

MANHATTAN

Guide 1

# GMAT

## Fractions, Decimals, & Percents

**Includes 6  
Free Online  
Exams  
& More!**

*Clarifies Concepts and Builds Computation Skills*

*Teaches Problem Solving & Data Sufficiency Strategies*

*Includes Practice Problems with Detailed Explanations*

*Updated for The Official Guide for GMAT® Review, 13th Ed.*

FIFTH EDITION



Eric Caballero, Manhattan GMAT Instructor

**99th Percentile Instructors • Content-Based Curriculum**

GMAT and GMAC are registered trademarks of the Graduate Management Admission Council which neither sponsors nor endorses this product.



# MANHATTAN GMAT

# Fractions, Decimals, & Percents

## GMAT Strategy Guide

This guide provides an in-depth look at the variety of GMAT questions that test your knowledge of fractions, decimals, and percents. Learn to see the connections among these part-whole relationships and practice implementing strategic shortcuts.



*guide* **1**

Fractions, Decimals, & Percents GMAT Strategy Guide, Fifth Edition

10-digit International Standard Book Number: 1-935707-63-9

13-digit International Standard Book Number: 978-1-935707-63-9

eISBN: 978-1-937707-04-0

Copyright © 2012 MG Prep, Inc.

ALL RIGHTS RESERVED. No part of this work may be reproduced or used in any form or by any means—graphic, electronic, or mechanical, including photocopying, recording, taping, web distribution—without the prior written permission of the publisher, MG Prep, Inc.

Note: *GMAT*, *Graduate Management Admission Test*, *Graduate Management Admission Council*, and *GMAC* are all registered trademarks of the Graduate Management Admission Council, which neither sponsors nor is affiliated in any way with this product.

Layout Design: Dan McNaney and Cathy Huang

Cover Design: Eryn Williams and Dan McNaney

Cover Photography: Alli Ugosoli





## ***INSTRUCTIONAL GUIDE SERIES***

---

- |          |   |          |  |
|----------|---|----------|--|
| <b>0</b> | <b>GMAT Roadmap</b><br>(ISBN: 978-1-935707-69-1)                        | <b>5</b> | <b>Number Properties</b><br>(ISBN: 978-1-935707-65-3)                |
| <b>1</b> | <b>Fractions, Decimals, &amp; Percents</b><br>(ISBN: 978-1-935707-63-9) | <b>6</b> | <b>Critical Reasoning</b><br>(ISBN: 978-1-935707-61-5)               |
| <b>2</b> | <b>Algebra</b><br>(ISBN: 978-1-935707-62-2)                             | <b>7</b> | <b>Reading Comprehension</b><br>(ISBN: 978-1-935707-66-0)            |
| <b>3</b> | <b>Word Problems</b><br>(ISBN: 978-1-935707-68-4)                       | <b>8</b> | <b>Sentence Correction</b><br>(ISBN: 978-1-935707-67-7)              |
| <b>4</b> | <b>Geometry</b><br>(ISBN: 978-1-935707-64-6)                            | <b>9</b> | <b>Integrated Reasoning &amp; Essay</b><br>(ISBN: 978-1-935707-83-7) |

## ***SUPPLEMENTAL GUIDE SERIES***

---

### **Math GMAT Supplement Guides**

**Foundations of GMAT Math**

(ISBN: 978-1-935707-59-2)

**Advanced GMAT Quant**

(ISBN: 978-1-935707-15-8)

**Official Guide Companion**

(ISBN: 978-0-984178-01-8)

### **Verbal GMAT Supplement Guides**

**Foundations of GMAT Verbal**

(ISBN: 978-1-935707-01-9)



MANHATTAN  
GMAT

April 24th, 2012

Dear Student,

Thank you for picking up a copy of *Fractions, Decimals, & Percents*. I hope this book provides just the guidance you need to get the most out of your GMAT studies.

As with most accomplishments, there were many people involved in the creation of the book you are holding. First and foremost is Zeke Vanderhoek, the founder of Manhattan GMAT. Zeke was a lone tutor in New York when he started the company in 2000. Now, 12 years later, the company has instructors and offices nationwide and contributes to the studies and successes of thousands of students each year.

Our Manhattan GMAT Strategy Guides are based on the continuing experiences of our instructors and students. For this volume, we are particularly indebted to Dave Mahler and Stacey Koprince. Dave Mahler deserves special recognition for his contributions over the past number of years. Dan McNaney and Cathy Huang provided their design expertise to make the books as user-friendly as possible, and Noah Teitelbaum and Liz Krisher made sure all the moving pieces came together at just the right time. And there's Chris Ryan. Beyond providing additions and edits for this book, Chris continues to be the driving force behind all of our curriculum efforts. His leadership is invaluable. Finally, thank you to all of the Manhattan GMAT students who have provided input and feedback over the years. This book wouldn't be half of what it is without your voice.

