

Brief Edition

Susanna S. Epp

Discrete Mathematics

An Introduction to Mathematical Reasoning



List of Symbols

Subject	Symbol	Meaning	Page
Logic	$\sim p$	not p	25
	$p \wedge q$	p and q	25
	$p \vee q$	p or q	25
	$p \oplus q$ or p XOR q	p or q but not both p and q	28
	$P \equiv Q$	P is logically equivalent to Q	30
	$p \rightarrow q$	if p then q	39
	$p \leftrightarrow q$	p if and only if q	44
	\therefore	therefore	50
	$P(x)$	predicate in x	64
	$P(x) \Rightarrow Q(x)$	every element in the truth set for $P(x)$ is in the truth set for $Q(x)$	71
	$P(x) \Leftrightarrow Q(x)$	$P(x)$ and $Q(x)$ have identical truth sets	71
	\forall	for all	68
	\exists	there exists	70
	Number	$d \mid n$	d divides n
Theory	$d \nmid n$	d does not divide n	136
	$n \operatorname{div} d$	the integer quotient of n divided by d	145
	$n \operatorname{mod} d$	the integer remainder of n divided by d	145
	$ x $	the absolute value of x	151
	$\operatorname{gcd}(a, b)$	the greatest common divisor of a and b	387
	$x \cong y$	x is approximately equal to y	181
Sequences	\dots	and so forth	171
	$\sum_{k=m}^n a_k$	the summation from k equals m to n of a_k	174
	$\prod_{k=m}^n a_k$	the product from k equals m to n of a_k	177
	$n!$	n factorial	181
Set Theory	$a \in A$	a is an element of A	7
	$a \notin A$	a is not an element of A	7
	$\{a_1, a_2, \dots, a_n\}$	the set with elements a_1, a_2, \dots, a_n	7
	$\{x \in D \mid P(x)\}$	the set of all x in D for which $P(x)$ is true	8
	$\mathbf{R}, \mathbf{R}^-, \mathbf{R}^+, \mathbf{R}^{\operatorname{nonneg}}$	the sets of all real numbers, negative real numbers, positive real numbers, and nonnegative real numbers	7, 8
	$\mathbf{Z}, \mathbf{Z}^-, \mathbf{Z}^+, \mathbf{Z}^{\operatorname{nonneg}}$	the sets of all integers, negative integers, positive integers, and nonnegative integers	7, 8
	$\mathbf{Q}, \mathbf{Q}^-, \mathbf{Q}^+, \mathbf{Q}^{\operatorname{nonneg}}$	the sets of all rational numbers, negative rational numbers, positive rational numbers, and nonnegative rational numbers	7, 8
	\mathbf{N}	the set of natural numbers	8
	$A \subseteq B$	A is a subset of B	9
	$A \not\subseteq B$	A is not a subset of B	9
	$A = B$	A equals B	252
	$A \cup B$	A union B	254
	$A \cap B$	A intersect B	254
	$B - A$	the difference of B minus A	254
	A^c	the complement of A	254
	(x, y)	ordered pair	11
	(x_1, x_2, \dots, x_n)	ordered n -tuple	259
	$A \times B$	the Cartesian product of A and B	12
	$A_1 \times A_2 \times \dots \times A_n$	the Cartesian product of A_1, A_2, \dots, A_n	260
	\emptyset	the empty set	274
	$\mathcal{P}(A)$	the power set of A	259

Subject	Symbol	Meaning	Page
Counting and Probability	$N(A)$	the number of elements in set A	405
	$P(A)$	the probability of a set A	405
	$P(n, r)$	the number of r -permutations of a set of n elements	418
	$\binom{n}{r}$	n choose r , the number of r -combinations of a set of n elements, the number of r -element subsets of a set of n elements	182, 447
	ϵ	the null string	414
Functions	$f: X \rightarrow Y$	f is a function from X to Y	294
	$f(x)$	the value of f at x	294
	$x \xrightarrow{f} y$	f sends x to y	294
	$f(A)$	the image of A	305
	$f^{-1}(C)$	the inverse image of C	305
	I_x	the identity function on X	297
	b^x	b raised to the power x	312
	$\exp_b(x)$	b raised to the power x	312
	$\log_b(x)$	logarithm with base b of x	299
	F^{-1}	the inverse function of F	317
	$f \circ g$	the composition of g and f	322
Relations	$x R y$	x is related to y by R	14
	R^{-1}	the inverse relation of R	347
	$m \equiv n \pmod{d}$	m is congruent to n modulo d	363
	$[a]$	the equivalence class of a	364
	Z_n	the set of equivalence classes of integers modulo n	381
Graphs and Trees	$V(G)$	the set of vertices of a graph G	477
	$E(G)$	the set of edges of a graph G	477
	$\{v, w\}$	the edge joining v and w in a simple graph	483, 484
	K_n	complete graph on n vertices	484
	$K_{m,n}$	complete bipartite graph on (m, n) vertices	484
	$\deg(v)$	degree of vertex v	486
	$v_0 e_1 v_1 e_2 \cdots e_n v_n$	a walk from v_0 to v_n	495

