production Editor: *Rose Kernan*
Managing Editor: *Eileen Clark*
Art Director: *Heather Scott*
Assistant to Art Director: *John Christiana*
Copy Editor: *Patricia Daly*

Cover Designer: *Heather Scott*
Manufacturing Buyer: *Pat Brown*
Assistant Vice President of Production and
Manufacturing: *David W. Riccardi*
Editorial Assistant: *Kate Kalbni*
Interior Design: *Robert L. Kruse*
Page Layout: *Ginnie Masterson (PreTeX, Inc.)*
Art Production: *Blake MacLean (PreTeX, Inc.)*

Cover art: *Orange*, 1923, by Wassily Kandinsky (1866-1944), Lithograph in Colors. Source: Christie's Images



© 2000 by Prentice-Hall, Inc.
Simon & Schuster/A Viacom Company
Upper Saddle River, New Jersey 07458

The typesetting for this book was done with PreTeX, a preprocessor and macro package for the TeX typesetting system and the POSTSCRIPT page-description language. PreTeX is a trademark of PreTeX, Inc.; TeX is a trademark of the American Mathematical Society; POSTSCRIPT is a registered trademarks of Adobe Systems, Inc.

The authors and publisher of this book have used their best efforts in preparing this book. These efforts include the research, development, and testing of the theory and programs in the book to determine their effectiveness. The authors and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The authors and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-087697-6

Prentice-Hall International (U.K.) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

Contents

Preface	ix	
Synopsis	xii	
Course Structure	xiv	
Supplementary Materials	xv	
Book Production	xvi	
Acknowledgments	xvi	
1 Programming Principles	1	
1.1 Introduction	2	
1.2 The Game of Life	4	
1.2.1 Rules for the Game of Life	4	
1.2.2 Examples	5	
1.2.3 The Solution: Classes, Objects, and Methods	7	
1.2.4 Life: The Main Program	8	
1.3 Programming Style	10	
1.3.1 Names	10	
1.3.2 Documentation and Format	13	
1.3.3 Refinement and Modularity	15	
1.4 Coding, Testing, and Further Refinement	20	
1.4.1 Stubs	20	
1.4.2 Definition of the Class Life	22	
1.4.3 Counting Neighbors	23	
1.4.4 Updating the Grid	24	
1.4.5 Input and Output	25	
1.4.6 Drivers	27	
1.4.7 Program Tracing	28	
1.4.8 Principles of Program Testing	29	
1.5 Program Maintenance	34	
1.5.1 Program Evaluation	34	
1.5.2 Review of the Life Program	35	
1.5.3 Program Revision and Redevelopment	38	
1.6 Conclusions and Preview	39	
1.6.1 Software Engineering	39	
1.6.2 Problem Analysis	40	
1.6.3 Requirements Specification	41	
1.6.4 Coding	41	
Pointers and Pitfalls	45	
Review Questions	46	
References for Further Study	47	
C++	47	
Programming Principles	47	
The Game of Life	47	
Software Engineering	48	
2 Introduction to Stacks	49	
2.1 Stack Specifications	50	
2.1.1 Lists and Arrays	50	
2.1.2 Stacks	50	
2.1.3 First Example: Reversing a List	51	
2.1.4 Information Hiding	54	
2.1.5 The Standard Template Library	55	

2.2 Implementation of Stacks	57
2.2.1 Specification of Methods for Stacks	57
2.2.2 The Class Specification	60
2.2.3 Pushing, Popping, and Other Methods	61
2.2.4 Encapsulation	63
2.3 Application: A Desk Calculator	66
2.4 Application: Bracket Matching	69
2.5 Abstract Data Types and Their Implementations	71
2.5.1 Introduction	71
2.5.2 General Definitions	73
2.5.3 Refinement of Data Specification	74
Pointers and Pitfalls	76
Review Questions	76
References for Further Study	77

3 Queues 78

3.1 Definitions	79
3.1.1 Queue Operations	79
3.1.2 Extended Queue Operations	81
3.2 Implementations of Queues	84
3.3 Circular Implementation of Queues in C++	89
3.4 Demonstration and Testing	93
3.5 Application of Queues: Simulation	96
3.5.1 Introduction	96
3.5.2 Simulation of an Airport	96
3.5.3 Random Numbers	99
3.5.4 The Runway Class Specification	99
3.5.5 The Plane Class Specification	100
3.5.6 Functions and Methods of the Simulation	101
3.5.7 Sample Results	107
Pointers and Pitfalls	110
Review Questions	110
References for Further Study	111

4 Linked Stacks and Queues 112

4.1 Pointers and Linked Structures	113
4.1.1 Introduction and Survey	113
4.1.2 Pointers and Dynamic Memory in C++	116
4.1.3 The Basics of Linked Structures	122
4.2 Linked Stacks	127
4.3 Linked Stacks with Safeguards	131
4.3.1 The Destructor	131
4.3.2 Overloading the Assignment Operator	132
4.3.3 The Copy Constructor	135
4.3.4 The Modified Linked-Stack Specification	136
4.4 Linked Queues	137
4.4.1 Basic Declarations	137
4.4.2 Extended Linked Queues	139
4.5 Application: Polynomial Arithmetic	141
4.5.1 Purpose of the Project	141
4.5.2 The Main Program	141
4.5.3 The Polynomial Data Structure	144
4.5.4 Reading and Writing Polynomials	147
4.5.5 Addition of Polynomials	148
4.5.6 Completing the Project	150
4.6 Abstract Data Types and Their Implementations	152
Pointers and Pitfalls	154
Review Questions	155

5 Recursion 157

5.1 Introduction to Recursion	158
5.1.1 Stack Frames for Subprograms	158
5.1.2 Tree of Subprogram Calls	159
5.1.3 Factorials: A Recursive Definition	160
5.1.4 Divide and Conquer: The Towers of Hanoi	163
5.2 Principles of Recursion	170
5.2.1 Designing Recursive Algorithms	170
5.2.2 How Recursion Works	171
5.2.3 Tail Recursion	174
5.2.4 When Not to Use Recursion	176
5.2.5 Guidelines and Conclusions	180

