

# Fundamentals of Vehicle Dynamics

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## LIST OF SYMBOLS

$a$	Tire cornering stiffness parameter
$b$	Tire cornering stiffness parameter
$A$	Frontal area of a vehicle
$A_f$	Lateral force compliance steer coefficient on the front axle
$A_r$	Lateral force compliance steer coefficient on the rear axle
$a_x$	Acceleration in the x-direction
$a_y$	Acceleration in the lateral direction
$b$	Longitudinal distance from front axle to center of gravity
$c$	Longitudinal distance from center of gravity to rear axle
$C_\alpha$	Cornering stiffness of the tires on an axle
$C_\alpha'$	Cornering stiffness of one tire
$CC_\alpha$	Tire cornering coefficient
$C_\gamma$	Tire camber stiffness
$C_D$	Aerodynamic drag coefficient
$C_h$	Road surface rolling resistance coefficient
$C_L$	Aerodynamic lift coefficient
$C_{PM}$	Aerodynamic pitching moment coefficient
$C_{RM}$	Aerodynamic rolling moment coefficient
$C_{YM}$	Aerodynamic yawing moment coefficient
$C_s$	Suspension damping coefficient
$C_S$	Aerodynamic side force coefficient
$CP$	Center of pressure location of aerodynamic side force
$d$	Lateral distance between steering axis and center of tire contact at the ground
$d_h$	Distance from axle to the hitch point
$d_{ns}$	Distance from center of mass to the neutral steer point
$D$	Tire diameter
$DI$	Dynamic index
$D_x$	Linear deceleration
$D_A$	Aerodynamic drag force
$e$	Height of the pivot for an "equivalent torque arm"
	Drum brake geometry factor
$E[y^2]$	Mean square vibration response
$f$	Longitudinal length for an "equivalent torque arm"
$f_a$	Wheel hop resonant frequency (vertical)

## FUNDAMENTALS OF VEHICLE DYNAMICS

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$f_n$	Undamped natural frequency of a suspension system (Hz)
$f_r$	Rolling resistance coefficient
$F_b$	Braking force
	Vertical disturbance force on the sprung mass
$F_i$	Imbalance force in a tire
$F_x$	Force in the x-direction (tractive force)
$F_{xm}$	Maximum brake force on an axle
$F_{xt}$	Total force in the x-direction
$F_y$	Force in the y-direction (lateral force)
	Lateral force on an axle
$F_y'$	Lateral force on one tire
$F_z$	Force in the z-direction (vertical force)
$F_{zi}$	Vertical force on inside tire in a turn
$F_{zo}$	Vertical force on outside tire in a turn
$F_w$	Tire/wheel nonuniformity force on the unsprung mass
$g$	Acceleration of gravity (32.2 ft/sec <sup>2</sup> , 9.81 m/sec <sup>2</sup> )
$G$	Brake gain
$G_o$	Road roughness magnitude parameter
$G_z$	Power spectral density amplitude of road roughness
$G_{zs}$	Power spectral density amplitude of sprung mass acceleration
$h$	Center of gravity height
$h_a$	Height of the aerodynamic drag force
$h_h$	Hitch height
$h_l$	Height of the sprung mass center of gravity above the roll axis
$h_r$	Height of suspension roll center
$h_t$	Tire section height
$HP$	Engine or brake horsepower
$HP_A$	Aerodynamic horsepower
$HP_R$	Rolling resistance horsepower
$HP_{RL}$	Road load horsepower
$H_v$	Response gain function
$I_d$	Moment of inertia of the driveshaft
$I_e$	Moment of inertia of the engine
$I_t$	Moment of inertia of the transmission
$I_w$	Moment of inertia of the wheels
$I_{xx}$	Moment of inertia about the x-axis