DISCRETE MATHEMATICS

DISCRETE MATHEMATICS

AN INTRODUCTION TO MATHEMATICAL REASONING

SUSANNA S. EPP
DePaul University



Australia • Brazil • Japan • Korea • Mexico • Singapore • Spain • United Kingdom • United States

This is an electronic version of the print textbook. Due to electronic rights restrictions, some third party content may be suppressed. Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. The publisher reserves the right to remove content from this title at any time if subsequent rights restrictions require it. For valuable information on pricing, previous editions, changes to current editions, and alternate formats, please visit www.cengage.com/highered to search by ISBN#, author, title, or keyword for materials in your areas of interest.

Cover Photo: *The stones are discrete objects placed one on top of another like a chain of careful reasoning. A person who decides to build such a tower aspires to the heights and enjoys playing with a challenging problem. Choosing the stones takes both a scientific and an aesthetic sense. Getting them to balance requires patient effort and careful thought. And the tower that results is beautiful. A perfect metaphor for discrete mathematics!*

Discrete Mathematics: An Introduction to Mathematical Reasoning
Susanna S. Epp

Publisher: Richard Stratton
Senior Sponsoring Editor: Molly Taylor
Assistant Editor: Shaylin Walsh
Editorial Assistant: Alexander Gontar
Associate Media Editor: Andrew Coppola
Senior Marketing Manager: Jennifer Pursley Jones
Marketing Communications Manager: Mary Anne Payumo
Marketing Coordinator: Michael Ledesma
Content Project Manager: Alison Eigel Zade
Senior Art Director: Jill Ort
Senior Print Buyer: Diane Gibbons
Right Acquisition Specialists: Timothy Sisler and Don Schlotman
Production Management and Composition: Integra
Photo Manager: Chris Althof, Bill Smith Group
Cover Designer: Hanh Luu
Cover Image: GettyImages.com

© 2011 Brooks/Cole Cengage Learning
WCN: 02-200-203

ALL RIGHTS RESERVED. No part of this work covered by the copyright herein may be reproduced, transmitted, stored, or used in any form or by any means graphic, electronic, or mechanical, including but not limited to photocopying, recording, scanning, digitizing, taping, Web distribution, information networks, or information storage and retrieval systems, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without the prior written permission of the publisher.

For product information and technology assistance, contact us at
Cengage Learning Customer & Sales Support, 1-800-354-9706.

For permission to use material from this text or product,
submit all requests online at www.cengage.com/permissions.

Further permissions questions can be emailed to
permissionrequest@cengage.com.

Library of Congress Control Number: 2010940881

Student Edition:
ISBN-13: 978-0-495-82617-0
ISBN-10: 0-495-82617-0

Brooks/Cole
20 Channel Center Street
Boston, MA 02210
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at: international.cengage.com/region.

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

For your course and learning solutions, visit
www.cengage.com

Purchase any of our products at your local college store or at our preferred online store www.cengagebrain.com.

Printed in The United States of America
3 4 5 6 17 16 15 14

To my students, in appreciation for all they have taught me

CONTENTS

Chapter 1	Speaking Mathematically	1
1.1	<i>Variables</i>	1
	Using Variables in Mathematical Discourse; Introduction to Universal, Existential, and Conditional Statements	
1.2	<i>The Language of Sets</i>	6
	The Set-Roster and Set-Builder Notations; Subsets; Cartesian Products	
1.3	<i>The Language of Relations and Functions</i>	13
	Definition of a Relation from One Set to Another; Arrow Diagram of a Relation; Definition of Function; Function Machines; Equality of Functions	
Chapter 2	The Logic of Compound Statements	23
2.1	<i>Logical Form and Logical Equivalence</i>	23
	Statements; Compound Statements; Truth Values; Evaluating the Truth of More General Compound Statements; Logical Equivalence; Tautologies and Contradictions; Summary of Logical Equivalences	
2.2	<i>Conditional Statements</i>	38
	Logical Equivalences Involving \rightarrow ; Representation of <i>If-Then</i> As <i>Or</i> ; The Negation of a Conditional Statement; The Contrapositive of a Conditional Statement; The Converse and Inverse of a Conditional Statement; <i>Only If</i> and the Biconditional; Necessary and Sufficient Conditions; Remarks	
2.3	<i>Valid and Invalid Arguments</i>	50
	Modus Ponens and Modus Tollens; Additional Valid Argument Forms: Rules of Inference; Fallacies; Contradictions and Valid Arguments; Summary of Rules of Inference	
Chapter 3	The Logic of Quantified Statements	63
3.1	<i>Predicates and Quantified Statements I</i>	63
	The Universal Quantifier: \forall ; The Existential Quantifier: \exists ; Formal Versus Informal Language; Universal Conditional Statements; Equivalent Forms of Universal and Existential Statements; Implicit Quantification; Tarski's World	

