Fundamentals of Vehicle Dynamics

Thomas D. Gillespie

Published by:
Society of Automotive Engineers, Inc.
400 Commonwealth Drive
Warrendale, PA 15096-0001

LIST OF SYMBOLS

a	Tire cornering stiffness parameter
b	Tire cornering stiffness parameter
A	Frontal area of a vehicle
A_f	Lateral force compliance steer coefficient on the front axle
A_r	Lateral force compliance steer coefficient on the rear axle
a_x	Acceleration in the x-direction
a_y	Acceleration in the lateral direction
b	Longitudinal distance from front axle to center of gravity
c	Longitudinal distance from center of gravity to rear axle
C_α	Cornering stiffness of the tires on an axle
C_α'	Cornering stiffness of one tire
CC_α	Tire cornering coefficient
C_γ	Tire camber stiffness
C_D	Aerodynamic drag coefficient
C_h	Road surface rolling resistance coefficient
C_L	Aerodynamic lift coefficient
C_{PM}	Aerodynamic pitching moment coefficient
C_{RM}	Aerodynamic rolling moment coefficient
C_{YM}	Aerodynamic yawing moment coefficient
C_s	Suspension damping coefficient
C_S	Aerodynamic side force coefficient
CP	Center of pressure location of aerodynamic side force
d	Lateral distance between steering axis and center of tire contact at the ground
d_h	Distance from axle to the hitch point
d_{ns}	Distance from center of mass to the neutral steer point
D	Tire diameter
DI	Dynamic index
D_x	Linear deceleration
D_A	Aerodynamic drag force
e	Height of the pivot for an "equivalent torque arm"
	Drum brake geometry factor
$E[y^2]$	Mean square vibration response
f	Longitudinal length for an "equivalent torque arm"
f_a	Wheel hop resonant frequency (vertical)

f_n	Undamped natural frequency of a suspension system (Hz)
f_r	Rolling resistance coefficient
F_b	Braking force
	Vertical disturbance force on the sprung mass
F_i	Imbalance force in a tire
F_x	Force in the x-direction (tractive force)
F_{xm}	Maximum brake force on an axle
F_{xt}	Total force in the x-direction
F_y	Force in the y-direction (lateral force)
	Lateral force on an axle
F_y'	Lateral force on one tire
F_z	Force in the z-direction (vertical force)
F_{zi}	Vertical force on inside tire in a turn
F_{zo}	Vertical force on outside tire in a turn
F_w	Tire/wheel nonuniformity force on the unsprung mass
g	Acceleration of gravity (32.2 ft/sec^2, 9.81 m/sec^2)
G	Brake gain
G_o	Road roughness magnitude parameter
G_z	Power spectral density amplitude of road roughness
G_{zs}	Power spectral density amplitude of sprung mass acceleration
h	Center of gravity height
h_a	Height of the aerodynamic drag force
h_h	Hitch height
h_1	Height of the sprung mass center of gravity above the roll axis
h_r	Height of suspension roll center
h_t	Tire section height
HP	Engine or brake horsepower
HP_A	Aerodynamic horsepower
HP_R	Rolling resistance horsepower
HP_{RL}	Road load horsepower
H_v	Response gain function
I_d	Moment of inertia of the driveshaft
I_e	Moment of inertia of the engine
I_t	Moment of inertia of the transmission
I_w	Moment of inertia of the wheels
I_{xx}	Moment of inertia about the x-axis