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$I_{yy}$	Moment of inertia about the y-axis
$I_{zz}$	Moment of inertia about the z-axis
$k$	Radius of gyration
$K$	Understeer gradient
$K_{at}$	Understeer gradient due to aligning torque
$K_{llt}$	Understeer gradient due to lateral load transfer on the axles
$K_{lfcS}$	Understeer gradient due to lateral force compliance steer
$K_s$	Vertical stiffness of a suspension
$K_{SS}$	Steering system stiffness
$K_{strg}$	Understeer gradient due to the steering system
$K_t$	Vertical stiffness of a tire
$K_\phi$	Suspension roll stiffness
$L$	Wheelbase
$L_A$	Aerodynamic lift force
$m$	Drum brake geometry parameter
$M$	Mass of the vehicle
$M_{AT}$	Moment around the steer axis due to tire aligning torques
$M_L$	Moment around the steer axis due to tire lateral forces
$M_R$	Equivalent mass of the rotating components
$M_{SA}$	Moment around the steer axis due to front-wheel-drive forces and torques
$M_T$	Moment around the steer axis due to tire tractive forces
$M_V$	Moment around the steer axis due to tire vertical forces
$M_\phi$	Rolling moment
$n$	Drum brake geometry parameter
$N$	Normal force
$N_t$	Numerical ratio of the transmission
$N_f$	Numerical ratio of the final drive
$N_{tf}$	Numerical ratio of the combined transmission and final drive
$NSP$	Neutral steer point
$p$	Pneumatic trail
$P_a$	Brake application pressure/effort
$P_{atm}$	Atmospheric pressure
$P_f$	Front brake application pressure
$P_r$	Rear brake application pressure
$P_s$	Static pressure
$P_t$	Total pressure

PM	Aerodynamic pitching moment
$\mathbf{p}$	Roll velocity about the x-axis of the vehicle
$\mathbf{q}$	Pitch velocity about the y-axis of the vehicle
q	Dynamic pressure
$\mathbf{r}$	Yaw velocity about the z-axis of the vehicle
r	Rolling radius of the tires
$r_k$	Ratio of tire to suspension stiffness
R	Radius of turn
$R_h$	Hitch force
$R_g$	Grade force
$R_x$	Rolling resistance force
$R_{RL}$	Road load
RM	Aerodynamic rolling moment
RR	Ride rate of a tire/suspension system
$R_\phi$	Roll rate of the sprung mass
s	Lateral separation between suspension springs
$S_A$	Aerodynamic side force
$S_O$	Spectral density of white-noise
SD	Stopping distance
t	Tread
$t_s$	Length of time of a brake application
$T_a$	Torque in the axle
$T_b$	Brake torque
$T_c$	Torque at the clutch
$T_d$	Torque in the driveshaft
$T_e$	Torque of the engine
$T_{sf}$	Roll torque in a front suspension
$T_{sr}$	Roll torque in a rear suspension
$T_{amb}$	Ambient temperature
$T_x$	Torque about the x-axis
V	Forward velocity
$V_w$	Ambient wind velocity
$V_f$	Final velocity resulting from a brake application
$V_o$	Initial velocity in a brake application
w	Tire section width
W	Weight of the vehicle

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LIST OF SYMBOLS

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$W_a$	Axle weight
$W_d$	Dynamic load transfer
$W_f$	Dynamic weight on the front axle
$W_r$	Dynamic weight on the rear axle
$W_{rr}$	Dynamic weight on the right rear wheel
$W_{fs}$	Static weight on the front axle
$W_{rs}$	Static weight on the rear axle
$W_y$	Lateral weight transfer on an axle
$x$	Forward direction on the longitudinal axis of the vehicle
$y$	Lateral direction out the right side of the vehicle
$YM$	Aerodynamic yawing moment
$z$	Vertical direction with respect to the plane of the vehicle
$X$	Forward direction of travel
$Y$	Lateral direction of travel
$Z$	Vertical direction of travel
	Vertical displacement of the sprung mass
$Z_r$	Road profile elevation
$Z_u$	Vertical displacement of the unsprung mass
$\alpha$	Tire slip angle
	Coefficient in the pitch plane equations
$\alpha_{cw}$	Aerodynamic wind angle
$\alpha_d$	Rotational acceleration of the driveshaft
$\alpha_e$	Rotational acceleration of the engine
$\alpha_w$	Rotational acceleration of the wheels
$\alpha_x$	Rotational acceleration about the x-axis
$\beta$	Sideslip angle
	Rotation angle of a U-joint
	Coefficient in the pitch plane equations
$\gamma$	Camber angle
	Coefficient in the pitch plane equations
$\gamma_g$	Wheel camber with respect to the ground
$\gamma_b$	Wheel camber with respect to the vehicle body
$\delta$	Steer angle
$\delta_c$	Compliance steer
$\delta_i$	Steer angle of the inside wheel in a turn