5.3 Backtracking: Postponing the Work	183
5.3.1 Solving the Eight-Queens Puzzle	183
5.3.2 Example: Four Queens	184
5.3.3 Backtracking	185
5.3.4 Overall Outline	186
5.3.5 Refinement: The First Data Structure and Its Methods	188
5.3.6 Review and Refinement	191
5.3.7 Analysis of Backtracking	194
5.4 Tree-Structured Programs: Look-Ahead in Games	198
5.4.1 Game Trees	198
5.4.2 The Minimax Method	199
5.4.3 Algorithm Development	201
5.4.4 Refinement	203
5.4.5 Tic-Tac-Toe	204
Pointers and Pitfalls	209
Review Questions	210
References for Further Study	211

6 Lists and Strings 212

6.1 List Definition	213
6.1.1 Method Specifications	214
6.2 Implementation of Lists	217
6.2.1 Class Templates	218
6.2.2 Contiguous Implementation	219
6.2.3 Simply Linked Implementation	221
6.2.4 Variation: Keeping the Current Position	225
6.2.5 Doubly Linked Lists	227
6.2.6 Comparison of Implementations	230
6.3 Strings	233
6.3.1 Strings in C++	233
6.3.2 Implementation of Strings	234
6.3.3 Further String Operations	238
6.4 Application: A Text Editor	242
6.4.1 Specifications	242
6.4.2 Implementation	243
6.5 Linked Lists in Arrays	251
6.6 Application: Generating Permutations	260
Pointers and Pitfalls	265
Review Questions	266
References for Further Study	267

7 Searching 268

7.1 Searching: Introduction and Notation	269
7.2 Sequential Search	271
7.3 Binary Search	278
7.3.1 Ordered Lists	278
7.3.2 Algorithm Development	280
7.3.3 The Forgetful Version	281
7.3.4 Recognizing Equality	284
7.4 Comparison Trees	286
7.4.1 Analysis for $n = 10$	287
7.4.2 Generalization	290
7.4.3 Comparison of Methods	294
7.4.4 A General Relationship	296
7.5 Lower Bounds	297
7.6 Asymptotics	302
7.6.1 Introduction	302
7.6.2 Orders of Magnitude	304
7.6.3 The Big-O and Related Notations	310
7.6.4 Keeping the Dominant Term	311
Pointers and Pitfalls	314
Review Questions	315
References for Further Study	316

8 Sorting 317

8.1 Introduction and Notation	318
8.1.1 Sortable Lists	319
8.2 Insertion Sort	320
8.2.1 Ordered Insertion	320
8.2.2 Sorting by Insertion	321
8.2.3 Linked Version	323
8.2.4 Analysis	325
8.3 Selection Sort	329
8.3.1 The Algorithm	329
8.3.2 Contiguous Implementation	330
8.3.3 Analysis	331
8.3.4 Comparisons	332
8.4 Shell Sort	333
8.5 Lower Bounds	336

8.6 Divide-and-Conquer Sorting	339
8.6.1 The Main Ideas	339
8.6.2 An Example	340
8.7 Mergesort for Linked Lists	344
8.7.1 The Functions	345
8.7.2 Analysis of Mergesort	348
8.8 Quicksort for Contiguous Lists	352
8.8.1 The Main Function	352
8.8.2 Partitioning the List	353
8.8.3 Analysis of Quicksort	356
8.8.4 Average-Case Analysis of Quicksort	358
8.8.5 Comparison with Mergesort	360
8.9 Heaps and Heapsort	363
8.9.1 Two-Way Trees as Lists	363
8.9.2 Development of Heapsort	365
8.9.3 Analysis of Heapsort	368
8.9.4 Priority Queues	369
8.10 Review: Comparison of Methods	372
Pointers and Pitfalls	375
Review Questions	376
References for Further Study	377
9 Tables and Information Retrieval	379
9.1 Introduction:	
Breaking the $\lg n$ Barrier	380
9.2 Rectangular Tables	381
9.3 Tables of Various Shapes	383
9.3.1 Triangular Tables	383
9.3.2 Jagged Tables	385
9.3.3 Inverted Tables	386
9.4 Tables: A New Abstract Data Type	388
9.5 Application: Radix Sort	391
9.5.1 The Idea	392
9.5.2 Implementation	393
9.5.3 Analysis	396
9.6 Hashing	397
9.6.1 Sparse Tables	397
9.6.2 Choosing a Hash Function	399
9.6.3 Collision Resolution with Open Addressing	401
9.6.4 Collision Resolution by Chaining	406
9.7 Analysis of Hashing	411

9.8 Conclusions:	
Comparison of Methods	417
9.9 Application:	
The Life Game Revisited	418
9.9.1 Choice of Algorithm	418
9.9.2 Specification of Data Structures	419
9.9.3 The Life Class	421
9.9.4 The Life Functions	421
Pointers and Pitfalls	426
Review Questions	427
References for Further Study	428

10 Binary Trees _____ 429

10.1 Binary Trees	430
10.1.1 Definitions	430
10.1.2 Traversal of Binary Trees	432
10.1.3 Linked Implementation of Binary Trees	437
10.2 Binary Search Trees	444
10.2.1 Ordered Lists and Implementations	446
10.2.2 Tree Search	447
10.2.3 Insertion into a Binary Search Tree	451
10.2.4 Treesort	453
10.2.5 Removal from a Binary Search Tree	455
10.3 Building a Binary Search Tree	463
10.3.1 Getting Started	464
10.3.2 Declarations and the Main Function	465
10.3.3 Inserting a Node	466
10.3.4 Finishing the Task	467
10.3.5 Evaluation	469
10.3.6 Random Search Trees and Optimality	470
10.4 Height Balance: AVL Trees	473
10.4.1 Definition	473
10.4.2 Insertion of a Node	477
10.4.3 Removal of a Node	484
10.4.4 The Height of an AVL Tree	485
10.5 Splay Trees:	
A Self-Adjusting Data Structure	490
10.5.1 Introduction	490
10.5.2 Splaying Steps	491
10.5.3 Algorithm Development	495

