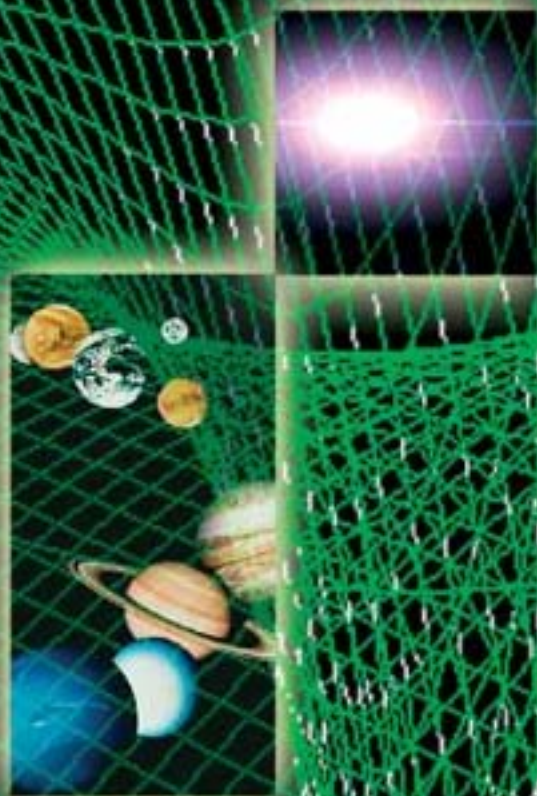


# Interplanetary Mission Analysis and Design



Stephen  
Kemble



---

# Interplanetary Mission Analysis and Design

---



---

Stephen Kemble

---

# Interplanetary Mission Analysis and Design

 Springer

Published in association with  
**Praxis Publishing**  
Chichester, UK

PRAXIS 

Mr Stephen Kemble  
Consultant  
EADS Astrium  
Stevenage  
Hertfordshire  
UK

---

SPRINGER-PRAXIS BOOKS IN ASTRONAUTICAL ENGINEERING  
SUBJECT *ADVISORY EDITOR*: John Mason M.Sc., B.Sc., Ph.D.

---

ISBN 10: 3-540-29913-0 Springer-Verlag Berlin Heidelberg New York

Springer is part of Springer-Science + Business Media ([springeronline.com](http://springeronline.com))

Bibliographic information published by Die Deutsche Bibliothek

Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;  
detailed bibliographic data are available from the Internet at <http://dnb.ddb.de>

Library of Congress Control Number: 2005935455

Apart from any fair dealing for the purposes of research or private study, or criticism or review, as permitted under the Copyright, Designs and Patents Act 1988, this publication may only be reproduced, stored or transmitted, in any form or by any means, with the prior permission in writing of the publishers, or in the case of reprographic reproduction in accordance with the terms of licences issued by the Copyright Licensing Agency. Enquiries concerning reproduction outside those terms should be sent to the publishers.

© Praxis Publishing Ltd, Chichester, UK, 2006  
Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Cover design: Jim Wilkie

Project copy editor: Bob Marriott

Project management: Originator Publishing Services, Gt Yarmouth, Norfolk, UK

Printed on acid-free paper

---

# Contents

<b>Preface</b> . . . . .	ix
<b>Acknowledgements</b> . . . . .	xiii
<b>List of figures</b> . . . . .	xv
<b>List of tables</b> . . . . .	xxvii
<b>Nomenclature</b> . . . . .	xxxii
<b>1 Interplanetary missions</b> . . . . .	1
1.1 Fundamentals of interplanetary missions . . . . .	1
1.1.1 Interplanetary transfers . . . . .	1
1.1.2 Lambert's problem . . . . .	4
1.1.3 Solutions to Lambert's problem for an interplanetary transfer . . . . .	10
1.1.4 Transfer types . . . . .	16
1.1.5 Launch opportunities . . . . .	19
1.1.6 Multi-revolution transfers . . . . .	20
1.2 Leaving a planet . . . . .	23
1.2.1 Escape orbits . . . . .	23
1.2.2 Orbiting a planet . . . . .	25
1.2.3 Intermediate launch and apogee raising . . . . .	25
1.2.4 Interplanetary departure implications . . . . .	38
1.3 Planet orbit selection and insertion . . . . .	43
1.3.1 Planetary approach and capture . . . . .	43
1.3.2 Target orbit options . . . . .	43
1.4 Transfers through the Solar System . . . . .	49
1.4.1 Mercury . . . . .	49
1.4.2 Venus . . . . .	49