$\delta_o$	Steer angle of the outside wheel in a turn
$\Delta$	Off-tracking distance in a turn
$\epsilon$	Roll steer coefficient
	Inclination of the roll axis
$\zeta$	Moment arm related to tire force yaw damping
	Half-shaft angle on a front-wheel drive
$\zeta_s$	Damping ratio of the suspension
$\eta_b$	Braking efficiency
$\eta_t$	Efficiency of the transmission
$\eta_f$	Efficiency of the final drive
$\eta_{tf}$	Combined efficiency of the transmission and final drive
$\theta$	Pitch angle
	Angle of a U-joint
$\theta_p$	Body pitch due to acceleration squat or brake dive
$\Theta$	Grade angle
$\lambda$	Lateral inclination angle of the steer axis (kingpin inclination angle)
$\mu$	Coefficient of friction
$\mu_p$	Peak coefficient of friction
$\mu_s$	Sliding coefficient of friction
$\nu$	Wavenumber of road roughness spectrum
$\xi$	Fraction of the drive force developed on the front axle of a 4WD
	Fraction of the brake force developed on the front axle
	Rear steer proportioning factor on a 4WS vehicle
$\rho$	Density of air
$\upsilon$	Caster angle of the steer axis
$\phi$	Roll angle
$\varphi$	Road cross-slope angle
$\chi$	Ratio of unsprung to sprung mass
$\psi$	Heading angle
	Yaw angle
$\omega$	Rotational speed
$\omega_d$	Damped natural frequency of a suspension system (radians/second)
	Rotational speed of the driveshaft
$\omega_e$	Rotational speed of the engine
$\omega_j$	Rotational speed at the input of a U-joint
$\omega_n$	Undamped natural frequency of a suspension system (radians/second)

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$\omega_o$	Rotational speed at the output of a U-joint
$\omega_u$	Natural frequency of the unsprung mass
$\omega_w$	Rotational speed of the wheels



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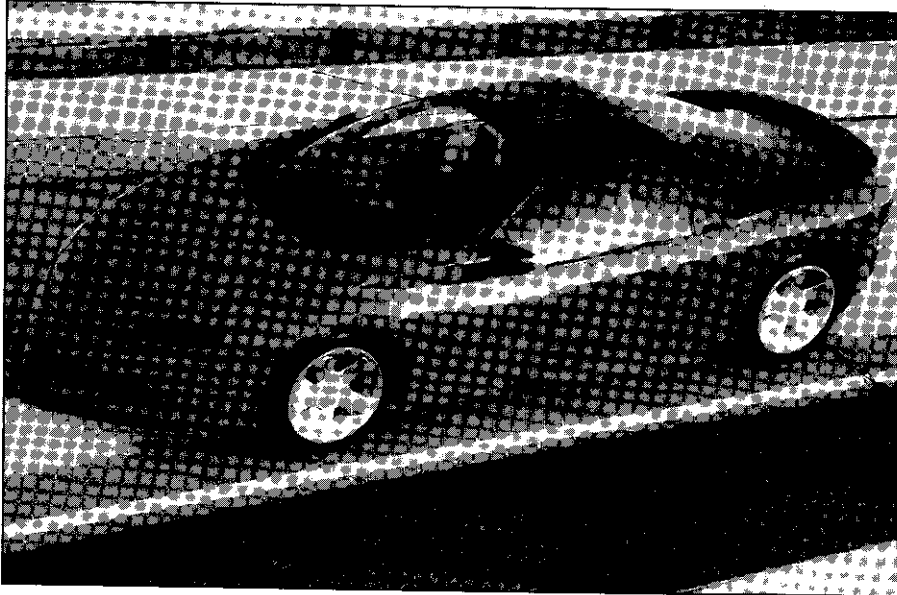
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# CHAPTER 1