- 3.2 *Predicates and Quantified Statements II* 75**
 Negations of Quantified Statements; Negations of Universal Conditional Statements; The Relation among \forall , \exists , \wedge , and \vee ; Vacuous Truth of Universal Statements; Variants of Universal Conditional Statements; Necessary and Sufficient Conditions, Only If
- 3.3 *Statements with Multiple Quantifiers* 84**
 Translating from Informal to Formal Language; Ambiguous Language; Negations of Multiply-Quantified Statements; Order of Quantifiers
- 3.4 *Arguments with Quantified Statements* 95**
 Universal Modus Ponens; Use of Universal Modus Ponens in a Proof; Universal Modus Tollens; Proving Validity of Arguments with Quantified Statements; Using Diagrams to Test for Validity; Creating Additional Forms of Argument; Remark on the Converse and Inverse Errors

Chapter 4 Elementary Number Theory and Methods of Proof 109

- 4.1 *Direct Proof and Counterexample I: Introduction* 110**
 Definitions; Proving Existential Statements; Disproving Universal Statements by Counterexample; Proving Universal Statements; Directions for Writing Proofs of Universal Statements; Variations among Proofs; Common Mistakes; Getting Proofs Started; Showing That an Existential Statement Is False; Conjecture, Proof, and Disproof
- 4.2 *Direct Proof and Counterexample II: Rational Numbers* 127**
 More on Generalizing from the Generic Particular; Proving Properties of Rational Numbers; Deriving New Mathematics from Old
- 4.3 *Direct Proof and Counterexample III: Divisibility* 134**
 Proving Properties of Divisibility; Counterexamples and Divisibility; The Unique Factorization of Integers Theorem
- 4.4 *Direct Proof and Counterexample IV: Division into Cases and the Quotient-Remainder Theorem* 144**
 Discussion of the Quotient-Remainder Theorem and Examples; *div* and *mod*; Alternative Representations of Integers and Applications to Number Theory; Absolute Value and the Triangle Inequality
- 4.5 *Indirect Argument: Contradiction and Contraposition* 154**
 Proof by Contradiction; Argument by Contraposition; Relation between Proof by Contradiction and Proof by Contraposition; Proof as a Problem-Solving Tool
- 4.6 *Indirect Argument: Two Classical Theorems* 163**
 The Irrationality of $\sqrt{2}$; Are There Infinitely Many Prime Numbers?; When to Use Indirect Proof; Open Questions in Number Theory

Chapter 5 Sequences, Mathematical Induction, and Recursion 171

5.1 Sequences 171

Explicit Formulas for Sequences; Summation Notation; Product Notation; Properties of Summations and Products; Change of Variable; Factorial and n Choose r Notation

5.2 Mathematical Induction I 185

Principle of Mathematical Induction; Sum of the First n Integers; Proving an Equality; Deducing Additional Formulas; Sum of a Geometric Sequence

5.3 Mathematical Induction II 199

Comparison of Mathematical Induction and Inductive Reasoning; Proving Divisibility Properties; Proving Inequalities; A Problem with Trominoes

5.4 Strong Mathematical Induction and the Well-Ordering Principle for the Integers 209

Strong Mathematical Induction; Binary Representation of Integers; The Well-Ordering Principle for the Integers

5.5 Defining Sequences Recursively 222

Definition of Recurrence Relation; Examples of Recursively Defined Sequences; Recursive Definitions of Sum and Product

5.6 Solving Recurrence Relations by Iteration 236

The Method of Iteration; Using Formulas to Simplify Solutions Obtained by Iteration; Checking the Correctness of a Formula by Mathematical Induction; Discovering That an Explicit Formula Is Incorrect

Chapter 6 Set Theory 249

6.1 Set Theory: Definitions and the Element Method of Proof 249

Subsets; Proof and Disproof; Set Equality; Venn Diagrams; Operations on Sets; The Empty Set; Partitions of Sets; Power Sets; Cartesian Products

