

GREAT
DISCOVERIES

EVERYTHING
AND MORE

*A Compact
History of ∞*

DAVID FOSTER
WALLACE



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Everything and More

A Compact History of ∞



ATLAS BOOKS



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For my mother and father

οὐκ ἔστι ἐν τῇ κεφαλῇ, ἀλλ' ἐν ᾧ ἡ κεφαλή ἐστιν.

Small But Necessary Foreword

Unfortunately this is a Foreword you actually have to read—and first—in order to understand certain structural idiosyncrasies and bits of what almost look like code in the main text. Of the latter the most frequent is a boldface ‘IYI’. This, be apprised, is not a tic or typo but instead stands for the clause *If you’re interested*, which was getting used over and over so many times in early drafts that what eventually happened is that through sheer repetition it evolved from a natural-language phrase for introducing some clause into an abstract extratextual sign—IYI—that now serves to classify certain chunks of text in a particular way. Which way will now be justified and explained.

Like the other booklets in this ‘Great Discoveries’ series, *Everything and More* is a piece of pop technical writing. Its subject is a set of mathematical achievements that are extremely abstract and technical, but also extremely profound and interesting, and beautiful. The aim is to discuss these achievements in such a way that they’re vivid and

comprehensible to readers who do not have pro-grade technical backgrounds and expertise. To make the math beautiful—or at least to get the reader to see how someone might find it so. Which of course all sounds very nice, except there's a hitch: just how technical can the presentation get without either losing the reader or burying her in endless little definitions and explanatory asides? Plus if you assume, as seems plausible, that some readers are going to have much stronger tech backgrounds than others, how can the discussion be pitched so that it's accessible to the neophyte without being dull or annoying to somebody who's had a lot of college math?

In the following document, the boldface 'TYI' designates bits of material that can be perused, glanced at, or skipped altogether if the reader wants. Meaning skipped without serious loss. Over half the document's footnotes are probably IYI, as well as several different ¶s and even a couple subsections of the main text. Some of the optional bits are digressions or bits of historical ephemera¹; some are definitions or explanations that a math-savvy reader won't need to waste time on. Most IYI-grade chunks, though, are designed for readers with strong technical backgrounds, or unusual interest in actual math, or preternatural patience, or all three; they (the

¹ IYI Here's a good example of an IYI factoid. Your author here is someone with a medium-strong amateur interest in math and formal systems. He is also someone who disliked and did poorly in every math course he ever took, save one, which wasn't even in college, but which was taught by one of those rare specialists who can make the abstract alive and urgent, and who actually really talks to you when he's lecturing, and of whom anything that's good about this booklet is a pale and well-meant imitation.

chunks) provide a more detailed look at stuff that the main discussion glosses or breezes through.

There are other abbreviations in the booklet, too. Some are just to save space. Others are the consequence of a strange stylistic problem in tech writing, which is that the same words often have to get used over and over in a way that would be terribly clunky in regular prose—the thing is that some technical words have highly specific denotations that no synonym can capture. Which means that, especially respecting certain high-tech proper nouns, abbreviation is the only way to achieve any kind of variation at all. None of that is really your problem. All the booklet’s abbreviations are contextualized in such a way that it ought to be totally clear what they stand for; but in case of authorial foul-ups or unnecessary confusion, here is a list of the main ones, which can be flipped back and referred to if necessary:

1-1C	= One-to-One Correspondence
A.C.	= Axiom of Choice
A.S.T.	= Axiomatic Set Theory
<i>ATH</i>	= Fourier’s <i>Analytic Theory of Heat</i>
B.T.	= Binomial Theorem
B.W.T.	= Bolzano-Weierstrass Theorem
“C. and I.R.”	= Dedekind’s “Continuity and Irrational Numbers”
C.H.	= Continuum Hypothesis
C.P.	= Cartesian Product
D.B.P.	= Divine Brotherhood of Pythagoras
D.E.	= Differential Equation
D.P.	= Diagonal Proof
E.G.	= EMERGENCY GLOSSARY