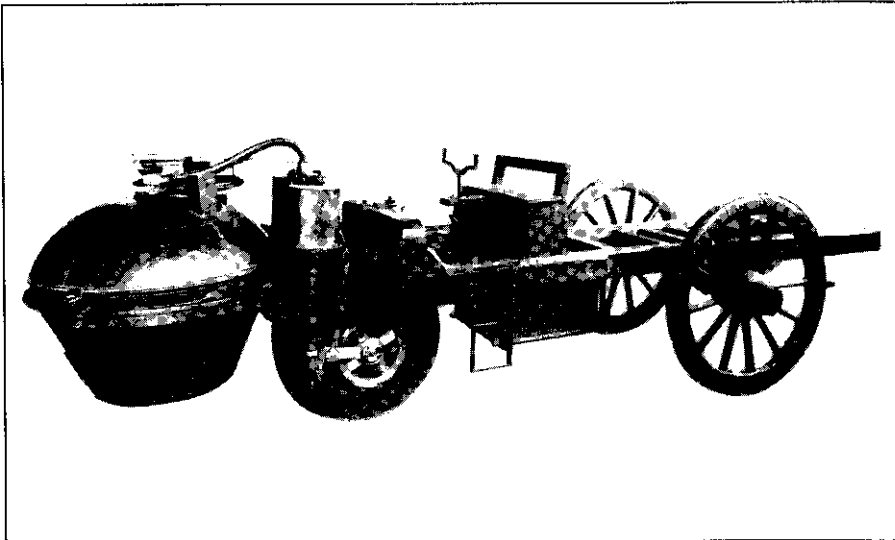
## INTRODUCTION



*The next-generation Camaro. (Photo courtesy of Chevrolet Motor Division.)*

### DAWN OF THE MOTOR VEHICLE AGE

The dawn of the motor vehicle age occurred around 1769 when the French military engineer, Nicholas Joseph Cugnot (1725-1804), built a three-wheeled, steam-driven vehicle for the purpose of pulling artillery pieces [1]. Within a few years an improved model was built, only to cause the first automotive accident when it ran into a wall! This was followed by a steam-powered vehicle built in 1784 by the Scottish engineer, James Watt (1736-1819), which proved unworkable. By 1802, Richard Trevithick (1771-1833), an Englishman, developed a steam coach that traveled from Cornwall to London. The coach met its demise by burning one night after Trevithick forgot to extinguish the boiler fire. Nevertheless, the steam coach business thrived in England until about 1865 when competition from the railroads and strict antispeed laws brought it to an end [2].



*Fig. 1.1 First motor vehicle, circa 1769, built by Cugnot. (Photo courtesy of Smithsonian Institution.)*

The first practical automobiles powered by gasoline engines arrived in 1886 with the credit generally going to Karl Benz (1844-1929) and Gottlieb Daimler (1834-1900) working independently. Over the next decade, automotive vehicles were developed by many other pioneers with familiar names such as Rene Panhard, Emile Levassor, Armand Peugeot, Frank and Charles Duryea, Henry Ford, and Ransom Olds. By 1908 the automotive industry was well established in the United States with Henry Ford manufacturing the Model T and the General Motors Corporation being founded. In Europe the familiar companies like Daimler, Opel, Renault, Benz, and Peugeot were becoming recognized as automotive manufacturers. By 1909, over 600 makes of American cars had been identified [3].