6.2 Properties of Sets 264

Set Identities; Proving Set Identities; Proving That a Set Is the Empty Set

6.3 Disproofs and Algebraic Proofs 279

Disproving an Alleged Set Property; Problem-Solving Strategy; The Number of Subsets of a Set; “Algebraic” Proofs of Set Identities

6.4 Boolean Algebras and Russell's Paradox 286

Boolean Algebras; Description of Russell's Paradox

Chapter 7 Functions 294

7.1 Functions Defined on General Sets 294

Additional Function Terminology; More Examples of Functions; Checking Whether a Function Is Well Defined; Functions Acting on Sets

7.2 One-to-One and Onto, Inverse Functions 305

One-to-One Functions; One-to-One Functions on Infinite Sets; Onto Functions; Onto Functions on Infinite Sets; Relations between Exponential and Logarithmic Functions; One-to-One Correspondences; Inverse Functions

7.3 Composition of Functions 322

Definition and Examples; Composition of One-to-One Functions; Composition of Onto Functions

7.4 Cardinality and Sizes of Infinity 333

Definition of Cardinal Equivalence; Countable Sets; The Search for Larger Infinities; The Cantor Diagonalization Process

Chapter 8 Relations 345

8.1 Relations on Sets 345

Additional Examples of Relations; The Inverse of a Relation; Directed Graph of a Relation

8.2 Reflexivity, Symmetry, and Transitivity 351

Reflexive, Symmetric, and Transitive Properties; Properties of Relations on Infinite Sets

8.3 Equivalence Relations 360

The Relation Induced by a Partition; Definition of an Equivalence Relation; Equivalence Classes of an Equivalence Relation

8.4 Modular Arithmetic and \mathbf{Z}_n 374

Properties of Congruence Modulo n ; Modular Arithmetic; Applications; \mathbf{Z}_n ; Definition of a Commutative Ring

8.5 The Euclidean Algorithm and Applications 387

The Euclidean Algorithm; Extending the Euclidean Algorithm; Euclid's Lemma; the Diophantine Equation $ax + by = c$; Multiplication in \mathbf{Z}_n ; Definition of Field

Chapter 9 Counting and Probability 403

- 9.1 *Introduction* 404
 Definition of Sample Space and Event; Probability in the Equally Likely Case; Counting the Elements of Lists
- 9.2 *Possibility Trees and the Multiplication Rule* 410
 Possibility Trees; The Multiplication Rule; When the Multiplication Rule Is Difficult or Impossible to Apply; Permutations; Permutations of Selected Elements
- 9.3 *Counting Elements of Disjoint Sets: The Addition Rule* 424
 The Addition Rule; The Difference Rule; The Inclusion/Exclusion Rule
- 9.4 *The Pigeonhole Principle* 435
 Statement and Discussion of the Principle; Applications; Decimal Expansions of Fractions; Generalized Pigeonhole Principle; Proof of the Pigeonhole Principle
- 9.5 *Counting Subsets of a Set: Combinations* 446
 r -Combinations; Ordered and Unordered Selections; Relation between Permutations and Combinations; Permutation of a Set with Repeated Elements; Some Advice about Counting
- 9.6 *Pascal's Formula and the Binomial Theorem* 462
 Combinatorial Formulas; Pascal's Triangle; Algebraic and Combinatorial Proofs of Pascal's Formula; The Binomial Theorem and Algebraic and Combinatorial Proofs for It; Applications

Chapter 10 Graphs and Trees 476

- 10.1 *Graphs: Definitions and Basic Properties* 476
 Basic Terminology and Examples of Graphs; Special Graphs; The Concept of Degree
- 10.2 *Trails, Paths, and Circuits* 493
 Definitions; Connectedness; Euler Circuits; Hamiltonian Circuits
- 10.3 *Trees* 512
 Definition and Examples of Trees; Characterizing Trees
- 10.4 *Rooted Trees* 523
 Definition and Examples of Rooted Trees; Binary Trees and Their Properties

Appendix A Properties of the Real Numbers A-1

Appendix B Solutions and Hints to Selected Exercises A-4

Index I-1

PREFACE

My purpose in developing this book was to provide a clear, accessible treatment of the essential aspects of discrete mathematics with a special focus on introducing students to mathematical proof. The book is based on the fourth edition of my *Discrete Mathematics with applications*, which has been used successfully by students at hundreds of institutions in North and South America, Europe, the Middle East, Asia, and Australia.

This version was written in response to requests by its users interested in a shorter, more streamlined treatment of the subject. Like its predecessor, however, its goal is to lay a mathematical foundation for upper-level courses in mathematics and computer science. The book may be used by students either before or after a course in calculus; a good background in algebra is the only prerequisite.