E.V.T.	= Weierstrass's Extreme Values Theorem
F.T.C.	= Fundamental Theorem of the Calculus
G.C.P.F.S.	= General Convergence Problem of Fourier Series
L.A.P.	= Limited Abstraction Principle
LEM	= Law of the Excluded Middle
N. & L.	= Newton and Leibniz
N.L.	= Number Line
N.S.T.	= Naïve Set Theory
O.O.M.	= Plato's One Over Many argument
P.I.	= Principle of Induction
<i>P of the I</i>	= Bolzano's <i>Paradoxes of the Infinite</i>
P.S.A.	= Power Set Axiom
P.T.	= Pythagorean Theorem
R.L.	= Real Line
TNS	= Galileo's <i>Two New Sciences</i>
U.A.P.	= Unlimited Abstraction Principle
U.T.	= Uniqueness Theorem
VC	= Vicious Circle
VIR	= Vicious Infinite Regress
VNB	= Von Neumann–Bernays system of axioms for set theory
V.S.P.	= Vibrating String Problem
W.E.	= Wave Equation
ZFS	= Zermelo–Fraenkel–Skolem system of axioms for set theory
Z.P.	= Zeno's Paradox

§1a. There is such a thing as an historian of mathematics. Here is a nice opening-type quotation from one such historian in the 1930s:

One conclusion appears to be inescapable: without a consistent theory of the mathematical infinite there is no theory of irrationals; without a theory of irrationals there is no mathematical analysis in any form even remotely resembling what we now have; and finally, without analysis the major part of mathematics—including geometry and most of applied mathematics—as it now exists would cease to exist. The most important task confronting mathematicians would therefore seem to be the construction of a satisfactory theory of the infinite. Cantor attempted this, with what success will be seen later.

The sexy math terms don't matter for now. The Cantor of the last line is Prof. Georg F. L. P. Cantor, b. 1845, a naturalized German of the merchant class and the acknowledged father of abstract set theory and transfinite math. Some historians have argued back and forth about whether he was Jewish. 'Cantor' is just Latin for singer.

G. F. L. P. Cantor is the most important mathematician of the nineteenth century and a figure of great complexity and pathos. He was in and out of mental hospitals for much of his later adulthood and died in a sanitarium in Halle¹ in 1918. K. Gödel, the most important mathematician of the twentieth century, also died as the result of mental illness. L. Boltzmann,

¹ IYI Halle, a literal salt mine just upriver from Leipzig, is best known as Handel's hometown.

the most important mathematical physicist of the nineteenth century, committed suicide. And so on. Historians and pop scholars tend to spend a lot of time on Cantor's psychiatric problems and on whether and how they were connected to his work on the mathematics of ∞ .

At Paris's 2nd International Congress of Mathematicians in 1900, Prof. D. Hilbert, then the world's #1 mathematician, described Georg Cantor's transfinite numbers as "the finest product of mathematical genius" and "one of the most beautiful realizations of human activity in the domain of the purely intelligible."

Here is a quotation from G. K. Chesterton: "Poets do not go mad; but chess players do. Mathematicians go mad, and cashiers; but creative artists very seldom. I am not attacking logic: I only say that this danger does lie in logic, not in imagination." Here also is a snippet from the flap copy for a recent pop bio of Cantor: "In the late nineteenth century, an extraordinary mathematician languished in an asylum. . . . The closer he came to the answers he sought, the further away they seemed. Eventually it drove him mad, as it had mathematicians before him."

The cases of great mathematicians with mental illness have enormous resonance for modern pop writers and filmmakers. This has to do mostly with the writers'/directors' own prejudices and receptivities, which in turn are functions of what you could call our era's particular archetypal template. It goes without saying that these templates change over time. The Mentally Ill Mathematician seems now in some ways to be what the Knight Errant, Mortified Saint, Tortured Artist, and Mad Scientist have been for other eras: sort of our

Prometheus, the one who goes to forbidden places and returns with gifts we all can use but he alone pays for. That's probably a bit overblown, at least in most cases.² But Cantor fits the template better than most. And the reasons for this are a lot more interesting than whatever his problems and symptoms were.³

Merely knowing about Cantor's accomplishments is different from appreciating them, which latter is the general project here and involves seeing transfinite math as kind of like a tree, one with its roots in the ancient Greek paradoxes of continuity and incommensurability and its branches entwined in the modern crises over math's foundations—Brouwer and Hilbert and Russell and Frege and Zermelo and Gödel and Cohen et al. The names right now are less important than the tree thing, which is the main sort of overview-trope you'll be asked to keep in mind.