In the early decades of the 1900s, most of the engineering energy of the automotive industry went into invention and design that would yield faster, more comfortable, and more reliable vehicles. The speed capability of motor vehicles climbed quickly in the embryonic industry as illustrated by the top speeds of some typical production cars, as shown in Figure 1.2.

In general, motor vehicles achieved high speed capability well before good paved roads existed on which to use it. With higher speeds the dynamics of the vehicles, particularly turning and braking, assumed greater importance as an engineering concern. The status of automotive engineering during this period was characterized in the reminiscences of Maurice Olley [4] as follows:

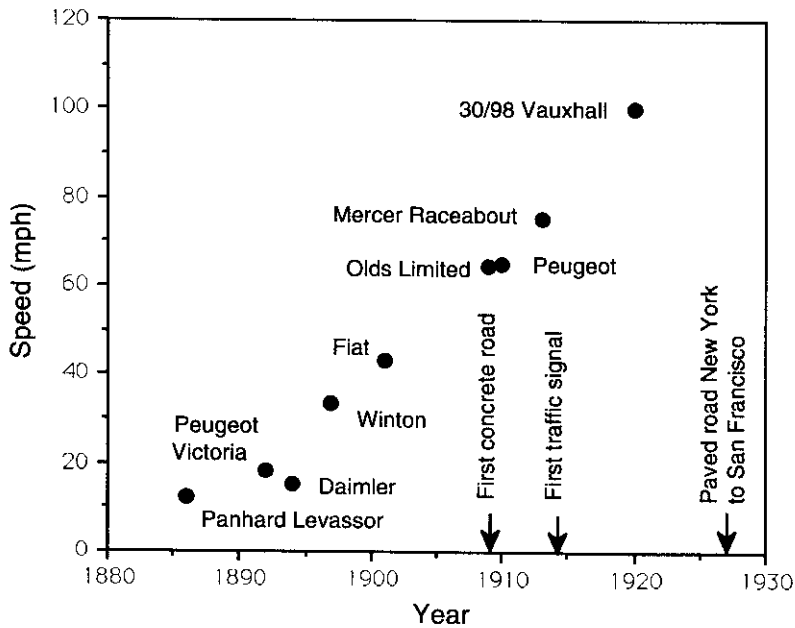


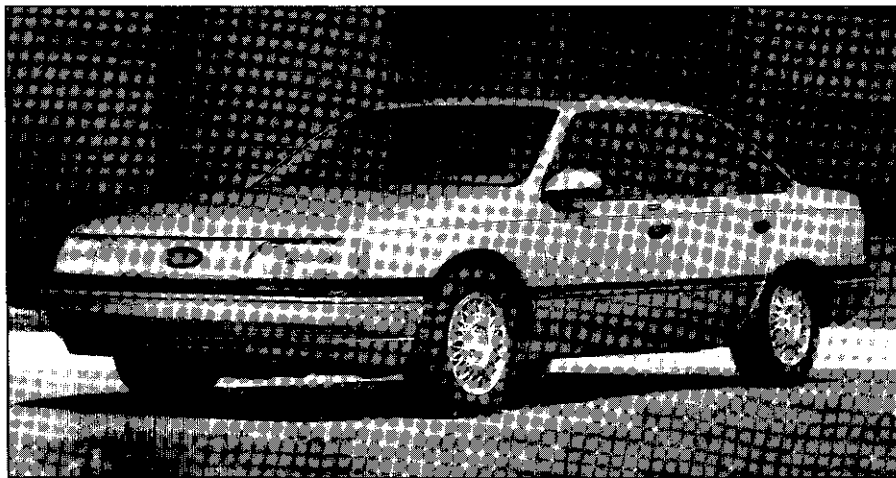
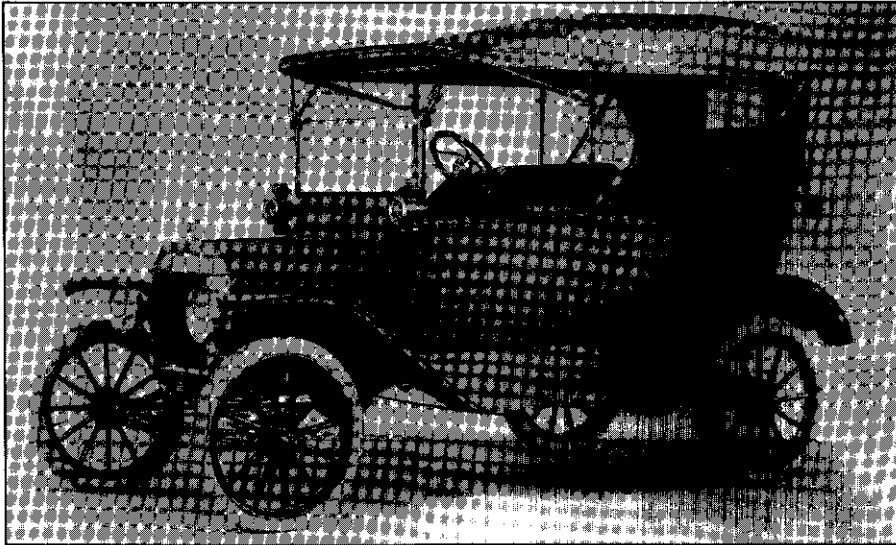
Fig. 1.2 Travel speeds of production automobiles.

*“There had been sporadic attempts to make the vehicle ride decently, but little had been done. The rear passengers still functioned as ballast, stuck out behind the rear wheels. Steering was frequently unstable and the front axle with front brakes made shimmy almost inevitable. The engineers had made all the parts function excellently, but when put together the whole was seldom satisfactory.”*

One of the first engineers to write on automotive dynamics was Frederick William Lanchester (1868-1946). (In a 1908 paper [5] he observed that a car with tiller steering “oversteers” if the centrifugal force on the driver’s hands pushes toward greater steer angle [6].) Steering shimmy problems were prevalent at that time as well [7, 8]. But, as described by Segel [6], the understanding of both turning behavior and the shimmy problems was hampered by a lack of knowledge about tire mechanics in these early years.