At Manhattan GMAT, we continually aspire to provide the best instructors and resources possible. We hope that you will find our commitment manifest in this book. If you have any questions or comments, please email me at [dgonzalez@manhattanprep.com](mailto:dgonzalez@manhattanprep.com). I'll look forward to reading your comments, and I'll be sure to pass them along to our curriculum team.

Thanks again, and best of luck preparing for the GMAT!

Sincerely,

Dan Gonzalez  
President  
Manhattan GMAT

# HOW TO ACCESS YOUR ONLINE RESOURCES

## If you...

### ➤ are a registered Manhattan GMAT student

and have received this book as part of your course materials, you have **AUTOMATIC** access to **ALL** of our online resources. This includes all practice exams, question banks, and online updates to this book. To access these resources, follow the instructions in the Welcome Guide provided to you at the start of your program. Do **NOT** follow the instructions below.



### ➤ purchased this book from the Manhattan GMAT online store or at one of our centers

1. Go to: <http://www.manhattangmat.com/practicecenter.cfm>.
2. Log in using the username and password used when your account was set up.



### ➤ purchased this book at a retail location

1. Create an account with Manhattan GMAT at the website: <https://www.manhattangmat.com/createaccount.cfm>.
2. Go to: <http://www.manhattangmat.com/access.cfm>.
3. Follow the instructions on the screen.

Your one year of online access begins on the day that you register your book at the above URL.

You only need to register your product **ONCE** at the above URL. To use your online resources any time **AFTER** you have completed the registration process, log in to the following URL: <http://www.manhattangmat.com/practicecenter.cfm>.

Please note that online access is nontransferable. This means that only **NEW** and **UNREGISTERED** copies of the book will grant you online access. Previously used books will **NOT** provide any online resources.



### ➤ purchased an eBook version of this book

1. Create an account with Manhattan GMAT at the website: <https://www.manhattangmat.com/createaccount.cfm>.
2. Email a copy of your purchase receipt to [books@manhattangmat.com](mailto:books@manhattangmat.com) to activate your resources. Please be sure to use the same email address to create an account that you used to purchase the eBook.



For any technical issues, email [books@manhattangmat.com](mailto:books@manhattangmat.com) or call 800-576-4628.

Please refer to the following page for a description of the online resources that come with this book.



# YOUR ONLINE RESOURCES

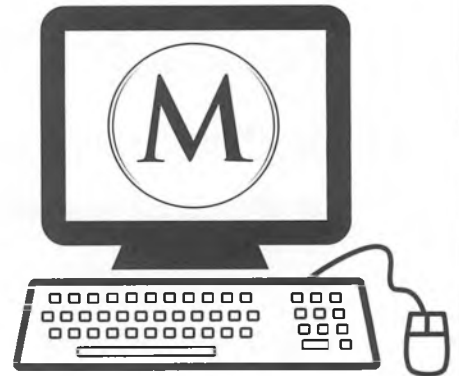
Your purchase includes **ONLINE ACCESS** to the following:

## ➤ 6 Computer-Adaptive Online Practice Exams

The 6 full-length computer-adaptive practice exams included with the purchase of this book are delivered online using Manhattan GMAT's proprietary computer-adaptive test engine. The exams adapt to your ability level by drawing from a bank of more than 1,200 unique questions of varying difficulty levels written by Manhattan GMAT's expert instructors, all of whom have scored in the 99th percentile on the Official GMAT. At the end of each exam you will receive a score, an analysis of your results, and the opportunity to review detailed explanations for each question. You may choose to take the exams timed or untimed.

The content presented in this book is updated periodically to ensure that it reflects the GMAT's most current trends and is as accurate as possible. You may view any known errors or minor changes upon registering for online access.

**Important Note:** The 6 computer adaptive online exams included with the purchase of this book are the SAME exams that you receive upon purchasing ANY book in the Manhattan GMAT Complete Strategy Guide Set.



## ➤ *Fractions, Decimals, & Percents* Online Question Bank

The Bonus Online Question Bank for *Fractions, Decimals, & Percents* consists of 25 extra practice questions (with detailed explanations) that test the variety of concepts and skills covered in this book. These questions provide you with extra practice beyond the problem sets contained in this book. You may use our online timer to practice your pacing by setting time limits for each question in the bank.