² IYI although so is the other, antipodal stereotype of mathematicians as nerdy little bowtied fissiparous creatures. In today's archetypology, the two stereotypes seem to play off each other in important ways.

³ In modern medical terms, it's fairly clear that G. F. L. P. Cantor suffered from manic-depressive illness at a time when nobody knew what this was, and that his polar cycles were aggravated by professional stresses and disappointments, of which Cantor had more than his share. Of course, this makes for less interesting flap copy than *Genius Driven Mad By Attempts To Grapple With ∞* . The truth, though, is that Cantor's work and its context are so totally interesting and beautiful that there's no need for breathless Prometheusizing of the poor guy's life. The real irony is that the view of ∞ as some forbidden zone or road to insanity—which view was very old and powerful and haunted math for 2000+ years—is precisely what Cantor's own work overturned. Saying that ∞ drove Cantor mad is sort of like mourning St. George's loss to the dragon: it's not only wrong but insulting.

§1b. Chesterton above is wrong in one respect. Or at least imprecise. The danger he's trying to name is not logic. Logic is just a method, and methods can't unhinge people. What Chesterton's really trying to talk about is one of logic's main characteristics—and mathematics'. Abstractness. Abstraction.

It is worth getting straight on the meaning of *abstraction*. It's maybe the single most important word for appreciating Cantor's work and the contexts that made it possible. Grammatically, the root form is the adjectival, from the L. *abstractus* = 'drawn away'. The *O.E.D.* has nine major definitions of the adjective, of which the most apposite is 4.a.: "Withdrawn or separated from matter, from material embodiment, from practice, or from particular examples. Opposed to *concrete*." Also of interest are the *O.E.D.*'s 4.b., "Ideal, distilled to its essence," and 4.c., "Abstruse."

Here is a quotation from Carl B. Boyer, who is more or less the Gibbon of math history⁴: "But what, after all, are the integers? Everyone thinks that he or she knows, for example, what the number three is—until he or she tries to define or explain it." W/t/t which it is instructive to talk to 1st- and 2nd-grade math teachers and find out how children are actually taught about integers. About what, for example, the number five is. First they are given, say, five oranges. Something they can touch or hold. Are asked to count them. Then they

⁴ FYI Boyer is joined at the top of the math-history food chain only by Prof. Morris Kline. Boyer's and Kline's major works are respectively *A History of Mathematics* and *Mathematical Thought from Ancient to Modern Times*. Both books are extraordinarily comprehensive and good and will be liberally cribbed from.

are given a picture of five oranges. Then a picture that combines the five oranges with the numeral '5' so they associate the two. Then a picture of just the numeral '5' with the oranges removed. The children are then engaged in verbal exercises in which they start talking about the integer 5 per se, as an object in itself, apart from five oranges. In other words they are systematically fooled, or awakened, into treating numbers as things instead of as symbols for things. Then they can be taught arithmetic, which comprises elementary relations between numbers. (You will note how this parallels the ways we are taught to use language. We learn early on that the noun 'five' means, symbolizes, the integer 5. And so on.)

Sometimes a kid will have trouble, the teachers say. Some children understand that the word 'five' stands for 5, but they keep wanting to know 5 *what?* 5 oranges, 5 pennies, 5 points? These children, who have no problem adding or subtracting oranges or coins, will nevertheless perform poorly on arithmetic tests. They cannot treat 5 as an object per se. They are often then remanded to Special Ed Math, where everything is taught in terms of groups or sets of actual objects rather than as numbers "withdrawn from particular examples."⁵

⁵ B. Russell has an interesting ¶ in this regard about high-school math, which is usually the next big jump in abstraction after arithmetic:

In the beginning of algebra, even the most intelligent child finds, as a rule, very great difficulty. The use of letters is a mystery, which seems to have no purpose except mystification. It is almost impossible, at first, not to think that every letter stands for some particular number, if only the teacher would reveal *what* number it stands for. The fact is, that in algebra the mind is first taught to consider general truths, truths which are not asserted to hold only of this or that particular thing, but of any one of a whole group of things. It is in

The point: The basic def. of 'abstract' for our purposes is going to be the somewhat concatenated 'removed from or transcending concrete particularity, sensuous experience'. Used in just this way, 'abstract' is a term from metaphysics. Implicit in all mathematical theories, in fact, is some sort of metaphysical position. The father of abstraction in mathematics: Pythagoras. The father of abstraction in metaphysics: Plato.

The *O.E.D.*'s other defs. are not irrelevant, though. Not just because modern math is abstract in the sense of being extremely abstruse and arcane and often hard to even look at on the page. Also essential to math is the sense in which abstracting something can mean reducing it to its absolute skeletal essence, as in the abstract of an article or book. As such, it can mean thinking hard about things that for the most part people can't think hard about—because it drives them crazy.

All this is just sort of warming up; the whole thing won't be like this. Here are two more quotations from towering figures. M. Kline: "One of the great Greek contributions to the very concept of mathematics was the conscious recognition and emphasis of the fact that mathematical entities are abstractions, ideas entertained by the mind and sharply distinguished from physical objects or pictures." F.d.I. Saussure: "What has escaped philosophers and logicians is that from the moment a system of symbols becomes independent

the power of understanding and discovering such truths that the mastery of the intellect over the whole world of things actual and possible resides; and ability to deal with the general as such is one of the gifts that a mathematical education should bestow.

of the objects designated it is itself subject to undergoing displacements that are incalculable for the logician.”

Abstraction has all kinds of problems and headaches built in, we all know. Part of the hazard is how we use nouns. We think of nouns' meanings in terms of denotations. Nouns stand for things—*man, desk, pen, David, head, aspirin*. A special kind of comedy results when there's confusion about what's a real noun, as in 'Who's on first?' or those *Alice in Wonderland* routines—'What can you see on the road?' 'Nothing.' 'What great eyesight! What does nothing look like?' The comedy tends to vanish, though, when the nouns denote abstractions, meaning general concepts divorced from particular instances. Many of these abstraction-nouns come from root verbs. 'Motion' is a noun, and 'existence'; we use words like this all the time. The confusion comes when we try to consider what exactly they mean. It's like Boyer's point about integers. What exactly do 'motion' and 'existence' denote? We know that concrete particular things exist, and that sometimes they move. Does motion per se exist? In what way? In what way do abstractions exist?

Of course, that last question is itself very abstract. Now you can probably feel the headache starting. There's a special sort of unease or impatience with stuff like this. Like 'What exactly is existence?' or 'What exactly do we mean when we talk about motion?' The unease is very distinctive and sets in only at a certain level in the abstraction process—because abstraction proceeds in levels, rather like exponents or dimensions. Let's say 'man' meaning some particular man is Level One. 'Man' meaning the species is Level Two. Something like 'humanity' or 'humanness' is Level Three; now we're talking about the abstract criteria for something qualifying as human.

And so forth. Thinking this way can be dangerous, weird. Thinking abstractly enough about anything . . . surely we've all had the experience of thinking about a word—'pen,' say—and of sort of saying the word over and over to ourselves until it ceases to denote; the very strangeness of calling something a pen begins to obtrude on the consciousness in a creepy way, like an epileptic aura.

As you probably know, much of what we now call analytic philosophy is concerned with Level Three— or even Four—grade questions like this. As in epistemology = 'What exactly is knowledge?'; metaphysics = 'What exactly are the relations between mental constructs and real-world objects?'; etc.⁶ It might be that philosophers and mathematicians, who spend a lot of time thinking (a) abstractly or (b) about abstractions or (c) both, are *eo ipso* rendered prone to mental illness. Or it might just be that people who are susceptible to mental illness are more prone to think about these sorts of things. It's a chicken-and-egg question. One thing is certain, though. It is a total myth that man is by nature curious and truth-hungry and wants, above all things, *to know*.⁷ Given certain recognized senses of 'to know,' there is in fact a great deal of stuff we do *not* want to know. Evidence for this is the enormous number of very basic questions and issues we do not like to think about abstractly.