I_{yy}	Moment of inertia about the y-axis
I_{zz}	Moment of inertia about the z-axis
k	Radius of gyration
K	Understeer gradient
K_{at}	Understeer gradient due to aligning torque
K_{llt}	Understeer gradient due to lateral load transfer on the axles
K_{lfcs}	Understeer gradient due to lateral force compliance steer
K_s	Vertical stiffness of a suspension
K_{ss}	Steering system stiffness
K_{strg}	Understeer gradient due to the steering system
K_t	Vertical stiffness of a tire
K_ϕ	Suspension roll stiffness
L	Wheelbase
L_A	Aerodynamic lift force
m	Drum brake geometry parameter
M	Mass of the vehicle
M_{AT}	Moment around the steer axis due to tire aligning torques
M_L	Moment around the steer axis due to tire lateral forces
M_r	Equivalent mass of the rotating components
M_{SA}	Moment around the steer axis due to front-wheel-drive forces and torques
M_T	Moment around the steer axis due to tire tractive forces
M_V	Moment around the steer axis due to tire vertical forces
M_ϕ	Rolling moment
n	Drum brake geometry parameter
N	Normal force
N_t	Numerical ratio of the transmission
N_f	Numerical ratio of the final drive
N_{tf}	Numerical ratio of the combined transmission and final drive
NSP	Neutral steer point
p	Pneumatic trail
P_a	Brake application pressure/effort
P_{atm}	Atmospheric pressure
P_f	Front brake application pressure
P_r	Rear brake application pressure
P_s	Static pressure
P_t	Total pressure

PM Aerodynamic pitching moment
p Roll velocity about the x-axis of the vehicle
q Pitch velocity about the y-axis of the vehicle
q Dynamic pressure
r Yaw velocity about the z-axis of the vehicle
r Rolling radius of the tires
r_k Ratio of tire to suspension stiffness
R Radius of turn
R_h Hitch force
R_g Grade force
R_x Rolling resistance force
R_{RL} Road load
RM Aerodynamic rolling moment
RR Ride rate of a tire/suspension system
R_ϕ Roll rate of the sprung mass
s Lateral separation between suspension springs
S_A Aerodynamic side force
S_o Spectral density of white-noise
SD Stopping distance
t Tread
t_s Length of time of a brake application
T_a Torque in the axle
T_b Brake torque
T_c Torque at the clutch
T_d Torque in the driveshaft
T_e Torque of the engine
T_{sf} Roll torque in a front suspension
T_{sr} Roll torque in a rear suspension
T_{amb} Ambient temperature
T_x Torque about the x-axis
V Forward velocity
V_w Ambient wind velocity
V_f Final velocity resulting from a brake application
V_o Initial velocity in a brake application
w Tire section width
W Weight of the vehicle