- 10.5.4 Amortized Algorithm Analysis: Introduction 505
- 10.5.5 Amortized Analysis of Splaying 509
- Pointers and Pitfalls 515
- Review Questions 516
- References for Further Study 518

11 Multiway Trees 520

- 11.1 Orchards, Trees, and Binary Trees 521
 - 11.1.1 On the Classification of Species 521
 - 11.1.2 Ordered Trees 522
 - 11.1.3 Forests and Orchards 524
 - 11.1.4 The Formal Correspondence 526
 - 11.1.5 Rotations 527
 - 11.1.6 Summary 527
- 11.2 Lexicographic Search Trees: Tries 530
 - 11.2.1 Tries 530
 - 11.2.2 Searching for a Key 530
 - 11.2.3 C++ Algorithm 531
 - 11.2.4 Searching a Trie 532
 - 11.2.5 Insertion into a Trie 533
 - 11.2.6 Deletion from a Trie 533
 - 11.2.7 Assessment of Tries 534
- 11.3 External Searching: B-Trees 535
 - 11.3.1 Access Time 535
 - 11.3.2 Multiway Search Trees 535
 - 11.3.3 Balanced Multiway Trees 536
 - 11.3.4 Insertion into a B-Tree 537
 - 11.3.5 C++ Algorithms: Searching and Insertion 539
 - 11.3.6 Deletion from a B-Tree 547
- 11.4 Red-Black Trees 556
 - 11.4.1 Introduction 556
 - 11.4.2 Definition and Analysis 557
 - 11.4.3 Red-Black Tree Specification 559
 - 11.4.4 Insertion 560
 - 11.4.5 Insertion Method Implementation 561
 - 11.4.6 Removal of a Node 565
- Pointers and Pitfalls 566
- Review Questions 567
- References for Further Study 568

12 Graphs 569

- 12.1 Mathematical Background 570
 - 12.1.1 Definitions and Examples 570
 - 12.1.2 Undirected Graphs 571
 - 12.1.3 Directed Graphs 571
- 12.2 Computer Representation 572
 - 12.2.1 The Set Representation 572
 - 12.2.2 Adjacency Lists 574
 - 12.2.3 Information Fields 575
- 12.3 Graph Traversal 575
 - 12.3.1 Methods 575
 - 12.3.2 Depth-First Algorithm 577
 - 12.3.3 Breadth-First Algorithm 578
- 12.4 Topological Sorting 579
 - 12.4.1 The Problem 579
 - 12.4.2 Depth-First Algorithm 580
 - 12.4.3 Breadth-First Algorithm 581
- 12.5 A Greedy Algorithm: Shortest Paths 583
 - 12.5.1 The Problem 583
 - 12.5.2 Method 584
 - 12.5.3 Example 585
 - 12.5.4 Implementation 586
- 12.6 Minimal Spanning Trees 587
 - 12.6.1 The Problem 587
 - 12.6.2 Method 589
 - 12.6.3 Implementation 590
 - 12.6.4 Verification of Prim's Algorithm 593
- 12.7 Graphs as Data Structures 594
- Pointers and Pitfalls 596
- Review Questions 597
- References for Further Study 597

13 Case Study: The Polish Notation 598

- 13.1 The Problem 599
 - 13.1.1 The Quadratic Formula 599
- 13.2 The Idea 601
 - 13.2.1 Expression Trees 601
 - 13.2.2 Polish Notation 603

13.3 Evaluation of Polish Expressions 604

- 13.3.1 Evaluation of an Expression in Prefix Form 605
- 13.3.2 C++ Conventions 606
- 13.3.3 C++ Function for Prefix Evaluation 607
- 13.3.4 Evaluation of Postfix Expressions 608
- 13.3.5 Proof of the Program: Counting Stack Entries 609
- 13.3.6 Recursive Evaluation of Postfix Expressions 612

13.4 Translation from Infix Form to Polish Form 617

13.5 An Interactive Expression Evaluator 623

- 13.5.1 Overall Structure 623
- 13.5.2 Representation of the Data: Class Specifications 625
- 13.5.3 Tokens 629
- 13.5.4 The Lexicon 631
- 13.5.5 Expressions: Token Lists 634
- 13.5.6 Auxiliary Evaluation Functions 639
- 13.5.7 Graphing the Expression: The Class Plot 640
- 13.5.8 A Graphics-Enhanced Plot Class 643

References for Further Study 645

A **Mathematical Methods** 647

A.1 Sums of Powers of Integers 647

A.2 Logarithms 650

- A.2.1 Definition of Logarithms 651
- A.2.2 Simple Properties 651
- A.2.3 Choice of Base 652
- A.2.4 Natural Logarithms 652
- A.2.5 Notation 653
- A.2.6 Change of Base 654
- A.2.7 Logarithmic Graphs 654
- A.2.8 Harmonic Numbers 656

A.3 Permutations, Combinations, Factorials 657

- A.3.1 Permutations 657
- A.3.2 Combinations 657
- A.3.3 Factorials 658

A.4 Fibonacci Numbers 659

A.5 Catalan Numbers 661

- A.5.1 The Main Result 661
- A.5.2 The Proof by One-to-One Correspondences 662
- A.5.3 History 664
- A.5.4 Numerical Results 665

References for Further Study 665

B **Random Numbers** 667

B.1 Introduction 667

B.2 Strategy 668

B.3 Program Development 669

References for Further Study 673

C **Packages and Utility Functions** 674

C.1 Packages and C++ Translation Units 674

C.2 Packages in the Text 676

C.3 The Utility Package 678

C.4 Timing Methods 679

D **Programming Precepts, Pointers, and Pitfalls** 681

D.1 Choice of Data Structures and Algorithms 681

- D.1.1 Stacks 681
- D.1.2 Lists 681
- D.1.3 Searching Methods 682
- D.1.4 Sorting Methods 682
- D.1.5 Tables 682
- D.1.6 Binary Trees 683
- D.1.7 General Trees 684
- D.1.8 Graphs 684