⁶ **IYI** According to most sources, G. F. L. P. Cantor was not just a mathematician—he had an actual Philosophy of the Infinite. It was weird and quasi-religious and, not surprisingly, abstract. At one point Cantor tried to switch his U. Halle job from the math dept. to philosophy; the request was turned down. Admittedly, this was not one of his stabler periods.

⁷ **IYI** The source of this pernicious myth is Aristotle, who is in certain respects the villain of our whole Story—q.v. §2 sub.

Theory: The dreads and dangers of abstract thinking are a big reason why we now all like to stay so busy and bombarded with stimuli all the time. Abstract thinking tends most often to strike during moments of quiet repose. As in for example the early morning, especially if you wake up slightly before your alarm goes off, when it can suddenly and for no reason occur to you that you've been getting out of bed every morning without the slightest doubt that the floor would support you. Lying there now considering the matter, it appears at least theoretically possible that some flaw in the floor's construction or its molecular integrity could make it buckle, or that even some aberrant bit of quantum flux or something could cause you to melt right through. Meaning it doesn't seem logically impossible or anything. It's not like you're actually scared that the floor might give way in a moment when you really do get out of bed. It's just that certain moods and lines of thinking are more abstract, not just focused on whatever needs or obligations you're going to get out of bed to attend to. This is just an example. The abstract question you're lying there considering is whether you are truly justified in your confidence about the floor. The initial answer, which is yes, lies in the fact that you've gotten out of bed in the morning thousands—actually well over ten thousand times so far, and each time the floor has supported you. It's the same way you're also justified in believing that the sun will come up, that your wife will know your name, that when you feel a certain set of sensations it means you're getting ready to sneeze, & c. Because they've happened over and over before. The principle involved is really the only way we can predict any of the phenomena we just automatically count on without having to think about them. And the vast bulk of

daily life is composed of these sorts of phenomena; and without this confidence based on past experience we'd all go insane, or at least we'd be unable to function because we'd have to stop and deliberate about every last little thing. It's a fact: life as we know it would be impossible without this confidence. Still, though: Is the confidence actually justified, or just highly convenient? This is abstract thinking, with its distinctive staircase-shaped graph, and you're now several levels up. You're no longer thinking just about the floor and your weight, or about your confidence re same and how necessary to basic survival this kind of confidence seems to be. You're now thinking about some more general rule, law, or principle by which this unconsidered confidence in all its myriad forms and intensities is in fact justified instead of being just a series of weird clonic jerks or reflexes that propel you through the day. Another sure sign it's abstract thinking: You haven't moved yet. It feels like tremendous energy and effort is being expended and you're still lying perfectly still. All this is just going on in your mind. It's extremely weird; no wonder most people don't like it. It suddenly makes sense why the insane are so often represented as grabbing their head or beating it against something. If you had the right classes in school, however, you might now recall that the rule or principle you want does exist—its official name is the Principle of Induction. It is the fundamental precept of modern science. Without the Principle of Induction, experiments couldn't confirm a hypothesis, and nothing in the physical universe could be predicted with any confidence at all. There could be no natural laws or scientific truths. The P.I. states that if something x has happened in certain particular circumstances n times in the past, we are justified in believing that the same