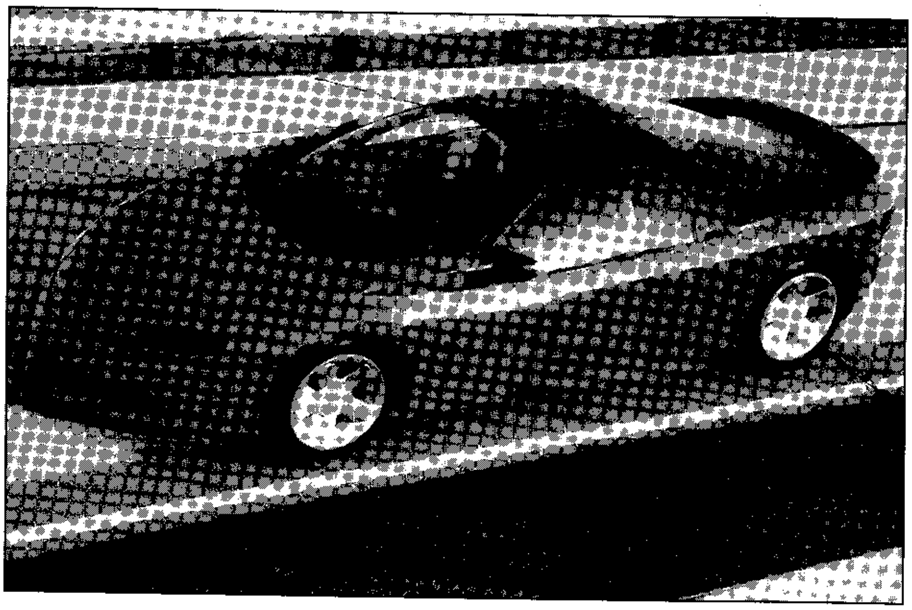
W_a	Axle weight
W_d	Dynamic load transfer
W_f	Dynamic weight on the front axle
W_r	Dynamic weight on the rear axle
W_{rr}	Dynamic weight on the right rear wheel
W_{fs}	Static weight on the front axle
W_{rs}	Static weight on the rear axle
W_y	Lateral weight transfer on an axle
x	Forward direction on the longitudinal axis of the vehicle
y	Lateral direction out the right side of the vehicle
YM	Aerodynamic yawing moment
z	Vertical direction with respect to the plane of the vehicle
X	Forward direction of travel
Y	Lateral direction of travel
Z	Vertical direction of travel
	Vertical displacement of the sprung mass
Z_r	Road profile elevation
Z_u	Vertical displacement of the unsprung mass
α	Tire slip angle
	Coefficient in the pitch plane equations
α_{cw}	Aerodynamic wind angle
α_d	Rotational acceleration of the driveshaft
α_e	Rotational acceleration of the engine
α_w	Rotational acceleration of the wheels
α_x	Rotational acceleration about the x-axis
β	Sideslip angle
	Rotation angle of a U-joint
	Coefficient in the pitch plane equations
γ	Camber angle
	Coefficient in the pitch plane equations
γ_g	Wheel camber with respect to the ground
γ_b	Wheel camber with respect to the vehicle body
δ	Steer angle
δ_c	Compliance steer
δ_i	Steer angle of the inside wheel in a turn

δ_o	Steer angle of the outside wheel in a turn
Δ	Off-tracking distance in a turn
ε	Roll steer coefficient
	Inclination of the roll axis
ζ	Moment arm related to tire force yaw damping
	Half-shaft angle on a front-wheel drive
ζ_s	Damping ratio of the suspension
η_b	Braking efficiency
η_t	Efficiency of the transmission
η_f	Efficiency of the final drive
η_{tf}	Combined efficiency of the transmission and final drive
θ	Pitch angle
	Angle of a U-joint
θ_p	Body pitch due to acceleration squat or brake dive
Θ	Grade angle
λ	Lateral inclination angle of the steer axis (kingpin inclination angle)
μ	Coefficient of friction
μ_p	Peak coefficient of friction
μ_s	Sliding coefficient of friction
ν	Wavenumber of road roughness spectrum
ξ	Fraction of the drive force developed on the front axle of a 4WD
	Fraction of the brake force developed on the front axle
	Rear steer proportioning factor on a 4WS vehicle
ρ	Density of air
υ	Caster angle of the steer axis
ϕ	Roll angle
φ	Road cross-slope angle
χ	Ratio of unsprung to sprung mass
ψ	Heading angle
	Yaw angle
ω	Rotational speed
ω_d	Damped natural frequency of a suspension system (radians/second)
	Rotational speed of the driveshaft
ω_e	Rotational speed of the engine
ω_i	Rotational speed at the input of a U-joint
ω_n	Undamped natural frequency of a suspension system (radians/second)

ω_o Rotational speed at the output of a U-joint
ω_u Natural frequency of the unsprung mass
ω_w Rotational speed of the wheels

TABLE OF CONTENTS

CHAPTER 1
INTRODUCTION



The next-generation Camaro. (Photo courtesy of Chevrolet Motor Division.)

DAWN OF THE MOTOR VEHICLE AGE

The dawn of the motor vehicle age occurred around 1769 when the French military engineer, Nicholas Joseph Cugnot (1725-1804), built a three-wheeled, steam-driven vehicle for the purpose of pulling artillery pieces [1]. Within a few years an improved model was built, only to cause the first automotive accident when it ran into a wall! This was followed by a steam-powered vehicle built in 1784 by the Scottish engineer, James Watt (1736-1819), which proved unworkable. By 1802, Richard Trevithick (1771-1833), an Englishman, developed a steam coach that traveled from Cornwall to London. The coach met its demise by burning one night after Trevithick forgot to extinguish the boiler fire. Nevertheless, the steam coach business thrived in England until about 1865 when competition from the railroads and strict antispeed laws brought it to an end [2].