In 1931, a test device—a tire dynamometer—was built which could measure the necessary mechanical properties of the pneumatic tire for the understandings to be developed [9]. Only then could engineers like Lanchester [10], Olley [11], Rieckert and Schunk [12], Rocard [13], Segel [14] and others develop mechanistic explanations of the turning behavior of automobiles which lays the groundwork for much of our understanding today.

The industry has now completed its first century. Engineers have achieved dramatic advancements in the technologies employed in automobiles from the Model T to the Taurus (Figure 1.3). More than ever, dynamics plays an important role in vehicle design and development. A number of textbooks have been written to help the engineer in this discipline [15 - 24], but there remains a need for books that lay out the fundamental aspects of vehicle dynamics. This book attempts to fill that need.



*Fig. 1.3 Eighty years of progress from the Model T to the Taurus. (Photos courtesy of Henry Ford Museum and Ford Motor Company.)*

## INTRODUCTION TO VEHICLE DYNAMICS

It has often been said that the primary forces by which a high-speed motor vehicle is controlled are developed in four patches—each the size of a man's hand—where the tires contact the road. This is indeed the case. A knowledge of the forces and moments generated by pneumatic (rubber) tires at the ground is essential to understanding highway vehicle dynamics. Vehicle dynamics in its broadest sense encompasses all forms of conveyance—ships, airplanes, railroad trains, track-laying vehicles, as well as rubber-tired vehicles. The principles involved in the dynamics of these many types of vehicles are diverse and extensive. Therefore, this book focuses only on rubber-tired vehicles. Most of the discussion and examples will concentrate on the automobile, although the principles are directly applicable to trucks and buses, large and small. Where warranted, trucks will be discussed separately when the functional design or performance qualities distinguish them from the automobile.