## ➤ Online Updates to the Contents in this Book

The content presented in this book is updated periodically to ensure that it reflects the GMAT's most current trends. You may view all updates, including any known errors or changes, upon registering for online access.

# TABLE *of* CONTENTS

<b>1. Digits &amp; Decimals</b>	<b>11</b>
Problem Set	21
<b>2. Fractions</b>	<b>25</b>
Problem Set	37
<b>3. Percents</b>	<b>41</b>
Problem Set	51
<b>4. Ratios</b>	<b>61</b>
Problem Set	69
<b>5. FDPs</b>	<b>73</b>
Problem Set	79
<b>6. FDP Strategies</b>	<b>83</b>
Problem Set	99
<b>7. Extra FDPs</b>	<b>107</b>
Problem Set	119
<b>Appendix A: Official Guide Problem Sets</b>	<b>125</b>

*guide* **1**



# Chapter 1

*of*

**Fractions, Decimals, & Percents**

**Digits & Decimals**

# *In This Chapter...*

*Digits*

*Decimals*

*Place Value*

*Rounding to the Nearest Place Value*

*Adding Zeroes to Decimals*

*Powers of 10: Shifting the Decimal*

*Decimal Operations*

# Chapter 1:

## Digits & Decimals

### Digits

---

Every number is composed of digits. There are only ten digits in our number system: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The term digit refers to one building block of a number; it does not refer to a number itself. For example, 356 is a number composed of three digits: 3, 5, and 6.

Integers can be classified by the number of digits they contain. For example:

2, 7, and  $-8$  are each single-digit numbers (they are each composed of one digit).

43, 63, and  $-14$  are each double-digit numbers (composed of two digits).

500,000 and  $-468,024$  are each six-digit numbers (composed of six digits).

789,526,622 is a nine-digit number (composed of nine digits).

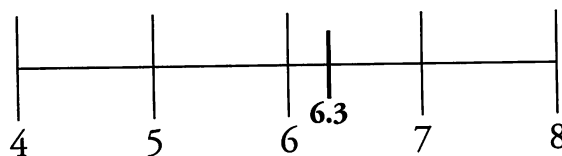
Non-integers are not generally classified by the number of digits they contain, since you can always add any number of zeroes at the end, on the right side of the decimal point:

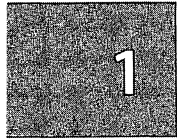
$$9.1 = 9.10 = 9.100$$

### Decimals

---

GMAT math goes beyond an understanding of the properties of integers (which include the counting numbers, such as 1, 2, 3, and their negative counterparts, such as  $-1$ ,  $-2$ ,  $-3$ , and 0). The GMAT also tests your ability to understand the numbers that fall in between the integers. Such numbers can be expressed as decimals. For example, the decimal 6.3 falls between the integers 6 and 7:





Some other examples of decimals include:

- Decimals less than -1: -3.65, -12.01, -145.9
- Decimals between -1 and 0: -0.65, -0.8912, -0.076
- Decimals between 0 and 1: 0.65, 0.8912, 0.076
- Decimals greater than 1: 3.65, 12.01, 145.9

Note that an integer can be expressed as a decimal by adding the decimal point and the digit 0. For example:

$$8 = 8.0 \qquad -123 = -123.0 \qquad 400 = 400.0$$

## Place Value

Every digit in a number has a particular place value depending on its location within the number. For example, in the number 452, the digit 2 is in the ones (or “units”) place, the digit 5 is in the tens place, and the digit 4 is in the hundreds place. The name of each location corresponds to the “value” of that place. Thus:

- 2 is worth two “units” (two “ones”), or  $2 (= 2 \times 1)$ ;
- 5 is worth five tens, or  $50 (= 5 \times 10)$ ; and
- 4 is worth four hundreds, or  $400 (= 4 \times 100)$ .

You can now write the number 452 as the **sum** of these products:

$$452 = 4 \times 100 + 5 \times 10 + 2 \times 1$$

6	9	2	5	6	7	8	9	1	0	2	3	.	8	3	4	7
H	T	O	H	T	O	H	T		H	T	U		T	H	T	T
U	E	N	U	E	N	U	E		U	E	N		E	U	H	E
N	N	E	N	N	E	N	N		N	N	I		N	N	O	N
D			D			D			D	S			T	D	U	
R			R			R			R		O		H	R	S	
E			E			E			E		R		S	E	A	T
D			D			D			D		S		S	D	N	H
	B	B	M	M	M	T	T	T			O			S	H	S
	I	I	I	I	I	H	H	H			N			S	S	A
	L	L	L	L	L	O	O	O			E			S	S	N
	L	L	L	L	L	U	U	U			S			S	S	D
	I	I	I	I	I	S	S	S			O			S	S	T
	O	O	O	O	O	A	A	A			N			S	S	H
	N	N	N	N	N	N	N	N			S			S	S	S
	S	S	S	S	S	D	D	D			S			S	S	S

The chart to the left analyzes the place value of all the digits in the number **692,567,891,023.8347**.