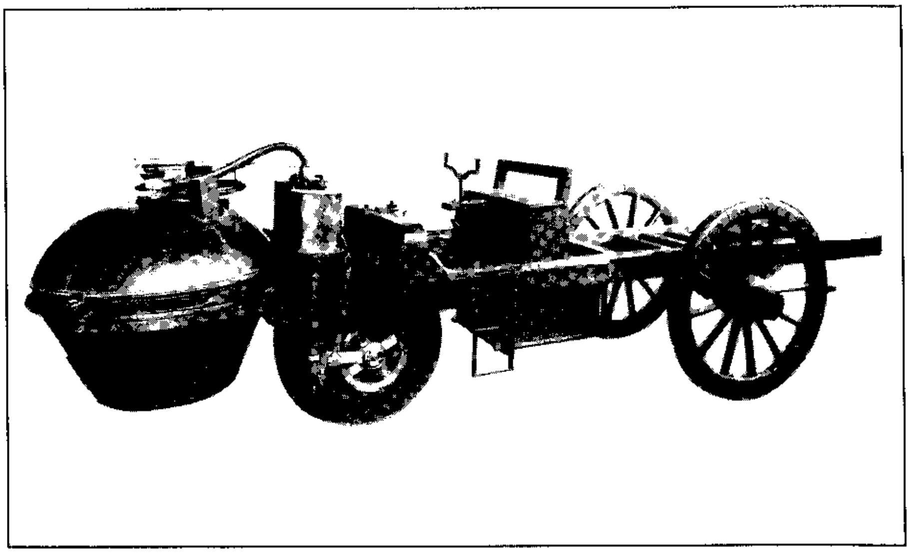


Fig. 1.1 First motor vehicle, circa 1769, built by Cugnot. (Photo courtesy of Smithsonian Institution.)

The first practical automobiles powered by gasoline engines arrived in 1886 with the credit generally going to Karl Benz (1844-1929) and Gottlieb Daimler (1834-1900) working independently. Over the next decade, automotive vehicles were developed by many other pioneers with familiar names such as Rene Panhard, Emile Levassor, Armand Peugeot, Frank and Charles Duryea, Henry Ford, and Ransom Olds. By 1908 the automotive industry was well established in the United States with Henry Ford manufacturing the Model T and the General Motors Corporation being founded. In Europe the familiar companies like Daimler, Opel, Renault, Benz, and Peugeot were becoming recognized as automotive manufacturers. By 1909, over 600 makes of American cars had been identified [3].

In the early decades of the 1900s, most of the engineering energy of the automotive industry went into invention and design that would yield faster, more comfortable, and more reliable vehicles. The speed capability of motor vehicles climbed quickly in the embryonic industry as illustrated by the top speeds of some typical production cars, as shown in Figure 1.2.

In general, motor vehicles achieved high speed capability well before good paved roads existed on which to use it. With higher speeds the dynamics of the vehicles, particularly turning and braking, assumed greater importance as an engineering concern. The status of automotive engineering during this period was characterized in the reminiscences of Maurice Olley [4] as follows:

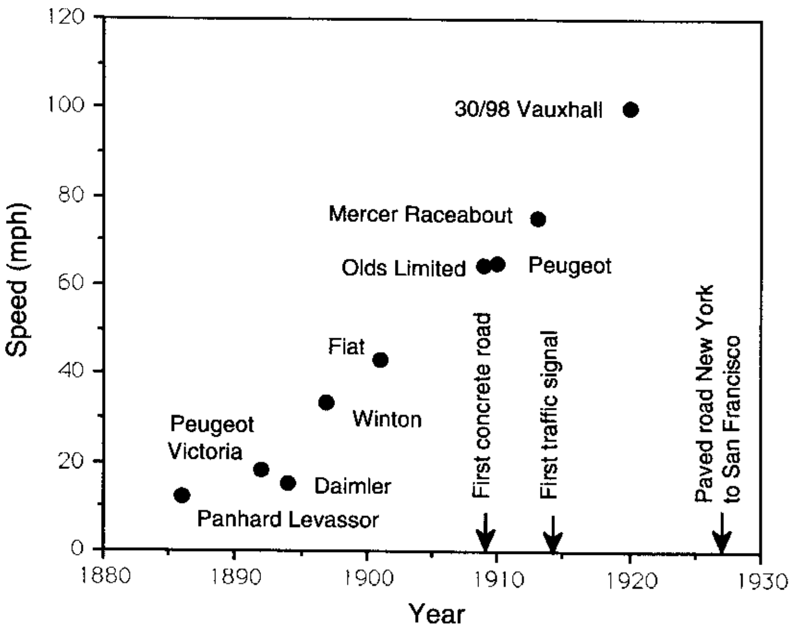


Fig. 1.2 Travel speeds of production automobiles.

> *"There had been sporadic attempts to make the vehicle ride decently, but little had been done. The rear passengers still functioned as ballast, stuck out behind the rear wheels. Steering was frequently unstable and the front axle with front brakes made shimmy almost inevitable. The engineers had made all the parts function excellently, but when put together the whole was seldom satisfactory."*

One of the first engineers to write on automotive dynamics was Frederick William Lanchester (1868-1946). (In a 1908 paper [5] he observed that a car with tiller steering "oversteers" if the centrifugal force on the driver's hands pushes toward greater steer angle [6].) Steering shimmy problems were prevalent at that time as well [7, 8]. But, as described by Segel [6], the understanding of both turning behavior and the shimmy problems was hampered by a lack of knowledge about tire mechanics in these early years.

In 1931, a test device—a tire dynamometer—was built which could measure the necessary mechanical properties of the pneumatic tire for the understandings to be developed [9]. Only then could engineers like Lanchester [10], Olley [11], Rieckert and Schunk [12], Rocard [13], Segel [14] and others develop mechanistic explanations of the turning behavior of automobiles which lays the groundwork for much of our understanding today.

The industry has now completed its first century. Engineers have achieved dramatic advancements in the technologies employed in automobiles from the Model T to the Taurus (Figure 1.3). More than ever, dynamics plays an important role in vehicle design and development. A number of textbooks have been written to help the engineer in this discipline [15 - 24], but there remains a need for books that lay out the fundamental aspects of vehicle dynamics. This book attempts to fill that need.