Inasmuch as the performance of a vehicle—the motions accomplished in accelerating, braking, cornering and ride—is a response to forces imposed, much of the study of vehicle dynamics must involve the study of how and why the forces are produced. The dominant forces acting on a vehicle to control performance are developed by the tire against the road. Thus it becomes necessary to develop an intimate understanding of the behavior of tires, characterized by the forces and moments generated over the broad range of conditions over which they operate. Studying tire performance without a thorough understanding of its significance to the vehicle is unsatisfying, as is the inverse. Therefore, the relevant properties of tires are introduced at appropriate points in the early chapters of the text, while the reader is referred to Chapter 10 for a more comprehensive discussion of tire properties.

At the outset it is worth noting that the term “handling” is often used interchangeably with cornering, turning, or directional response, but there are nuances of difference between these terms. Cornering, turning, and directional response refer to objective properties of the vehicle when changing direction and sustaining lateral acceleration in the process. For example, cornering ability may be quantified by the level of lateral acceleration that can be sustained in a stable condition, or directional response may be quantified by the time required for lateral acceleration to develop following a steering input. Handling, on the other hand, adds to this the vehicle qualities that feed back to the driver affecting the ease of the driving task or affecting the driver's ability to maintain control. Handling implies, then, not only the vehicle's explicit capabilities, but its contributions as well to the system performance of the driver/vehicle combination. Throughout the book the various terms will be used in a manner most appropriate to the discussion at hand.

Understanding vehicle dynamics can be accomplished at two levels—the empirical and the analytical. The empirical understanding derives from trial and error by which one learns which factors influence vehicle performance, in which way, and under what conditions. The empirical method, however, can often lead to failure. Without a mechanistic understanding of how changes in vehicle design or properties affect performance, extrapolating past experience to new conditions may involve unknown factors which may produce a new result, defying the prevailing rules of thumb. For this reason (and because they are methodical by nature), engineers favor the analytical approach. The analytical approach attempts to describe the mechanics of interest based on the known laws of physics so that an analytical model can be established. In the simpler cases these models can be represented by algebraic or differential equations that relate forces or motions of interest to control inputs and vehicle or tire properties. These equations then allow one to evaluate the role of each vehicle property in the phenomenon of interest. The existence of the model thereby provides a means to identify the important factors, the way in which they operate, and under what conditions. The model provides a predictive capability as well, so that changes necessary to reach a given performance goal can be identified.

It might be noted at this point that analytical methods also are not foolproof because they usually only approximate reality. As many have experienced, the assumptions that must be made to obtain manageable models may often prove fatal to an application of the analysis, and on occasion engineers have been found to be wrong. Therefore, it is very important for the engineer to understand the assumptions that have been made in modeling any aspect of dynamics to avoid these errors.

In the past, many of the shortcomings of analytical methods were a consequence of the mathematical limitations in solving problems. Before the advent of computers, analysis was only considered successful if the “problem” could be reduced to a closed form solution. That is, only if the mathematical expression could be manipulated to a form which allowed the analyst to extract relationships between the variables of interest. To a large extent this limited the functionality of the analytical approach to solution of problems in vehicle dynamics. The existence of large numbers of components, systems, sub-systems, and nonlinearities in vehicles made comprehensive modeling virtually impossible, and the only utility obtained came from rather simplistic models of certain mechanical systems. Though useful, the simplicity of the models often constituted deficiencies that handicapped the engineering approach in vehicle development.