Notice that the place values to the left of the decimal all end in “-s,” while the place values to the right of the decimal all end in “-ths.” This is because the suffix “-ths” gives these places (to the right of the decimal) a fractional value.



Analyze the end of the preceding number: **0.8347**:

8 is in the tenths place, giving it a value of 8 tenths, or  $\frac{8}{10}$ .

3 is in the hundredths place, giving it a value of 3 hundredths, or  $\frac{3}{100}$ .

4 is in the thousandths place, giving it a value of 4 thousandths, or  $\frac{4}{1,000}$ .

7 is in the ten thousandths place, giving it a value of 7 ten thousandths, or  $\frac{7}{10,000}$ .

To use a concrete example, 0.8 might mean eight tenths of one dollar, which would be 8 dimes or 80 cents. Additionally, 0.03 might mean three hundredths of one dollar, which would be 3 pennies or 3 cents.

## Rounding to the Nearest Place Value

The GMAT occasionally requires you to round a number to a specific place value.

What is 3.681 rounded to the nearest tenth?

First, find the digit located in the specified place value. The digit 6 is in the tenths place.

Second, look at the right-digit-neighbor (the digit immediately to the right) of the digit in question. In this case, 8 is the right-digit-neighbor of 6. If the right-digit-neighbor is 5 or greater, round the digit in question UP. Otherwise, leave the digit alone. In this case, since 8 is greater than 5 the digit in question (6) must be rounded up to 7. Thus, 3.681 rounded to the nearest tenth equals 3.7. Note that all the digits to the right of the right-digit-neighbor are irrelevant when rounding.

Rounding appears on the GMAT in the form of questions such as this:

If  $x$  is the decimal  $8.1d5$ , with  $d$  as an unknown digit, and  $x$  rounded to the nearest tenth is equal to 8.1, which digits could not be the value of  $d$ ?

In order for  $x$  to be 8.1 when rounded to the nearest tenth, the right-digit-neighbor,  $d$ , must be less than 5. Therefore,  $d$  cannot be 5, 6, 7, 8 or 9.

1

## Adding Zeroes to Decimals

Adding zeroes to the end of a decimal or taking zeroes away from the end of a decimal does not change the value of the decimal. For example:

$$3.6 = 3.60 = 3.6000$$

Be careful, however, not to add or remove any zeroes from within a number. Doing so will change the value of the number:

$$7.01 \neq 7.1$$

## Powers of 10: Shifting the Decimal

Place values continually decrease from left to right by powers of 10. Understanding this can help you understand the following shortcuts for multiplication and division.

<b>In words</b>	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
<b>In numbers</b>	1,000	100	10	1	0.1	0.01	0.001
<b>In powers of ten</b>	$10^3$	$10^2$	$10^1$	$10^0$	$10^{-1}$	$10^{-2}$	$10^{-3}$

When you multiply any number by a positive power of 10, move the decimal to the right the specified number of places. This makes positive numbers larger:

$$3.9742 \times 10^3 = 3,974.2 \quad \text{Move the decimal to the right 3 spaces.}$$

$$89.507 \times 10 = 895.07 \quad \text{Move the decimal to the right 1 space.}$$

When you divide any number by a positive power of 10, move the decimal to the left the specified number of places. This makes positive numbers smaller:

$$4,169.2 \div 10^2 = 41.692 \quad \text{Move the decimal to the left 2 spaces.}$$

$$89.507 \div 10 = 8.9507 \quad \text{Move the decimal to the left 1 space.}$$

Note that if you need to add zeroes in order to shift a decimal, you should do so:

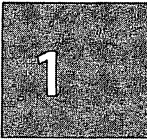
$$2.57 \times 10^6 = 2,570,000 \quad \text{Add 4 zeroes at the end.}$$

$$14.29 \div 10^5 = 0.0001429 \quad \text{Add 3 zeroes at the beginning.}$$

Finally, note that negative powers of 10 reverse the regular process:

$$6,782.01 \times 10^{-3} = 6.78201 \quad 53.0447 \div 10^{-2} = 5,304.47$$

You can think about these processes as **trading decimal places for powers of 10**.