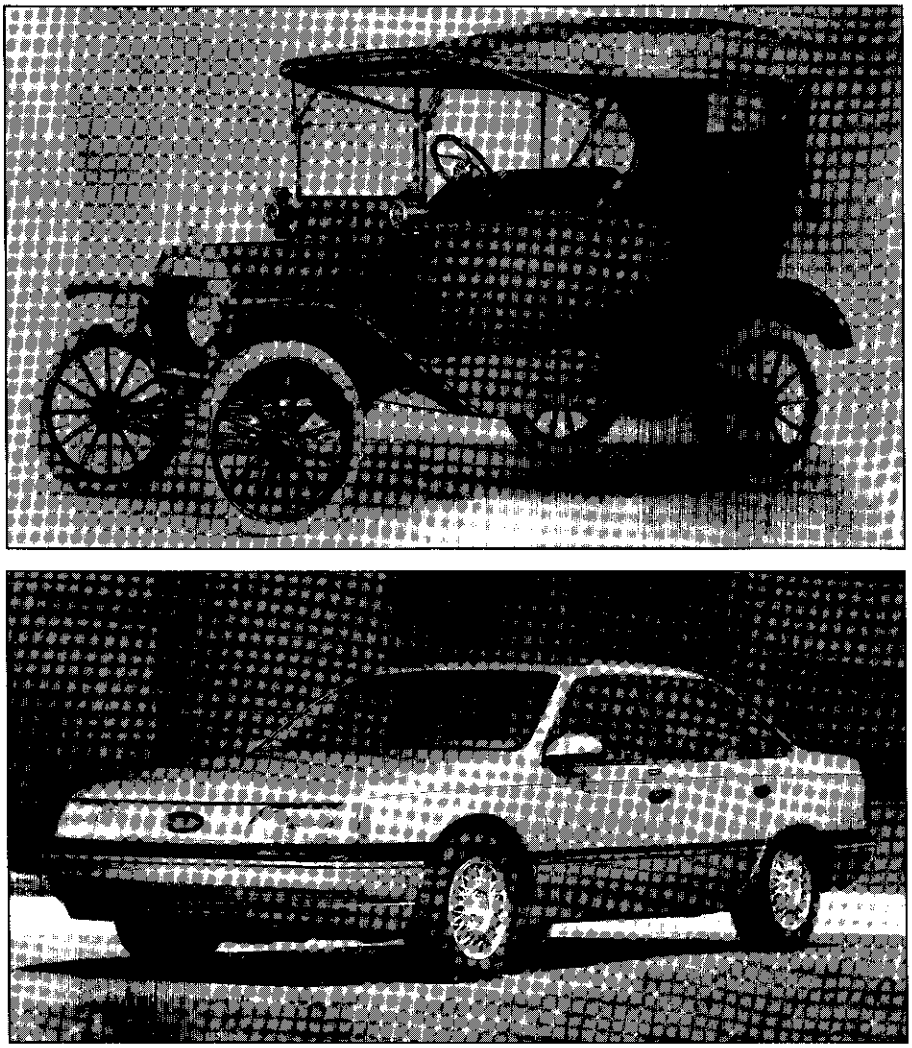


Fig. 1.3 Eighty years of progress from the Model T to the Taurus. (Photos courtesy of Henry Ford Museum and Ford Motor Company.)

INTRODUCTION TO VEHICLE DYNAMICS

It has often been said that the primary forces by which a high-speed motor vehicle is controlled are developed in four patches—each the size of a man's hand—where the tires contact the road. This is indeed the case. A knowledge of the forces and moments generated by pneumatic (rubber) tires at the ground is essential to understanding highway vehicle dynamics. Vehicle dynamics in its broadest sense encompasses all forms of conveyance—ships, airplanes, railroad trains, track-laying vehicles, as well as rubber-tired vehicles. The principles involved in the dynamics of these many types of vehicles are diverse and extensive. Therefore, this book focuses only on rubber-tired vehicles. Most of the discussion and examples will concentrate on the automobile, although the principles are directly applicable to trucks and buses, large and small. Where warranted, trucks will be discussed separately when the functional design or performance qualities distinguish them from the automobile.

Inasmuch as the performance of a vehicle—the motions accomplished in accelerating, braking, cornering and ride—is a response to forces imposed, much of the study of vehicle dynamics must involve the study of how and why the forces are produced. The dominant forces acting on a vehicle to control performance are developed by the tire against the road. Thus it becomes necessary to develop an intimate understanding of the behavior of tires, characterized by the forces and moments generated over the broad range of conditions over which they operate. Studying tire performance without a thorough understanding of its significance to the vehicle is unsatisfying, as is the inverse. Therefore, the relevant properties of tires are introduced at appropriate points in the early chapters of the text, while the reader is referred to Chapter 10 for a more comprehensive discussion of tire properties.