Recent curricular recommendations from the Institute for Electrical and Electronic Engineers Computer Society (IEEE-CS) and the Association for Computing Machinery (ACM) include discrete mathematics as the largest portion of “core knowledge” for computer science students and state that students should take at least a one-semester course in the subject as part of their first-year studies, with a two-semester course preferred when possible. This book includes most of the topics recommended by those organizations; the ones that it omits are often covered elsewhere in computer science programs. Coverage of sets, relations, and functions is foundational for all mathematics and computer science courses; inclusion of basic number theory provides background for students’ future study of abstract algebra, certain computer algorithms, and cryptography; extensive work with quantifiers is especially useful for students who will take a course in real analysis or go on to study artificial intelligence, database theory, or techniques for establishing program correctness; and discussion of counting principles, graph theory, and the calculation of the likelihood of events prepares the way for further study of combinatorics and probability.

At one time, most of the topics in discrete mathematics were taught only to upper-level undergraduates. Discovering how to present these topics in ways that can be understood by first- and second-year students was the major and most interesting challenge of the work I have done. The presentation has been developed over a long period of experimentation during which my students were in many ways my teachers. Their questions, comments, and written work continue to show me what concepts and techniques cause them difficulty, and their reaction to my exposition shows me what works to build their understanding and to encourage their interest.

Themes of a Discrete Mathematics Course

Discrete mathematics describes processes that consist of a sequence of individual steps. This contrasts with calculus, which describes processes that change in a continuous fashion. Whereas the ideas of calculus were fundamental to the science and technology of the industrial revolution, the ideas of discrete mathematics underlie the science and technology of the computer age. Important themes of a first course in discrete mathematics are logic and proof, induction and recursion, discrete structures, and combinatorics and discrete probability.

Logic and Proof Probably the most important goal of a first course in discrete mathematics is to help students develop the ability to think abstractly. This means learning

to use logically valid forms of argument and avoid common logical errors, appreciating what it means to reason from definitions, knowing how to use both direct and indirect argument to derive new results from those already known to be true, and being able to work with symbolic representations as if they were concrete objects.

Induction and Recursion An exciting development of recent years has been the increased appreciation for the power and beauty of “recursive thinking.” To think recursively means to address a problem by assuming that similar problems of a smaller nature have already been solved and figuring out how to put those solutions together to solve the larger problem. Such thinking is used in modeling biological and financial systems, developing data management algorithms, and analyzing algorithms. Recurrence relations that result from recursive thinking often give rise to formulas that are verified by mathematical induction.

Discrete Structures Discrete mathematical structures are the abstract structures that describe, categorize, and reveal the underlying relationships among discrete mathematical objects. Those studied in this book are the sets of integers and rational numbers, general sets, functions, relations, and graphs and trees. In addition, the book includes brief introductions to Boolean algebras and commutative rings and fields.

Combinatorics and Discrete Probability Combinatorics is the mathematics of counting and arranging objects, and probability is the study of laws concerning the measurement of random or chance events. Discrete probability focuses on situations involving discrete sets of objects, such as finding the likelihood of obtaining a certain number of heads when an unbiased coin is tossed a certain number of times. Skill in using combinatorics and probability is needed in almost every discipline where mathematics is applied, from economics to biology, to computer science, to chemistry and physics, to business management.

Special Features of This Book

Mathematical Reasoning The feature that most distinguishes this book from other discrete mathematics texts is that it teaches—explicitly but in a way that is accessible to first- and second-year college and university students—the unspoken logic and reasoning that underlie mathematical thought. For many years I taught an intensively interactive transition-to-abstract-mathematics course to mathematics and computer science majors. This experience showed me that while it is possible to teach the majority of students to understand and construct straightforward mathematical arguments, the obstacles to doing so cannot be passed over lightly. To be successful, a text for such a course must address students’ difficulties with logic and language directly and at some length. It must also include enough concrete examples and exercises to enable students to develop the mental models needed to conceptualize more abstract problems. The treatment of logic and proof in this book blends common sense and rigor in a way that explains the essentials, yet avoids overloading students with formal detail.

Spiral Approach to Concept Development A number of concepts in this book appear in increasingly more sophisticated forms in successive chapters to help students develop the ability to deal effectively with increasing levels of abstraction. For example, by the time students encounter the theory behind modular arithmetic and the solutions for linear Diophantine equations in Sections 8.4 and 8.5, they have been introduced to the logic of mathematical discourse in Chapters 1, 2, and 3, learned the basic methods of proof and the concepts of *mod* and *div* in Chapter 4, explored *mod* and *div* as functions

in Chapter 7, and become familiar with equivalence relations in Sections 8.2 and 8.3. This approach builds in useful review and develops mathematical maturity in natural stages.