D.2 Recursion 685

D.3 Design of Data Structures 686

D.4 Algorithm Design and Analysis 687

D.5 Programming 688

D.6 Programming with Pointer Objects 689

D.7 Debugging and Testing 690

D.8 Maintenance 690

Index 693

Preface

THE APPRENTICE CARPENTER may want only a hammer and a saw, but a master builder employs many precision tools. Computer programming likewise requires sophisticated tools to cope with the complexity of real applications, and only practice with these tools will build skill in their use. This book treats structured problem solving, object-oriented programming, data abstraction, and the comparative analysis of algorithms as fundamental tools of program design. Several case studies of substantial size are worked out in detail, to show how all the tools are used together to build complete programs.

Many of the algorithms and data structures we study possess an intrinsic elegance, a simplicity that cloaks the range and power of their applicability. Before long the student discovers that vast improvements can be made over the naïve methods usually used in introductory courses. Yet this elegance of method is tempered with uncertainty. The student soon finds that it can be far from obvious which of several approaches will prove best in particular applications. Hence comes an early opportunity to introduce truly difficult problems of both intrinsic interest and practical importance and to exhibit the applicability of mathematical methods to algorithm verification and analysis.

Many students find difficulty in translating abstract ideas into practice. This book, therefore, takes special care in the formulation of ideas into algorithms and in the refinement of algorithms into concrete programs that can be applied to practical problems. The process of data specification and abstraction, similarly, comes before the selection of data structures and their implementations.

We believe in progressing from the concrete to the abstract, in the careful development of motivating examples, followed by the presentation of ideas in a more general form. At an early stage of their careers most students need reinforcement from seeing the immediate application of the ideas that they study, and they require the practice of writing and running programs to illustrate each important concept that they learn. This book therefore contains many sample programs, both short

functions and complete programs of substantial length. The exercises and programming projects, moreover, constitute an indispensable part of the book. Many of these are immediate applications of the topic under study, often requesting that programs be written and run, so that algorithms may be tested and compared. Some are larger projects, and a few are suitable for use by a small group of students working together.

Our programs are written in the popular object-oriented language C++. We take the view that many object-oriented techniques provide natural implementations for basic principles of data-structure design. In this way, C++ allows us to construct safe, efficient, and simple implementations of data-structures. We recognize that C++ is sufficiently complex that students will need to use the experience of a data structures course to develop and refine their understanding of the language. We strive to support this development by carefully introducing and explaining various object-oriented features of C++ as we progress through the book. Thus, we begin [Chapter 1](#) assuming that the reader is comfortable with the elementary parts of C++ (essentially, with the C subset), and gradually we add in such object-oriented elements of C++ as classes, methods, constructors, inheritance, dynamic memory management, destructors, copy constructors, overloaded functions and operations, templates, virtual functions, and the STL. Of course, our primary focus is on the data structures themselves, and therefore students with relatively little familiarity with C++ will need to supplement this text with a C++ programming text.

SYNOPSIS

Programming Principles

By working through the first large project (CONWAY's game of Life), [Chapter 1](#) expounds principles of object-oriented program design, top-down refinement, review, and testing, principles that the student will see demonstrated and is expected to follow throughout the sequel. At the same time, this project provides an opportunity for the student to review the syntax of elementary features of C++, the programming language used throughout the book.

Introduction to Stacks

[Chapter 2](#) introduces the first data structure we study, the stack. The chapter applies stacks to the development of programs for reversing input, for modelling a desk calculator, and for checking the nesting of brackets. We begin by utilizing the STL stack implementation, and later develop and use our own stack implementation. A major goal of [Chapter 2](#) is to bring the student to appreciate the ideas behind information hiding, encapsulation and data abstraction and to apply methods of top-down design to data as well as to algorithms. The chapter closes with an introduction to abstract data types.

Queues

Queues are the central topic of [Chapter 3](#). The chapter expounds several different implementations of the abstract data type and develops a large application program showing the relative advantages of different implementations. In this chapter we introduce the important object-oriented technique of inheritance.

Linked Stacks and Queues

[Chapter 4](#) presents linked implementations of stacks and queues. The chapter begins with a thorough introduction to pointers and dynamic memory management in C++. After exhibiting a simple linked stack implementation, we discuss

destructors, copy constructors, and overloaded assignment operators, all of which are needed in the safe C++ implementation of linked structures.

Recursion Chapter 5 continues to elucidate stacks by studying their relationship to problem solving and programming with recursion. These ideas are reinforced by exploring several substantial applications of recursion, including backtracking and tree-structured programs. This chapter can, if desired, be studied earlier in a course than its placement in the book, at any time after the completion of Chapter 2.

Lists and Strings More general lists with their linked and contiguous implementations provide the theme for Chapter 6. The chapter also includes an encapsulated string implementation, an introduction to C++ templates, and an introduction to algorithm analysis in a very informal way.

Searching Chapter 7, Chapter 8, and Chapter 9 present algorithms for searching, sorting, and table access (including hashing), respectively. These chapters illustrate the

Sorting interplay between algorithms and the associated abstract data types, data structures, and implementations. The text introduces the “big- O ” and related notations for elementary algorithm analysis and highlights the crucial choices to be made regarding best use of space, time, and programming effort. These choices require

Tables and Information Retrieval that we find analytical methods to assess algorithms, and producing such analyses is a battle for which combinatorial mathematics must provide the arsenal. At an elementary level we can expect students neither to be well armed nor to possess the mathematical maturity needed to hone their skills to perfection. Our goal, therefore, is to help students recognize the importance of such skills in anticipation of later chances to study mathematics.