For instance, all of the following numbers equal 110,700:

$$\begin{array}{rcl} 110.7 & \times & 10^3 \\ 11.07 & \times & 10^4 \\ 1.107 & \times & 10^5 \\ 0.1107 & \times & 10^6 \\ 0.01107 & \times & 10^7 \end{array}$$

The first number gets smaller by a factor of 10 as you move the decimal one place to the left, but the second number gets bigger by a factor of 10 to compensate.

## Decimal Operations

---

### Addition & Subtraction

To add or subtract decimals, make sure to line up the decimal points. Then add zeroes to make the right sides of the decimals the same length:

$$4.319 + 221.8$$

$$\begin{array}{r} \text{Line up the} \\ \text{decimal points} \\ \text{and add zeroes.} \end{array} \quad \begin{array}{r} 4.319 \\ + 221.800 \\ \hline 226.119 \end{array}$$

$$10 - 0.063$$

$$\begin{array}{r} \text{Line up the} \\ \text{decimal points} \\ \text{and add zeroes.} \end{array} \quad \begin{array}{r} 10.000 \\ - 0.063 \\ \hline 9.937 \end{array}$$

**Addition and subtraction: Line up the decimal points!**

### Multiplication

To multiply decimals, ignore the decimal point until the end. Just multiply the numbers as you would if they were whole numbers. Then count the total number of digits to the right of the decimal point in the factors. The product should have the same number of digits to the right of the decimal point.

$$0.02 \times 1.4$$

$$\begin{array}{r} \text{Multiply normally:} \\ 14 \\ \times 2 \\ \hline 28 \end{array}$$

There are 3 digits to the right of the decimal point in the factors (0 and 2 in the first factor and 4 in the second factor). Therefore, move the decimal point 3 places to the left in the product:  $28 \rightarrow 0.028$ .

**Multiplication: In the factors, count all the digits to the right of the decimal point—then put that many digits to the right of the decimal point in the product.**

If the product ends with 0, count it in this process:  $0.8 \times 0.5 = 0.40$ , since  $8 \times 5 = 40$ .



1

If you are multiplying a very large number and a very small number, the following trick works to simplify the calculation: move the decimals **in the opposite direction** the same number of places.

$$0.0003 \times 40,000 = ?$$

Move the decimal point **right** four places on the 0.0003  $\rightarrow$  3

Move the decimal point **left** four places on the 40,000  $\rightarrow$  4

$$0.0003 \times 40,000 = 3 \times 4 = 12$$

The reason this technique works is that you are multiplying and then dividing by the same power of 10. In other words, you are **trading decimal places** in one number for decimal places in another number. This is just like trading decimal places for powers of 10, as you saw earlier.

## Division

If there is a decimal point in the dividend (the inner number) only, you can simply bring the decimal point straight up to the answer and divide normally:

$$12.42 \div 3 = 4.14$$

$$\begin{array}{r} 4.14 \\ 3 \overline{)12.42} \\ \underline{12} \phantom{00} \\ 04 \phantom{00} \\ \underline{3} \phantom{00} \\ 12 \phantom{00} \\ \underline{12} \phantom{00} \\ 00 \phantom{00} \end{array}$$

However, if there is a decimal point in the divisor (the outer number), you should shift the decimal point in both the divisor and the dividend to make the *divisor* a whole number. Then, bring the decimal point up and divide:

$$12.42 \div 0.3 \rightarrow 124.2 \div 3 = 41.4$$

$$\begin{array}{r} 41.4 \\ 3 \overline{)124.2} \\ \underline{12} \phantom{00} \\ 04 \phantom{00} \\ \underline{3} \phantom{00} \\ 12 \phantom{00} \\ \underline{12} \phantom{00} \\ 00 \phantom{00} \end{array}$$

Move the decimal one space to the right to make 0.3 a whole number. Then, move the decimal one space to the right in 12.42 to make it 124.2.

**Division: Divide by whole numbers!**



You can always simplify division problems that involve decimals by shifting the decimal point **in the same direction** in both the divisor and the dividend, even when the division problem is expressed as a fraction:

$$\frac{0.0045}{0.09} = \frac{45}{900}$$

Move the decimal 4 spaces to the right to make both the numerator and the denominator whole numbers.

Note that this is essentially the same process as simplifying a fraction. You are simply multiplying the numerator and denominator of the fraction by a power of 10—in this case,  $10^4$ , or 10,000.

Keep track of how you move the decimal point! To simplify multiplication, you can move decimals in opposite directions. But to simplify division, you move decimals in the same direction.