Support for the Student Students at colleges and universities inevitably have to learn a great deal on their own. Though it is often frustrating, learning to learn through self-study is a crucial step toward eventual success in a professional career. This book has a number of features to facilitate students' transition to independent learning.

Worked Examples

The book contains over 300 worked examples, which are written using a problem-solution format and are keyed in type and in difficulty to the exercises. Many solutions for the proof problems are developed in two stages: first a discussion of how one might come to think of the proof or disproof and then a summary of the solution, which is enclosed in a box. This format allows students to read the problem and skip immediately to the summary, if they wish, only going back to the discussion if they have trouble understanding the summary. The format also saves time for students who are rereading the text in preparation for an examination.

Marginal Notes and Test Yourself Questions

Notes about issues of particular importance and cautionary comments to help students avoid common mistakes are included in the margins throughout the book. Questions designed to focus attention on the main ideas of each section are located between the text and the exercises. For convenience, the questions use a fill-in-the-blank format, and the answers are found immediately after the exercises.

Exercises

The book contains almost 2000 exercises. The sets at the end of each section have been designed so that students with widely varying backgrounds and ability levels will find some exercises they can be sure to do successfully and also some exercises that will challenge them.

Solutions for Exercises

To provide adequate feedback for students between class sessions, Appendix B contains a large number of complete solutions to exercises. Students are strongly urged not to consult solutions until they have tried their best to answer the questions on their own. Once they have done so, however, comparing their answers with those given can lead to significantly improved understanding. In addition, many problems, including some of the most challenging, have partial solutions or hints so that students can determine whether they are on the right track and make adjustments if necessary. There are also plenty of exercises without solutions to help students learn to grapple with mathematical problems in a realistic environment.

Reference Features

My rationale for screening statements of definitions and theorems, for putting titles on exercises, and for giving the meanings of symbols and a list of reference formulas in the endpapers is to make it easier for students to use this book for review in a current course and as a reference in later ones. Figures and tables are included where doing so would help readers to a better understanding. In most, a second color is used to highlight meaning.

Support for the Instructor I have received a great deal of valuable feedback from instructors who have used editions of *Discrete Mathematics with Applications*, on which this book is based. Many aspects of this book have been improved through their

suggestions. In addition to the following items, there is additional instructor support on the book's website, described later in the preface.

Exercises

The large variety of exercises at all levels of difficulty allows instructors great freedom to tailor a course to the abilities of their students. Exercises with solutions in the back of the book have numbers in blue, and those whose solutions are given in a separate *Student Solutions Manual and Study Guide* have numbers that are a multiple of three. There are exercises of every type represented in this book that have no answer in either location to enable instructors to assign whatever mixture they prefer of exercises with and without answers. The ample number of exercises of all kinds gives instructors a significant choice of problems to use for review assignments and exams. Instructors are invited to use the many exercises stated as questions rather than in "prove that" form to stimulate class discussion on the role of proof and counterexample in problem solving.

Flexible Sections

Most sections are divided into subsections so that an instructor who is pressed for time can choose to cover certain subsections only and either omit the rest or leave them for the students to study on their own. The division into subsections also makes it easier for instructors to break up sections if they wish to spend more than one day on them.

Presentation of Proof Methods

It is inevitable that the proofs and disproofs in this book will seem easy to instructors. Many students, however, find them difficult. In showing students how to discover and construct proofs and disproofs, I have tried to describe the kinds of approaches that mathematicians use when confronting challenging problems in their own research.

Instructor Solutions

Complete instructor solutions to all exercises are available to anyone teaching a course from this book via Cengage's Solution Builder service. Instructors can sign up for access at www.cengage.com/solutionbuilder.

Companion Website

www.cengage.com/math/epp

A website has been developed for this book that contains information and materials for both students and instructors. It includes:

- descriptions and links to many sites on the Internet with accessible information about discrete mathematical topics,
- links to applets that illustrate or provide practice in the concepts of discrete mathematics,
- additional examples and exercises with solutions,
- review guides for the chapters of the book.

A special section for instructors contains:

- suggestions about how to approach the material of each chapter,
- solutions for all exercises not fully solved in Appendix B,
- ideas for projects and writing assignments,
- PowerPoint slides,
- review sheets and additional exercises for quizzes and exams.