Binary Trees Binary trees are surely among the most elegant and useful of data structures. Their study, which occupies Chapter 10, ties together concepts from lists, searching, and sorting. As recursively defined data structures, binary trees afford an excellent opportunity for the student to become comfortable with recursion applied both to data structures and algorithms. The chapter begins with elementary topics and progresses as far as such advanced topics as splay trees and amortized algorithm analysis.

Multiway Trees Chapter 11 continues the study of more sophisticated data structures, including tries, B-trees, and red-black trees.

Graphs Chapter 12 introduces graphs as more general structures useful for problem solving, and introduces some of the classical algorithms for shortest paths and minimal spanning trees in graphs.

Case Study: The Polish Notation The case study in Chapter 13 examines the Polish notation in considerable detail, exploring the interplay of recursion, trees, and stacks as vehicles for problem solving and algorithm development. Some of the questions addressed can serve as an informal introduction to compiler design. As usual, the algorithms are fully developed within a functioning C++ program. This program accepts as input an expression in ordinary (infix) form, translates the expression into postfix form, and evaluates the expression for specified values of the variable(s). Chapter 13 may be studied anytime after the completion of Section 10.1.

The appendices discuss several topics that are not properly part of the book’s subject but that are often missing from the student’s preparation.

Mathematical Methods Appendix A presents several topics from discrete mathematics. Its final two sections, Fibonacci numbers and Catalan numbers, are more advanced and not

needed for any vital purpose in the text, but are included to encourage combinatorial interest in the more mathematically inclined.

Random Numbers Appendix B discusses pseudorandom numbers, generators, and applications, a topic that many students find interesting, but which often does not fit anywhere in the curriculum.

Packages and Utility Functions Appendix C catalogues the various utility and data-structure packages that are developed and used many times throughout this book. Appendix C discusses declaration and definition files, translation units, the utility package used throughout the book, and a package for calculating CPU times.

Programming Precepts, Pointers, and Pitfalls Appendix D, finally, collects all the Programming Precepts and all the Pointers and Pitfalls scattered through the book and organizes them by subject for convenience of reference.

COURSE STRUCTURE

prerequisite The prerequisite for this book is a first course in programming, with experience using the elementary features of C++. However, since we are careful to introduce sophisticated C++ techniques only gradually, we believe that, used in conjunction with a supplementary C++ textbook and extra instruction and emphasis on C++ language issues, this text provides a data structures course in C++ that remains suitable even for students whose programming background is in another language such as C, Pascal, or Java.

A good knowledge of high school mathematics will suffice for almost all the algorithm analyses, but further (perhaps concurrent) preparation in discrete mathematics will prove valuable. Appendix A reviews all required mathematics.

content This book is intended for courses such as the ACM Course CS2 (*Program Design and Implementation*), ACM Course CS7 (*Data Structures and Algorithm Analysis*), or a course combining these. Thorough coverage is given to most of the ACM/IEEE knowledge units¹ on data structures and algorithms. These include:

- AL1 Basic data structures, such as arrays, tables, stacks, queues, trees, and graphs;
- AL2 Abstract data types;
- AL3 Recursion and recursive algorithms;
- AL4 Complexity analysis using the big Oh notation;
- AL6 Sorting and searching; and
- AL8 Practical problem-solving strategies, with large case studies.

The three most advanced knowledge units, AL5 (complexity classes, NP-complete problems), AL7 (computability and undecidability), and AL9 (parallel and distributed algorithms) are not treated in this book.

¹ See *Computing Curricula 1991: Report of the ACM/IEEE-CS Joint Curriculum Task Force*, ACM Press, New York, 1990.

Most chapters of this book are structured so that the core topics are presented first, followed by examples, applications, and larger case studies. Hence, if time allows only a brief study of a topic, it is possible, with no loss of continuity, to move rapidly from chapter to chapter covering only the core topics. When time permits, however, both students and instructor will enjoy the occasional excursion into the supplementary topics and worked-out projects.

two-term course

A two-term course can cover nearly the entire book, thereby attaining a satisfying integration of many topics from the areas of problem solving, data structures, program development, and algorithm analysis. Students need time and practice to understand general methods. By combining the studies of data abstraction, data structures, and algorithms with their implementations in projects of realistic size, an integrated course can build a solid foundation on which, later, more theoretical courses can be built. Even if it is not covered in its entirety, this book will provide enough depth to enable interested students to continue using it as a reference in later work. It is important in any case to assign major programming projects and to allow adequate time for their completion.

SUPPLEMENTARY MATERIALS

A CD-ROM version of this book is anticipated that, in addition to the entire contents of the book, will include:

- ➔ All packages, programs, and other C++ code segments from the text, in a form ready to incorporate as needed into other programs;
- ➔ Executable versions (for DOS or Windows) of several demonstration programs and nearly all programming projects from the text;
- ➔ Brief outlines or summaries of each section of the text, suitable for use as a study guide.

These materials will also be available from the publisher's internet site. To reach these files with ftp, log in as user anonymous to the site `ftp.prenhall.com` and change to the directory

```
pub/esm/computer_science.s-041/kruse/cpp
```

Instructors teaching from this book may obtain, at no charge, an instructor's version on CD-ROM which, in addition to all the foregoing materials, includes:

- ➔ Brief teaching notes on each chapter;
- ➔ Full solutions to nearly all exercises in the textbook;
- ➔ Full source code to nearly all programming projects in the textbook;
- ➔ Transparency masters.

BOOK PRODUCTION

This book and its supplements were written and produced with software called `PreTeX`, a preprocessor and macro package for the `TeX` typesetting system.² `PreTeX`, by exploiting context dependency, automatically supplies much of the typesetting markup required by `TeX`. `PreTeX` also supplies several tools that greatly simplify some aspects of an author's work. These tools include a powerful cross-reference system, simplified typesetting of mathematics and computer-program listings, and automatic generation of the index and table of contents, while allowing the processing of the book in conveniently small files at every stage. Solutions, placed with exercises and projects, are automatically removed from the text and placed in a separate document.