Equivalently, by adding zeroes, you can express the numerator and the denominator as the same units, then simplify:

$$\frac{0.0045}{0.09} = \frac{45}{900} = 45 \text{ ten thousandths} \div 900 \text{ ten thousandths} = \frac{45}{900} = \frac{5}{100} = 0.05$$

## Powers and Roots

To square or cube a decimal, you can always simply multiply it by itself once or twice. However, to raise a decimal to a larger power, you can rewrite the decimal as the product of an integer and a power of 10, and then apply the exponent:

$$(0.5)^4 = ?$$

$$0.5 = 5 \times 10^{-1}$$

Rewrite the decimal.

$$(5 \times 10^{-1})^4 = 5^4 \times 10^{-4}$$

Apply the exponent to each part.

$$5^4 = 25^2 = 625$$

Compute the first part and combine.

$$625 \times 10^{-4} = 0.0625$$

Solve for roots of decimals the same way. Recall that a root is a number raised to a fractional power: a square root is a number raised to the  $1/2$  power, a cube root is a number raised to the  $1/3$  power, etc.:

$$\sqrt[3]{0.000027} = ?$$

Rewrite the decimal. Make the first number something you can take the cube root of easily:

$$0.000027 = 27 \times 10^{-6}$$

$$(0.000027)^{1/3} = (27 \times 10^{-6})^{1/3}$$

Write the root as a fractional exponent.

$$(27)^{1/3} \times (10^{-6})^{1/3} = (27)^{1/3} \times 10^{-2}$$

Apply the exponent to each part.

$$(27)^{1/3} = 3 \quad (\text{since } 3^3 = 27)$$

Compute the first part and combine.

$$3 \times 10^{-2} = 0.03$$

1

**Powers and roots: Rewrite the decimal using powers of 10!**

Once you understand the principles, you can take a shortcut by counting decimal places. For instance, the number of decimal places in the result of a cubed decimal is 3 times the number of decimal places in the original decimal:

$$(0.04)^3 = 0.000064 \qquad (0.04)^3 \qquad = 0.000064$$

*2 places* *2 × 3 = 6 places*

Likewise, the number of decimal places in a cube root is  $1/3$  the number of decimal places in the original decimal:

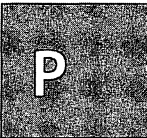
$$\sqrt[3]{0.000000008} = 0.002 \qquad \sqrt[3]{0.000000008} \qquad = 0.002$$

*9 places* *9 ÷ 3 = 3 places*

However, make sure that you can work with powers of 10 using exponent rules.

## Problem Set

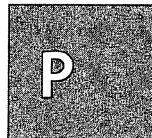
---



Solve each problem, applying the concepts and rules you learned in this section.

1. In the decimal,  $2.4d7$ ,  $d$  represents a digit from 0 to 9. If the value of the decimal rounded to the nearest tenth is less than 2.5, what are the possible values of  $d$ ?
2. If  $k$  is an integer, and if  $0.02468 \times 10^k$  is greater than 10,000, what is the least possible value of  $k$ ?
3. Which integer values of  $b$  would give the number  $2002 \div 10^{-b}$  a value between 1 and 100?
4. Simplify:  $(4 \times 10^{-2}) - (2.5 \times 10^{-3})$
5. What is  $4,563,021 \div 10^5$ , rounded to the nearest whole number?
6. Which integer values of  $j$  would give the number  $-37,129 \times 10^j$  a value between  $-100$  and  $-1$ ?
7. Simplify:  $\frac{0.00081}{0.09}$
8. Simplify:  $\sqrt[8]{0.00000256}$

## Solutions



1. **{0, 1, 2, 3, 4}**: If  $d$  is 5 or greater, the decimal rounded to the nearest tenth will be 2.5.

2. **6**: Multiplying 0.02468 by a positive power of 10 will shift the decimal point to the right. Simply shift the decimal point to the right until the result is greater than 10,000. Keep track of how many times you shift the decimal point. Shifting the decimal point 5 times results in 2,468. This is still less than 10,000. Shifting one more place yields 24,680, which is greater than 10,000.

3. **{-2, -3}**: In order to give 2002 a value between 1 and 100, you must shift the decimal point to change the number to 2.002 or 20.02. This requires a shift of either two or three places to the left. Remember that, while multiplication shifts the decimal point to the right, division shifts it to the left. To shift the decimal point 2 places to the left, you would divide by  $10^2$ . To shift it 3 places to the left, you would divide by  $10^3$ . Therefore, the exponent  $-b = \{2, 3\}$ , and  $b = \{-2, -3\}$ .