Student Solutions Manual and Study Guide

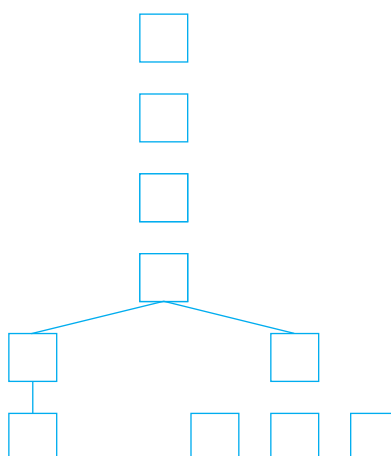
(ISBN-10: 0-495-82618-9; ISBN-13: 978-0-495-82618-7)

In writing this book, I strove to give sufficient help to students through the exposition in the text, the worked examples, and the exercise solutions, so that the book itself would provide all that a student would need to successfully master the material of the course. I believe that students who finish the study of this book with the ability to solve, on their own, all the exercises with full solutions in Appendix B will have developed an excellent command of the subject. Nonetheless, I have become aware that some students want the opportunity to obtain additional helpful materials. In response, I developed a Student Solutions Manual and Study Guide, available separately from this book, which contains complete solutions to every exercise that is not completely answered in Appendix B and whose number is divisible by 3. The guide also includes alternative explanations for some of the concepts and review questions for each chapter.

Organization

The following tree diagram shows, approximately, how the chapters of this book depend on each other. Chapters on different branches of the tree are sufficiently independent that instructors need to make at most minor adjustments if they skip chapters but follow paths along branches of the tree.

In most cases, covering only the core material in each chapter is adequate preparation for moving down the tree.



Acknowledgments

I owe a debt of gratitude to many people at DePaul University for their support and encouragement throughout the years I worked on *Discrete Mathematics with Applications*, on which this book is based. A number of my colleagues used early versions and various editions and provided many excellent suggestions for improvement. For this, I am thankful to Louis Aquila, J. Marshall Ash, Allan Berele, Jeffrey Bergen, William Chin, Barbara Cortzen, Constantine Georgakis, Sigrun Goes, Jerry Goldman, Lawrence Gluck, Leonid Krop, Carolyn Narasimhan, Walter Pranger, Eric Rieders, Ayse Sahin, Yuen-Fat Wong, and, most especially, Jeanne LaDuke. The thousands of students to whom I have taught discrete mathematics have had a profound influence on the book's form.

By sharing their thoughts and thought processes with me, they taught me how to teach them better. I am very grateful for their help. I owe the DePaul University administration, especially my dean, Charles Suchar, and my former deans, Michael Mezey and Richard Meister, a special word of thanks for considering the writing of this book a worthwhile scholarly endeavor.

My thanks to the reviewers for their valuable suggestions for editions of *Discrete Mathematics with Applications*: David Addis, Texas Christian University; Itshak Borosh, Texas A & M University; Douglas M. Campbell, Brigham Young University; David G. Cantor, University of California at Los Angeles; C. Patrick Collier, University of Wisconsin-Oshkosh; Kevan H. Croteau, Francis Marion University; Irinel Drogan, University of Texas at Arlington; Pablo Echeverria, Camden County College; Rachel Esselstein, California State University-Monterrey Bay; Henry A. Etlinger, Rochester Institute of Technology; Melvin J. Friske, Wisconsin Lutheran College; William Gasarch, University of Maryland; Ladnor Geissinger, University of North Carolina; Jerrold R. Griggs, University of South Carolina; Nancy Baxter Hastings, Dickinson College; Lillian Hupert, Loyola University Chicago; Joseph Kolibal, University of Southern Mississippi; Benny Lo, International Technological University; George Luger, University of New Mexico; Leonard T. Malinowski, Finger Lakes Community College; William Marion, Valparaiso University; Michael McClendon, University of Central Oklahoma; Steven Miller, Brown University; John F. Morrison, Towson State University; Paul Pederson, University of Denver; George Peck, Arizona State University; Roxy Peck, California Polytechnic State University, San Luis Obispo; Dix Pettey, University of Missouri; Anthony Ralston, State University of New York at Buffalo; Norman Richert, University of Houston–Clear Lake; John Roberts, University of Louisville; and George Schultz, St. Petersburg Junior College, Clearwater. Special thanks are due John Carroll, San Diego State University; Dr. Joseph S. Fulda; and Porter G. Webster, University of Southern Mississippi; Peter Williams, California State University at San Bernardino; and Jay Zimmerman, Towson University for their unusual thoroughness and their encouragement.