For a book such as this, `PreTeX`'s treatment of computer programs is its most important feature. Computer programs are not included with the main body of the text; instead, they are placed in separate, secondary files, along with any desired explanatory text, and with any desired typesetting markup in place. By placing tags at appropriate places in the secondary files, `PreTeX` can extract arbitrary parts of a secondary file, in any desired order, for typesetting with the text. Another utility removes all the tags, text, and markup, producing as its output a program ready to be compiled. The same input file thus automatically produces both typeset program listings and compiled program code. In this way, the reader gains increased confidence in the accuracy of the computer program listings appearing in the text. In fact, with just two exceptions, all of the programs developed in this book have been compiled and successfully tested under the `g++` and Borland C++ compilers (versions 2.7.2.1 and 5.0, respectively). The two exceptions are the first program in [Chapter 2](#) (which requires a compiler with a full ANSI C++ standard library) and the last program of [Chapter 13](#) (which requires a compiler with certain Borland graphics routines).

ACKNOWLEDGMENTS

Over the years, the Pascal and C antecedents of this book have benefitted greatly from the contributions of many people: family, friends, colleagues, and students, some of whom are noted in the previous books. Many other people, while studying these books or their translations into various languages, have kindly forwarded their comments and suggestions, all of which have helped to make this a better book.

We are happy to acknowledge the suggestions of the following reviewers, who have helped in many ways to improve the presentation in this book: KEITH VANDER LINDEN (Calvin College), JENS GREGOR (University of Tennessee), VICTOR BERRY (Boston University), JEFFERY LEON (University of Illinois at Chicago), SUSAN

² `TeX` was developed by DONALD E. KNUTH, who has also made many important research contributions to data structures and algorithms. (See the entries under his name in the index.)

HUTT (University of Missouri–Columbia), FRED HARRIS (University of Nevada), ZHI-LI ZHANG (University of Minnesota), and ANDREW SUNG (New Mexico Institute of Technology).

ALEX RYBA especially acknowledges the helpful suggestions and encouraging advice he has received over the years from WIM RUITENBURG and JOHN SIMMS of Marquette University, as well as comments from former students RICK VOGEL and JUN WANG.

It is a special pleasure for ROBERT KRUSE to acknowledge the continuing advice and help of PAUL MAILHOT of PreTeX, Inc., who was from the first an outstanding student, then worked as a dependable research assistant, and who has now become a valued colleague making substantial contributions in software development for book production, in project management, in problem solving for the publisher, the printer, and the authors, and in providing advice and encouragement in all aspects of this work. The CD-ROM versions of this book, with all their hypertext features (such as extensive cross-reference links and execution of demonstration programs from the text), are entirely his accomplishment.

Without the continuing enthusiastic support, faithful encouragement, and patience of the editorial staff of Prentice Hall, especially ALAN APT, Publisher, LAURA STEELE, Acquisitions Editor, and MARCIA HORTON, Editor in Chief, this project would never have been started and certainly could never have been brought to completion. Their help, as well as that of the production staff named on the copyright page, has been invaluable.

ROBERT L. KRUSE
ALEXANDER J. RYBA

Programming Principles

1

THIS CHAPTER summarizes important principles of good programming, especially as applied to large projects, and introduces methods such as object-oriented design and top-down design for discovering effective algorithms. In the process we raise questions in program design and data-storage methods that we shall address in later chapters, and we also review some of the elementary features of the language C++ by using them to write programs.

1.1 Introduction	2	1.4.8 Principles of Program Testing	29
1.2 The Game of Life	4	1.5 Program Maintenance	34
1.2.1 Rules for the Game of Life	4	1.5.1 Program Evaluation	34
1.2.2 Examples	5	1.5.2 Review of the Life Program	35
1.2.3 The Solution: Classes, Objects, and Methods	7	1.5.3 Program Revision and Redevelopment	38
1.2.4 Life: The Main Program	8	1.6 Conclusions and Preview	39
1.3 Programming Style	10	1.6.1 Software Engineering	39
1.3.1 Names	10	1.6.2 Problem Analysis	40
1.3.2 Documentation and Format	13	1.6.3 Requirements Specification	41
1.3.3 Refinement and Modularity	15	1.6.4 Coding	41
1.4 Coding, Testing, and Further Refinement	20	Pointers and Pitfalls	45
1.4.1 Stubs	20	Review Questions	46
1.4.2 Definition of the Class Life	22	References for Further Study	47
1.4.3 Counting Neighbors	23	C++	47
1.4.4 Updating the Grid	24	Programming Principles	47
1.4.5 Input and Output	25	The Game of Life	47
1.4.6 Drivers	27	Software Engineering	48
1.4.7 Program Tracing	28		

1.1 INTRODUCTION



The greatest difficulties of writing large computer programs are not in deciding what the goals of the program should be, nor even in finding methods that can be used to reach these goals. The president of a business might say, “Let’s get a computer to keep track of all our inventory information, accounting records, and personnel files, and let it tell us when inventories need to be reordered and budget lines are overspent, and let it handle the payroll.” With enough time and effort, a staff of systems analysts and programmers might be able to determine how various staff members are now doing these tasks and write programs to do the work in the same way.

problems of large programs

This approach, however, is almost certain to be a disastrous failure. While interviewing employees, the systems analysts will find some tasks that can be put on the computer easily and will proceed to do so. Then, as they move other work to the computer, they will find that it depends on the first tasks. The output from these, unfortunately, will not be quite in the proper form. Hence they need more programming to convert the data from the form given for one task to the form needed for another. The programming project begins to resemble a patchwork quilt. Some of the pieces are stronger, some weaker. Some of the pieces are carefully sewn onto the adjacent ones, some are barely tacked together. If the programmers are lucky, their creation may hold together well enough to do most of the routine work most of the time. But if any change must be made, it will have unpredictable consequences throughout the system. Later, a new request will come along, or an unexpected problem, perhaps even an emergency, and the programmers’ efforts will prove as effective as using a patchwork quilt as a safety net for people jumping from a tall building.