circumstances will produce x on the $(n + 1)$ th occasion. The P.I. is wholly respectable and authoritative, and it seems like a well-lit exit out of the whole problem. Until, that is, it happens to strike you (as can occur only in very abstract moods or when there's an unusual amount of time before the alarm goes off) that the P.I. is *itself* merely an abstraction from experience . . . and so now what exactly is it that justifies our confidence in the P.I.? This latest thought may or may not be accompanied by a concrete memory of several weeks spent on a relative's farm in childhood (long story). There were four chickens in a wire coop off the garage, the brightest of whom was called Mr. Chicken. Every morning, the farm's hired man's appearance in the coop area with a certain burlap sack caused Mr. Chicken to get excited and start doing warmup-pecks at the ground, because he knew it was feeding time. It was always around the same time t every morning, and Mr. Chicken had figured out that $t(\text{man} + \text{sack}) = \text{food}$, and thus was confidently doing his warmup-pecks on that last Sunday morning when the hired man suddenly reached out and grabbed Mr. Chicken and in one smooth motion wrung his neck and put him in the burlap sack and bore him off to the kitchen. Memories like this tend to remain quite vivid, if you have any. But with the thrust, lying here, being that Mr. Chicken appears now actually to have been correct—according to the Principle of Induction—in expecting nothing but breakfast from that $(n + 1)$ th appearance of man + sack at t . Something about the fact that Mr. Chicken not only didn't suspect a thing but appears to have been *wholly justified* in not suspecting a thing—this seems concretely creepy and upsetting. Finding some higher-level justification for your confidence in the P.I. seems much more urgent when you

realize that, without this justification, our own situation is basically indistinguishable from that of Mr. Chicken. But the conclusion, abstract as it is, seems inescapable: what justifies our confidence in the Principle of Induction is that it has always worked so well in the past, at least up to now. Meaning that our only real justification for the Principle of Induction is the Principle of Induction, which seems shaky and question-begging in the extreme.

The only way out of the potentially bedridden-for-life paralysis of this last conclusion is to pursue further abstract side-inquiries into what exactly 'justification' means and whether it's true that the only valid justifications for certain beliefs and principles are rational and noncircular. For instance, we know that in a certain number of cases every year cars suddenly veer across the centerline into oncoming traffic and crash head-on into people who were driving along not expecting to get killed; and thus we also know, on some level, that whatever confidence lets us drive on two-way roads is not 100% rationally justified by the laws of statistical probability. And yet 'rational justification' might not apply here. It might be more the fact that, if you cannot believe your car won't suddenly get crashed into out of nowhere, you just can't drive, and thus that your need/desire to be able to drive functions as a kind of 'justification' of your confidence.⁸

⁸ A compelling parallel here is the fact that most of us fly despite knowing that a definite percentage of commercial airliners crash every year. This gets into the various different kinds of knowing v. 'knowing,' though (see §1c below). Plus it involves etiquette, since commercial air travel is public and a kind of group confidence comes into play. This is why turning to inform your seatmate of the precise statistical probability of your

It would be better not to then start analyzing the various putative ‘justifications’ for your need/desire to be able to drive a car—at some point you realize that the process of abstract justification can, at least in principle, go on forever. The ability to halt a line of abstract thinking once you see it has no end is part of what usually distinguishes sane, functional people—people who when the alarm finally goes off can hit the floor without trepidation and plunge into the concrete business of the real workaday world—from the unhinged.

INTERPOLATION

The tactical reason for sometimes using ‘ ∞ ’ instead of ‘infinity’ in the natural-language text here is that the double-blink strangeness of ‘ ∞ ’ serves as a reminder that it’s not clear what we’re even talking about. Not yet. For instance, beware of thinking that ∞ is just an incredibly, unbelievably enormous number. There are, of course, many such numbers, especially in physics and astronomy—like, if in physics an ultranano-instant of 5×10^{-44} seconds is generally acknowledged to be the smallest time-interval in which the normal concept of continuous time applies (which it is), astronomical data indicates that there have been roughly 6×10^{60} such ultranano-instants since the Big Bang. That’s a 6 followed by 60 zeroes. We’ve all heard about numbers like this, which we usually imagine can

plane crashing is not false but cruel: you are messing with the delicate psychological infrastructure of her justification for flying.