I have also benefitted greatly from the suggestions of the many instructors who have generously offered me their ideas for improvement based on their experiences with *Discrete Mathematics with Applications*, especially Jonathan Goldstine, Pennsylvania State University; David Hecker, St. Joseph's University; Edward Huff, Northern Virginia Community College; Robert Messer, Albion College; Sophie Quigley, Ryerson University; Piotr Rudnicki, University of Alberta; Anwar Shiek, Diné College; Norton Starr, Amherst College; and Eng Wee, National University of Singapore. Production of the third edition received valuable assistance from Christopher Novak, University of Michigan, Dearborn, and Ian Crewe, Ascension Collegiate School. I am especially grateful for the many excellent suggestions for improvement made by Tom Jenkyns, Brock University.

I owe many thanks to the Brooks/Cole staff, especially my editors, Dan Seibert and Shaylin Walsh, for their advice and direction during the production process, and my previous editors, Stacy Green, Robert Pirtle, Barbara Holland, and Heather Bennett, for their encouragement and enthusiasm.

The older I get the more I realize the profound debt I owe my own mathematics teachers for shaping the way I perceive the subject. My first thanks must go to my husband, Helmut Epp, who, on a high school date (!), introduced me to the power and beauty of the field axioms and the view that mathematics is a subject with ideas as well as formulas and techniques. In my formal education, I am most grateful to Daniel Zelinsky and Ky Fan at Northwestern University and Izaak Wirszup, I. N. Herstein, and Irving Kaplansky at the University of Chicago, all of whom, in their own ways, helped lead me to appreciate the elegance, rigor, and excitement of mathematics.

To my family, I owe thanks beyond measure. I am grateful to my mother, whose keen interest in the workings of the human intellect started me many years ago on the

track that led ultimately to this book, and to my late father, whose devotion to the written word has been a constant source of inspiration. I thank my children and grandchildren for their affection and cheerful acceptance of the demands this book has placed on my life. And, most of all, I am grateful to my husband, who for many years has encouraged me with his faith in the value of this project and supported me with his love and his wise advice.

Susanna Epp

SPEAKING MATHEMATICALLY

Therefore O students study mathematics and do not build without foundations. —Leonardo da Vinci (1452–1519)

The aim of this book is to introduce you to a mathematical way of thinking that can serve you in a wide variety of situations. Often when you start work on a mathematical problem, you may have only a vague sense of how to proceed. You may begin by looking at examples, drawing pictures, playing around with notation, rereading the problem to focus on more of its details, and so forth. The closer you get to a solution, however, the more your thinking has to crystallize. And the more you need to understand, the more you need language that expresses mathematical ideas clearly, precisely, and unambiguously.

This chapter will introduce you to some of the special language that is a foundation for much mathematical thought, the language of variables, sets, relations, and functions. Think of the chapter like the exercises you would do before an important sporting event. Its goal is to warm up your mental muscles so that you can do your best.

1.1 Variables

A variable is sometimes thought of as a mathematical “John Doe” because you can use it as a placeholder when you want to talk about something but either (1) you imagine that it has one or more values but you don’t know what they are, or (2) you want whatever you say about it to be equally true for all elements in a given set, and so you don’t want to be restricted to considering only a particular, concrete value for it. To illustrate the first use, consider asking

Is there a number with the following property: doubling it and adding 3 gives the same result as squaring it?

In this sentence you can introduce a variable to replace the potentially ambiguous word “it”:

Is there a number x with the property that $2x + 3 = x^2$?

The advantage of using a variable is that it allows you to give a temporary name to what you are seeking so that you can perform concrete computations with it to help discover its possible values. To emphasize the role of the variable as a placeholder, you might write the following:

Is there a number \square with the property that $2 \cdot \square + 3 = \square^2$?

The emptiness of the box can help you imagine filling it in with a variety of different values, some of which might make the two sides equal and others of which might not.

- [click Winning the Clutter War](#)
- [read online Psychiatry \(2010 Edition\) \(Current Clinical Strategies\)](#)
- [read online Bread Matters: The State of Modern Bread and a Definitive Guide to Baking Your Own pdf, azw \(kindle\), epub, doc, mobi](#)
- [Pathological Altruism here](#)
- [Dead Beat \(Kate Brannigan Mystery Series, Book 1\) here](#)

- <http://unpluggedtv.com/lib/The-Philosophy-Book--Big-Ideas-Simply-Explained.pdf>
- <http://qolorea.com/library/Psychiatry--2010-Edition---Current-Clinical-Strategies-.pdf>
- <http://hasanetmekci.com/ebooks/Catalyst.pdf>
- <http://toko-gumilar.com/books/Pathological-Altruism.pdf>
- <http://diy-chirol.com/lib/The-Wiley-Blackwell-Companion-to-the-Sociology-of-Families.pdf>