At the outset it is worth noting that the term "handling" is often used interchangeably with cornering, turning, or directional response, but there are nuances of difference between these terms. Cornering, turning, and directional response refer to objective properties of the vehicle when changing direction and sustaining lateral acceleration in the process. For example, cornering ability may be quantified by the level of lateral acceleration that can be sustained in a stable condition, or directional response may be quantified by the time required for lateral acceleration to develop following a steering input. Handling, on the other hand, adds to this the vehicle qualities that feed back to the driver affecting the ease of the driving task or affecting the driver's ability to maintain control. Handling implies, then, not only the vehicle's explicit capabilities, but its contributions as well to the system performance of the driver/vehicle combination. Throughout the book the various terms will be used in a manner most appropriate to the discussion at hand.

Understanding vehicle dynamics can be accomplished at two levels—the empirical and the analytical. The empirical understanding derives from trial and error by which one learns which factors influence vehicle performance, in which way, and under what conditions. The empirical method, however, can often lead to failure. Without a mechanistic understanding of how changes in vehicle design or properties affect performance, extrapolating past experience to new conditions may involve unknown factors which may produce a new result, defying the prevailing rules of thumb. For this reason (and because they are methodical by nature), engineers favor the analytical approach. The analytical approach attempts to describe the mechanics of interest based on the known laws of physics so that an analytical model can be established. In the simpler cases these models can be represented by algebraic or differential equations that relate forces or motions of interest to control inputs and vehicle or tire properties. These equations then allow one to evaluate the role of each vehicle property in the phenomenon of interest. The existence of the model thereby provides a means to identify the important factors, the way in which they operate, and under what conditions. The model provides a predictive capability as well, so that changes necessary to reach a given performance goal can be identified.

It might be noted at this point that analytical methods also are not foolproof because they usually only approximate reality. As many have experienced, the assumptions that must be made to obtain manageable models may often prove fatal to an application of the analysis, and on occasion engineers have been found to be wrong. Therefore, it is very important for the engineer to understand the assumptions that have been made in modeling any aspect of dynamics to avoid these errors.

In the past, many of the shortcomings of analytical methods were a consequence of the mathematical limitations in solving problems. Before the advent of computers, analysis was only considered successful if the "problem" could be reduced to a closed form solution. That is, only if the mathematical expression could be manipulated to a form which allowed the analyst to extract relationships between the variables of interest. To a large extent this limited the functionality of the analytical approach to solution of problems in vehicle dynamics. The existence of large numbers of components, systems, subsystems, and nonlinearities in vehicles made comprehensive modeling virtually impossible, and the only utility obtained came from rather simplistic models of certain mechanical systems. Though useful, the simplicity of the models often constituted deficiencies that handicapped the engineering approach in vehicle development.

Today with the computational power available in desktop and mainframe computers, a major shortcoming of the analytical method has been overcome. It is now possible to assemble models (equations) for the behavior of individual components of a vehicle that can be integrated into comprehensive models of the overall vehicle, allowing simulation and evaluation of its behavior before being rendered in hardware. Such models can calculate performance that could not be solved for in the past. In cases where the engineer is uncertain of the importance of specific properties, those properties can be included in the model and their importance assessed by evaluating their influence on simulated behavior. This provides the engineer with a powerful new tool as a means to test our understanding of a complex system and investigate means of improving performance. In the end we are forced to confront all the variables that may influence the performance of interest, and recognize everything that is important.

FUNDAMENTAL APPROACH TO MODELING

The subject of "vehicle dynamics" is concerned with the movements of vehicles—automobiles, trucks, buses, and special-purpose vehicles—on a road surface. The movements of interest are acceleration and braking, ride, and turning. Dynamic behavior is determined by the forces imposed on the vehicle from the tires, gravity, and aerodynamics. The vehicle and its components are studied to determine what forces will be produced by each of these sources at a particular maneuver and trim condition, and how the vehicle will respond to these forces. For that purpose it is essential to establish a rigorous approach to modeling the systems and the conventions that will be used to describe motions.