IYI Depending on mood/time, it might strike you as interesting that people who cannot summon this strange faith in principles that cannot be rationally justified, and so cannot fly, are commonly referred to as having an ‘irrational fear’ of flying.

be conceived and manipulated only with really advanced super-cooled computers or something. Actually, there are plenty of numbers too big for any real or even theoretical computer to process. *Bremermann's Limit* is the operative term here. Given limits imposed by basic quantum theory, one H. Bremermann proved in 1962 that "No data processing system, whether artificial or living, can process more than 2×10^{47} bits per second per gram of its mass," which means that a hypothetical supercomputer the size of the earth (= c. 6×10^{27} grams) grinding away for as long as the earth has existed (= about 10^{10} years, with c. 3.14×10^7 seconds/year) can have processed at most 2.56×20^{92} bits, which number is known as Bremermann's Limit. Calculations involving numbers larger than 2.56×20^{92} are called *transcomputational problems*, meaning they're not even theoretically doable; and there are plenty of such problems in statistical physics, complexity theory, fractals, etc. All this is sexy but not quite germane. What's germane is: Take some such transcomputational number, imagine it's a grain of sand, conceive of a whole beach, or desert, or planet, or even galaxy filled with such sand, and not only will the corresponding 10^x number be $< \infty$, but its square will be $< \infty$, and $10^{(x^{(10^9)})}$ will be $< \infty$, and so on; and actually it's not even right to compare 10^x and ∞ arithmetically this way because they're not even in the same mathematical area code—even, as it were, the same dimension. And yet it's also true that some ∞ s are bigger than others, as in arithmetically bigger. All this will get discussed; the thing for now is that only after R. Dedekind and G. Cantor is it even possible to talk about infinite quantities and their arithmetic coherently, meaningfully. Hence the point of using '∞'.

IYI The ‘∞’ symbol itself is technically called the *lemniscate* (apparently from the Greek for ‘ribbon’) and was introduced to math by John Wallis in his 1655 *Arithmetica infinitorum*, which was one of the important preliminaries for Newton’s brand of calculus.⁹ Wallis’s contemporary Thomas Hobbes, something of a mathematical crank, complained in a review that *Arithmetica infinitorum* was too brutally abstract to even try to read, “a scab of symbols,” thereby speaking for generations of undergrads to follow. Other names for the lemniscate include ‘the love knot’ and ‘the Cartesian plane curve that satisfies the equation $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ ’. If, on the other hand, it’s treated trigonometrically and called ‘the curve that satisfies the polar equation $r^2 = a \cos 2\theta$,’ it is also known as *Bernoulli’s Lemniscate*.

END INTERPOLATION

§1c. Apropos the whole business of abstractness and nouns’ denotations, there is a syndrome that’s either a high-level abstraction or some type of strange nominal mutation. ‘Horse’ can mean this one horse right here, or it can mean the abstract concept, as in ‘Horse = hoofed mammal of family *Equidae*’. Same with the word ‘horn’; same with ‘forehead’. All these can be abstracted from particulars, but we still know they came from particulars. Except what about a unicorn, which seems to result from the combination of the concepts ‘horse,’ ‘horn,’ and ‘forehead’ and thus has its whole origin in the concatenation of abstractions? Meaning we can

⁹ **IYI** As it happens, the only thing that kept Wallis from actually inventing differential calculus in *A.i.* was his ignorance of the Binomial Theorem, which is essential to working with infinitesimals—see esp. §4 below.

conjoin and manipulate abstractions to form entities whose nouns have no particular denotations at all. Here the big problem becomes: In what way can we say a unicorn exists that is fundamentally different, less real, than the way abstractions like humanity or horn or integer exist? Which is once again the question: In what way do abstract entities exist, or do they exist at all except as ideas in human minds—i.e., are they metaphysical fictions? This sort of question can keep you in bed all day too. And it hangs over math from the beginning—what is the ontological status of mathematical entities and relations? Are mathematical realities discovered, or merely created, or somehow both? Here is M. Kline again: “The philosophical doctrines of the Greeks limited mathematics in another way. Throughout the classical period they believed that man does not create the mathematical facts: they preexist. He is limited to ascertaining and recording them.”

Plus here is another quotation from D. Hilbert, the great early champion of Cantor’s transfinities:

[T]he infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to. Can thought about things be so much different from things? Can thinking processes be so unlike the actual process of things? In short, can thought be so far removed from reality?

And it’s true: there is nothing more abstract than infinity. Meaning at least our fuzzy, intuitive, natural-language concept of ∞ . It’s sort of the ultimate in drawing away from actual experience. Take the single most ubiquitous and oppressive feature of the concrete world—namely that everything ends,

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