The main purpose of this book is to describe programming methods and tools that will prove effective for projects of realistic size, programs much larger than those ordinarily used to illustrate features of elementary programming. Since a piecemeal approach to large problems is doomed to fail, we must first of all adopt a consistent, unified, and logical approach, and we must also be careful to observe important principles of program design, principles that are sometimes ignored in writing small programs, but whose neglect will prove disastrous for large projects.

problem specification

The first major hurdle in attacking a large problem is deciding exactly what the problem is. It is necessary to translate vague goals, contradictory requests, and perhaps unstated desires into a precisely formulated project that can be programmed. And the methods or divisions of work that people have previously used are not necessarily the best for use in a machine. Hence our approach must be to determine overall goals, but precise ones, and then slowly divide the work into smaller problems until they become of manageable size.

program design

The maxim that many programmers observe, “First make your program work, then make it pretty,” may be effective for small programs, but not for large ones. Each part of a large program must be well organized, clearly written, and thoroughly understood, or else its structure will have been forgotten, and it can no longer be tied to the other parts of the project at some much later time, perhaps by another programmer. Hence we do not separate style from other parts of program design, but from the beginning we must be careful to form good habits.

Even with very large projects, difficulties usually arise not from the inability to find a solution but, rather, from the fact that there can be so many different methods and algorithms that might work that it can be hard to decide which is best, which may lead to programming difficulties, or which may be hopelessly inefficient. The greatest room for variability in algorithm design is generally in the way in which the data of the program are stored:

choice of data structures

- ➔ How they are arranged in relation to each other.
- ➔ Which data are kept in memory.
- ➔ Which are calculated when needed.
- ➔ Which are kept in files, and how the files are arranged.

A second goal of this book, therefore, is to present several elegant, yet fundamentally simple ideas for the organization and manipulation of data. Lists, stacks, and queues are the first three such organizations that we study. Later, we shall develop several powerful algorithms for important tasks within data processing, such as sorting and searching.

analysis of algorithms

When there are several different ways to organize data and devise algorithms, it becomes important to develop criteria to recommend a choice. Hence we devote attention to analyzing the behavior of algorithms under various conditions.

testing and verification

The difficulty of debugging a program increases much faster than its size. That is, if one program is twice the size of another, then it will likely not take twice as long to debug, but perhaps four times as long. Many very large programs (such as operating systems) are put into use still containing errors that the programmers have despaired of finding, because the difficulties seem insurmountable. Sometimes projects that have consumed years of effort must be discarded because it is impossible to discover why they will not work. If we do not wish such a fate for our own projects, then we must use methods that will

program correctness

- ➔ Reduce the number of errors, making it easier to spot those that remain.
- ➔ Enable us to verify in advance that our algorithms are correct.
- ➔ Provide us with ways to test our programs so that we can be reasonably confident that they will not misbehave.

Development of such methods is another of our goals, but one that cannot yet be fully within our grasp.

maintenance

Even after a program is completed, fully debugged, and put into service, a great deal of work may be required to maintain the usefulness of the program. In time there will be new demands on the program, its operating environment will change, new requests must be accommodated. For this reason, it is essential that a large project be written to make it as easy to understand and modify as possible.

C++

The programming language C++ is a particularly convenient choice to express the algorithms we shall encounter. The language was developed in the early 1980s, by Bjarne Stroustrup, as an extension of the popular C language. Most of the new features that Stroustrup incorporated into C++ facilitate the understanding and implementation of data structures. Among the most important features of C++ for our study of data structures are:

Highlights

- ➔ C++ allows **data abstraction**: This means that programmers can create new types to represent whatever collections of data are convenient for their applications.
- ➔ C++ supports **object-oriented design**, in which the programmer-defined types play a central role in the implementation of algorithms.
- ➔ Importantly, as well as allowing for object-oriented approaches, C++ allows for the use of the **top-down approach**, which is familiar to C programmers.
- ➔ C++ facilitates **code reuse**, and the construction of general purpose libraries. The language includes an extensive, efficient, and convenient standard library.
- ➔ C++ improves on several of the inconvenient and dangerous aspects of C.
- ➔ C++ maintains the efficiency that is the hallmark of the C language.

It is the combination of flexibility, generality and efficiency that has made C++ one of the most popular choices for programmers at the present time.

We shall discover that the general principles that underlie the design of all data structures are naturally implemented by the data abstraction and the object-oriented features of C++. Therefore, we shall carefully explain how these aspects of C++ are used and briefly summarize their syntax (grammar) wherever they first arise in our book. In this way, we shall illustrate and describe many of the features of C++ that do not belong to its small overlap with C. For the precise details of C++ syntax, consult a textbook on C++ programming—we recommend several such books in the references at the end of this chapter.

1.2 THE GAME OF LIFE

If we may take the liberty to abuse an old proverb,

One concrete problem is worth a thousand unapplied abstractions.



Throughout this chapter we shall concentrate on one case study that, while not large by realistic standards, illustrates both the principles of program design and the pitfalls that we should learn to avoid. Sometimes the example motivates general principles; sometimes the general discussion comes first; always it is with the view of discovering general principles that will prove their value in a range of practical applications. In later chapters we shall employ similar methods for larger projects.

The example we shall use is the game called **Life**, which was introduced by the British mathematician J. H. CONWAY in 1970.