Lumped Mass

A motor vehicle is made up of many components distributed within its exterior envelope. Yet, for many of the more elementary analyses applied to it, all components move together. For example, under braking, the entire vehicle slows down as a unit; thus it can be represented as one lumped mass located at its center of gravity (CG) with appropriate mass and inertia properties. For acceleration, braking, and most turning analyses, one mass is sufficient. For ride analysis, it is often necessary to treat the wheels as separate lumped masses. In that case the lumped mass representing the body is the "sprung mass," and the wheels are denoted as "unsprung masses."

For single mass representation, the vehicle is treated as a mass concentrated at its center of gravity (CG) as shown in Figure 1.4. The point mass at the CG, with appropriate rotational moments of inertia, is dynamically equivalent to the vehicle itself for all motions in which it is reasonable to assume the vehicle to be rigid.

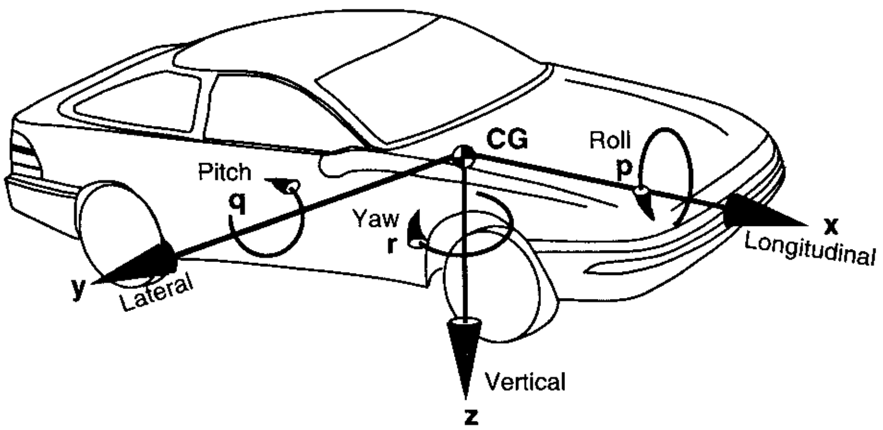


Fig. 1.4 SAE Vehicle Axis System.

Vehicle Fixed Coordinate System

On-board, the vehicle motions are defined with reference to a right-hand orthogonal coordinate system (the vehicle fixed coordinate system) which originates at the CG and travels with the vehicle. By SAE convention [25] the coordinates are:

x - Forward and on the longitudinal plane of symmetry

y - Lateral out the right side of the vehicle

z - Downward with respect to the vehicle

p - Roll velocity about the x axis

q - Pitch velocity about the y axis

r - Yaw velocity about the z axis

Motion Variables

Vehicle motion is usually described by the velocities (forward, lateral, vertical, roll, pitch and yaw) with respect to the vehicle fixed coordinate system, where the velocities are referenced to the earth fixed coordinate system.

sample content of Fundamentals of Vehicle Dynamics (R114) (Premiere Series Books)

- *read online Elusive Summits: Four expeditions in the Karakoram pdf, azw (kindle)*
- download online Gold: The Once and Future Money
- **read online Leading at a Higher Level: Blanchard on Leadership and Creating High Performing Organizations (Revised and Expanded Edition) book**
- download Leading at the Edge: Leadership Lessons from the Extraordinary Saga of Shackleton's Antarctic Expedition (2nd Edition)
- download online Big City Girl

- http://aircon.servicessingaporecompany.com/?lib/A-Wreath-for-Emmett-Till.pdf
- http://www.shreesaiexport.com/library/Gold--The-Once-and-Future-Money.pdf
- http://qolorea.com/library/Hipnosis-y-autohipnosis.pdf
- http://www.satilik-kopek.com/library/Leading-at-the-Edge--Leadership-Lessons-from-the-Extraordinary-Saga-of-Shackleton-s-Antarctic-Expedition--2nd-E
- http://studystrategically.com/freebooks/Big-City-Girl.pdf