Today with the computational power available in desktop and mainframe computers, a major shortcoming of the analytical method has been overcome. It is now possible to assemble models (equations) for the behavior of individual components of a vehicle that can be integrated into comprehensive models of the overall vehicle, allowing simulation and evaluation of its behavior before being rendered in hardware. Such models can calculate performance that could not be solved for in the past. In cases where the engineer is uncertain of the importance of specific properties, those properties can be included in the model and their importance assessed by evaluating their influence on simulated behavior. This provides the engineer with a powerful new tool as a means to test our understanding of a complex system and investigate means of improving performance. In the end we are forced to confront all the variables that may influence the performance of interest, and recognize everything that is important.

## FUNDAMENTAL APPROACH TO MODELING

The subject of “vehicle dynamics” is concerned with the movements of vehicles—automobiles, trucks, buses, and special-purpose vehicles—on a road surface. The movements of interest are acceleration and braking, ride, and turning. Dynamic behavior is determined by the forces imposed on the vehicle from the tires, gravity, and aerodynamics. The vehicle and its components are studied to determine what forces will be produced by each of these sources at a particular maneuver and trim condition, and how the vehicle will respond to these forces. For that purpose it is essential to establish a rigorous approach to modeling the systems and the conventions that will be used to describe motions.

### Lumped Mass

A motor vehicle is made up of many components distributed within its exterior envelope. Yet, for many of the more elementary analyses applied to it, all components move together. For example, under braking, the entire vehicle slows down as a unit; thus it can be represented as one lumped mass located at its center of gravity (CG) with appropriate mass and inertia properties. For acceleration, braking, and most turning analyses, one mass is sufficient. For ride analysis, it is often necessary to treat the wheels as separate lumped masses. In that case the lumped mass representing the body is the “sprung mass,” and the wheels are denoted as “unsprung masses.”

For single mass representation, the vehicle is treated as a mass concentrated at its center of gravity (CG) as shown in Figure 1.4. The point mass at

the CG, with appropriate rotational moments of inertia, is dynamically equivalent to the vehicle itself for all motions in which it is reasonable to assume the vehicle to be rigid.

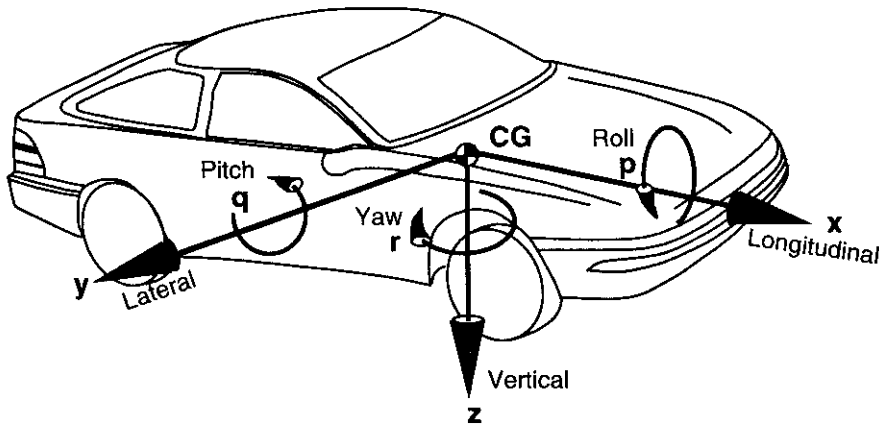


Fig. 1.4 SAE Vehicle Axis System.

### Vehicle Fixed Coordinate System

On-board, the vehicle motions are defined with reference to a right-hand orthogonal coordinate system (the vehicle fixed coordinate system) which originates at the CG and travels with the vehicle. By SAE convention [25] the coordinates are:

- x - Forward and on the longitudinal plane of symmetry
- y - Lateral out the right side of the vehicle
- z - Downward with respect to the vehicle
- p - Roll velocity about the x axis
- q - Pitch velocity about the y axis
- r - Yaw velocity about the z axis

### Motion Variables

Vehicle motion is usually described by the velocities (forward, lateral, vertical, roll, pitch and yaw) with respect to the vehicle fixed coordinate system, where the velocities are referenced to the earth fixed coordinate system.



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