4. **0.0375**: First, rewrite the numbers in standard notation by shifting the decimal point. Then, add zeroes, line up the decimal points, and subtract:

$$\begin{array}{r} 0.0400 \\ - 0.0025 \\ \hline 0.0375 \end{array}$$

5. **46**: To divide by a positive power of 10, shift the decimal point to the left. This yields 45.63021. To round to the nearest whole number, look at the tenths place. The digit in the tenths place, 6, is more than five. Therefore, the number is closest to 46.

6. **{-3, -4}**: In order to give  $-37,129$  a value between  $-100$  and  $-1$ , you must shift the decimal point to change the number to  $-37.129$  or  $-3.7129$ . This requires a shift of either three or four places to the left. Remember that multiplication by a positive power of 10 shifts the decimal point to the right. To shift the decimal point 3 places to the left, you would multiply by  $10^{-3}$ . To shift it 4 places to the left, you would multiply by  $10^{-4}$ . Therefore, the exponent  $j = \{-3, -4\}$ .

7. **0.009**: Shift the decimal point 2 spaces to eliminate the decimal point in the denominator:

$$\frac{0.00081}{0.09} = \frac{0.081}{9}$$

Then divide. First, drop the 3 decimal places:  $81 \div 9 = 9$ . Then put the 3 decimal places back: 0.009.

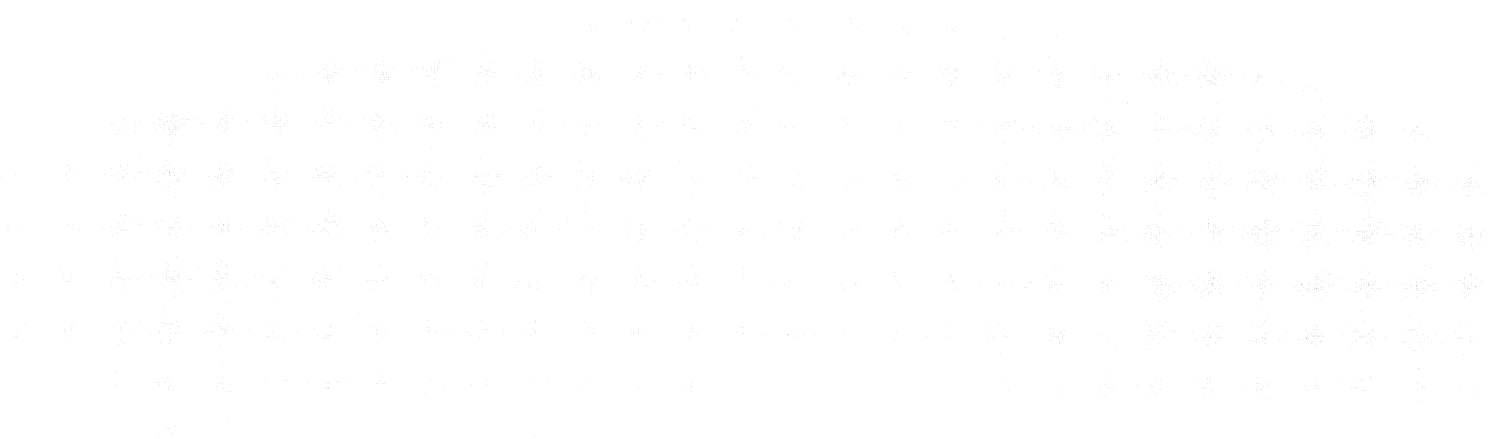
8. **0.2**: Write the expression as a decimal raised to a fractional power, using powers of 10 to separate the base from the exponent:  $(0.00000256)^{1/8} = (256)^{1/8} \times (10^{-8})^{1/8}$ . Next, compute each component separately and combine them at the finish:  $(256)^{1/8} \times (10^{-8})^{1/8} = 2 \times 10^{-1} = 0.2$ .