1.2.1 Rules for the Game of Life

definitions

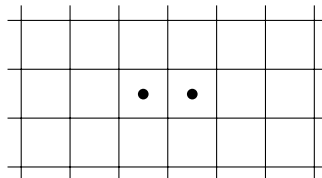
Life is really a simulation, not a game with players. It takes place on an unbounded rectangular grid in which each cell can either be occupied by an organism or not. Occupied cells are called **alive**; unoccupied cells are called **dead**. Which cells are alive changes from generation to generation according to the number of neighboring cells that are alive, as follows:

- transition rules*
1. The neighbors of a given cell are the eight cells that touch it vertically, horizontally, or diagonally.
 2. If a cell is alive but either has no neighboring cells alive or only one alive, then in the next generation the cell dies of loneliness.
 3. If a cell is alive and has four or more neighboring cells also alive, then in the next generation the cell dies of overcrowding.
 4. A living cell with either two or three living neighbors remains alive in the next generation.
 5. If a cell is dead, then in the next generation it will become alive if it has exactly three neighboring cells, no more or fewer, that are already alive. All other dead cells remain dead in the next generation.
 6. All births and deaths take place at exactly the same time, so that dying cells can help to give birth to another, but cannot prevent the death of others by reducing overcrowding; nor can cells being born either preserve or kill cells living in the previous generation.

configuration A particular arrangement of living and dead cells in a grid is called a **configuration**. The preceding rules explain how one configuration changes to another at each generation.

1.2.2 Examples

As a first example, consider the configuration



The counts of living neighbors for the cells are as follows:

0	0	0	0	0	0
0	1	2	2	1	0
0	1	•	•	1	0
0	1	2	2	1	0
0	0	0	0	0	0

6 Chapter 1 • Programming Principles

moribund example By rule 2 both the living cells will die in the coming generation, and rule 5 shows that no cells will become alive, so the configuration dies out.

On the other hand, the configuration

0	0	0	0	0	0
0	1	2	2	1	0
0	2	• 3	• 3	2	0
0	2	• 3	• 3	2	0
0	1	2	2	1	0
0	0	0	0	0	0

stability has the neighbor counts as shown. Each of the living cells has a neighbor count of three, and hence remains alive, but the dead cells all have neighbor counts of two or less, and hence none of them becomes alive.

The two configurations

0	0	0	0	0
1	2	3	2	1
1	• 1	• 2	• 1	1
1	2	3	2	1
0	0	0	0	0

and

0	1	1	1	0
0	2	• 1	2	0
0	3	• 2	3	0
0	2	• 1	2	0
0	1	1	1	0

alternation continue to alternate from generation to generation, as indicated by the neighbor counts shown.

It is a surprising fact that, from very simple initial configurations, quite complicated progressions of Life configurations can develop, lasting many generations, and it is usually not obvious what changes will happen as generations progress.

variety Some very small initial configurations will grow into large configurations; others will slowly die out; many will reach a state where they do not change, or where they go through a repeating pattern every few generations.

popularity Not long after its invention, MARTIN GARDNER discussed the Life game in his column in *Scientific American*, and, from that time on, it has fascinated many people, so that for several years there was even a quarterly newsletter devoted to related topics. It makes an ideal display for home microcomputers.

Our first goal, of course, is to write a program that will show how an initial configuration will change from generation to generation.

1.2.3 The Solution: Classes, Objects, and Methods

In outline, a program to run the Life game takes the form:

algorithm Set up a Life configuration as an initial arrangement of living and dead cells.
 Print the Life configuration.
 While the user wants to see further generations:
 Update the configuration by applying the rules of the Life game.
 Print the current configuration.

class The important thing for us to study in this algorithm is the Life configuration. In C++, we use a **class** to collect data and the methods used to access or change the data. Such a collection of data and methods is called an **object** belonging to the given class. For the Life game, we shall call the class Life, so that configuration becomes a Life **object**. We shall then use three methods for this object: initialize() will set up the initial configuration of living and dead cells; print() will print out the current configuration; and update() will make all the changes that occur in moving from one generation to the next.



C++ classes Every C++ class, in fact, consists of **members** that represent either variables or functions. The members that represent variables are called the **data members**; these are used to store data values. The members that represent functions belonging to a class are called the **methods** or **member functions**. The methods of a class are normally used to access or alter the data members.

methods

clients **Clients**, that is, user programs with access to a particular class, can declare and manipulate objects of that class. Thus, in the Life game, we shall declare a Life object by:

```
Life configuration;
```

member selection operator

We can now apply methods to work with configuration, using the C++ operator `.` (the member selection operator). For example, we can print out the data in configuration by writing:

```
configuration.print();
```



specifications

It is important to realize that, while writing a client program, we can use a C++ class so long as we know the **specifications** of each of its methods, that is, statements of precisely what each method does. We do not need to know how the data are actually stored or how the methods are actually programmed. For example, to use a Life object, we do not need to know exactly how the object is stored, or how the methods of the class Life are doing their work. This is our first example of an important programming strategy known as **information hiding**.

information hiding

private and public

When the time comes to implement the class Life, we shall find that more goes on behind the scenes: We shall need to decide how to store the data, and we shall need variables and functions to manipulate this data. All these variables and functions, however, are **private** to the class; the client program does not need to know what they are, how they are programmed, or have any access to them. Instead, the client program only needs the **public** methods that are specified and declared for the class.

- [20,000 Leagues under the Sea online](#)
- [click CBT Techmanual \(Classic Battletech\) book](#)
- [read online GameMastery Module: Seven Swords Of Sin here](#)
- [download online Invisible Sun \(Black Hole Sun, Book 2\) online](#)
- [read Harry Potter e a Ordem da FÃnix \(Harry Potter, Book 5\)](#)
- [download Art and Pornography: Philosophical Essays for free](#)

- <http://www.khoi.dk/?books/Wise-Words-and-Country-Ways-for-Cooks.pdf>
- <http://crackingscience.org/?library/Echo-Round-His-Bones.pdf>
- <http://www.freightunlocked.co.uk/lib/Kell-s-Legend--The-Clockwork-Vampire-Chronicles--Book-1-.pdf>
- <http://www.shreesaiexport.com/library/Invisible-Sun--Black-Hole-Sun--Book-2-.pdf>
- <http://www.mmastyles.com/books/On-Voluntary-Servitude--False-Consciousness-and-The-Theory-of-Ideology.pdf>
- <http://qolorea.com/library/Art-and-Pornography--Philosophical-Essays.pdf>