# Chapter 2

*of*

**Fractions, Decimals, & Percents**

## **Fractions**



# *In This Chapter...*

*Numerator and Denominator Rules*

*Simplifying Fractions*

*Converting Improper Fractions to Mixed Numbers*

*The Multiplication Shortcut*

*No Addition or Subtraction Shortcuts*

*Dividing Fractions: Use the Reciprocal*

*Division in Disguise*

*Comparing Fractions: Cross-Multiply*

*Never Split the Denominator*

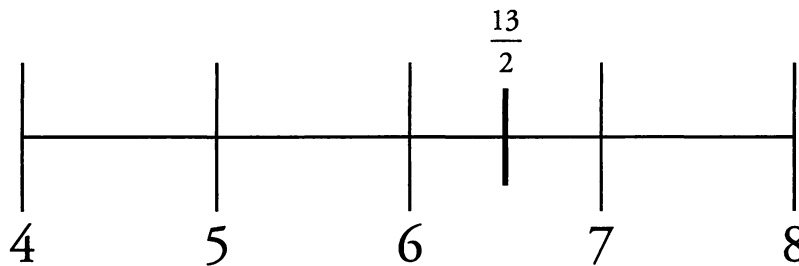


# Chapter 2:

## Fractions

Decimals are one way of expressing the numbers that fall in between the integers. Another way of expressing these numbers is fractions.

For example, the fraction  $\frac{13}{2}$ , which equals 6.5, falls between the integers 6 and 7:



**Proper fractions** are those that fall between 0 and 1. In proper fractions, the numerator is always smaller than the denominator. For example:

$$\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{7}{10}$$

**Improper fractions** are those that are greater than 1. In improper fractions, the numerator is greater than the denominator. For example:

$$\frac{5}{4}, \frac{13}{2}, \frac{11}{3}, \frac{101}{10}$$

Improper fractions can be rewritten as **mixed numbers**. A mixed number is an integer and a proper fraction. For example:

$$\frac{5}{4} = 1\frac{1}{4}$$

$$\frac{13}{2} = 6\frac{1}{2}$$

$$\frac{11}{3} = 3\frac{2}{3}$$

$$\frac{101}{10} = 10\frac{1}{10}$$

2

Although the preceding examples use positive fractions, note that fractions and mixed numbers can be negative as well.

## Numerator and Denominator Rules

Certain key rules govern the relationship between the numerator (the top number) and the denominator (the bottom number) of proper fractions. These rules are important to internalize, but keep in mind that, as written, they **only apply to positive fractions**.

As the **numerator** goes up, the fraction **increases**. If you increase the numerator of a fraction, while holding the denominator constant, the fraction increases in value:

$$\frac{1}{8} < \frac{2}{8} < \frac{3}{8} < \frac{4}{8} < \frac{5}{8} < \frac{6}{8} < \frac{7}{8} < \frac{8}{8} < \frac{9}{8} < \frac{10}{8} < \dots$$

As the **denominator** goes up, the fraction **decreases**. If you increase the denominator of a fraction, while holding the numerator constant, the fraction decreases in value as it approaches 0:

$$\frac{3}{2} > \frac{3}{3} > \frac{3}{4} > \frac{3}{5} > \frac{3}{6} \dots > \frac{3}{1,000} \dots \rightarrow 0$$

Adding the same number to **both** the numerator and the denominator brings the fraction **closer** to 1, regardless of the fraction's value.

If the fraction is originally smaller than 1, the fraction **increases** in value as it approaches 1:

$$\frac{1}{2} < \frac{1+1}{2+1} = \frac{2}{3} < \frac{2+9}{3+9} = \frac{11}{12} < \frac{11+1,000}{12+1,000} = \frac{1,011}{1,012}$$

Thus: 
$$\frac{1}{2} < \frac{2}{3} < \frac{11}{12} < \frac{1,011}{1,012} \dots \rightarrow 1$$

Conversely, if the fraction is originally larger than 1, the fraction **decreases** in value as it approaches 1:

$$\frac{3}{2} > \frac{3+1}{2+1} = \frac{4}{3} > \frac{4+9}{3+9} = \frac{13}{12} > \frac{13+1,000}{12+1,000} = \frac{1,013}{1,012}$$

Thus: 
$$\frac{3}{2} > \frac{4}{3} > \frac{13}{12} > \frac{1,013}{1,012} \dots \rightarrow 1$$

- [download Angry Lead Skies \(Garrett Files, Book 10\) pdf, azw \(kindle\)](#)
- [download City of Djinn: A Year in Delhi](#)
- [Earth \(21st Century Junior Library: Solar System\) for free](#)
- [Archaeology and World Religion pdf](#)
  
- <http://unpluggedtv.com/lib/Aristotle--Metaphysics-Theta--Translated-with-an-Introduction-and-Commentary.pdf>
- <http://tuscalaural.com/library/PowerPivot-for-the-Data-Analyst--Microsoft-Excel-2010--MrExcel-Library-.pdf>
- <http://serazard.com/lib/Jos---Saramago-En-Sus-Lectores.pdf>
- <http://hasanetmekci.com/ebooks/Archaeology-and-World-Religion.pdf>