1.4.3	Mars . . . . .	52
1.4.4	Jupiter . . . . .	53
1.4.5	Saturn . . . . .	55
1.4.6	Uranus, Neptune and Pluto . . . . .	56
1.5	Return missions to the planets . . . . .	58
1.5.1	Optimal stay times . . . . .	58
1.5.2	Case of an optimal stay time for a Mars return mission . . . . .	61
1.5.3	Short-stay time missions at Mars . . . . .	63
1.5.4	Short-duration Mars return missions. . . . .	67
<b>2</b>	<b>Spacecraft propulsion . . . . .</b>	<b>73</b>
2.1	Propulsion basics. . . . .	74
2.2	High-thrust systems . . . . .	77
2.2.1	Chemical propulsion systems. . . . .	77
2.2.2	Thermal rockets . . . . .	78
2.3	Low-thrust systems . . . . .	79
2.3.1	Electric propulsion . . . . .	80
2.3.2	Solar sails . . . . .	84
2.4	Choice of propulsion system . . . . .	86
<b>3</b>	<b>Optimisation. . . . .</b>	<b>89</b>
3.1	The trajectory optimisation problem . . . . .	89
3.2	Trajectory optimisation methods. . . . .	91
3.2.1	Indirect optimisation techniques. . . . .	91
3.2.2	Direct optimisation techniques. . . . .	95
3.2.3	An example of control parameterisation . . . . .	95
3.2.4	Techniques for solving direct optimisation problems . . . . .	97
3.2.5	Selection of appropriate techniques. . . . .	106
3.3	Application of direct trajectory optimisation methods. . . . .	107
3.3.1	Formulating the mathematical problem . . . . .	107
3.3.2	Evaluation of gradients . . . . .	115
3.3.3	Non-linear programming . . . . .	124
3.4	Combining system and trajectory optimisations: the optimal transport problem . . . . .	127
3.4.1	Propulsion system optimisation parameters. . . . .	127
<b>4</b>	<b>Special techniques . . . . .</b>	<b>135</b>
4.1	Motion in multi-body gravity fields. . . . .	135
4.1.1	The multi-body problem. . . . .	135
4.1.2	Identifying the dominant gravity field . . . . .	137
4.1.3	Motion in the three body problem . . . . .	141
4.2	Escape from a planet . . . . .	145
4.2.1	Analysis of escape. . . . .	146
4.2.2	Examples of escape . . . . .	155
4.3	Principles of gravity-assist manoeuvres. . . . .	168

4.3.1	Analysis of patched conics . . . . .	168
4.3.2	Plane-changing by gravity assist . . . . .	184
4.3.3	Multiple gravity assists and resonance . . . . .	194
4.3.4	Tisserand's criterion . . . . .	201
4.3.5	Multiple gravity assists for plane-changing . . . . .	206
4.3.6	Gravity assist at planetary moons for escape and capture . . . . .	213
4.3.7	Modelling gravity assist manoeuvres . . . . .	219
4.3.8	Anatomy of a gravity assist . . . . .	222
4.4	The variational equations of Lagrange and Gauss . . . . .	226
4.4.1	Orbital perturbations . . . . .	226
4.4.2	Lagrange's planetary equations . . . . .	226
4.4.3	Secular effects . . . . .	230
4.5	Low-thrust transfers . . . . .	233
4.5.1	Low-thrust transfer fundamentals . . . . .	233
4.6	Low thrust for planetary escape and capture . . . . .	245
4.6.1	Using thrust-coast arcs for energy gain . . . . .	245
4.6.2	Application of a low-thrust strategy for escaping Earth . . . . .	247
4.6.3	Planetary capture and orbit insertion with low thrust . . . . .	253
4.6.4	Optimal utilisation of low-thrust for interplanetary transfers . . . . .	253
4.7	Combining low thrust with gravity assist . . . . .	257
4.7.1	Use of manoeuvres between gravity assists . . . . .	257
4.7.2	The Earth gravity assist escape loop . . . . .	257
4.7.3	Examples of raising aphelion with single Earth gravity assist . . . . .	259
4.7.4	Examples of raising aphelion with double Earth gravity assist . . . . .	268
4.8	Using multi-body gravity perturbations . . . . .	272
4.8.1	The three-body problem . . . . .	272
4.8.2	The Lagrange libration points . . . . .	274
4.8.3	Orbits at the Lagrange libration points . . . . .	282
4.8.4	Transfers to the Lagrange libration points . . . . .	285
4.8.5	Gravity-assisted planetary escape and capture . . . . .	298
4.8.6	Use of low thrust and gravitational escape . . . . .	324
4.8.7	Summary of gravitational escape and capture techniques . . . . .	325
4.9	Aerocapture and aerobraking . . . . .	327
4.9.1	Aerocapture . . . . .	327
4.9.2	Aerobraking . . . . .	329
<b>5</b>	<b>Missions to the planets . . . . .</b>	<b>335</b>
5.1	Interplanetary missions using gravity assist . . . . .	335
5.1.1	Routes through the inner planets: mission to Mercury . . . . .	335
5.1.2	Messenger to Mercury . . . . .	352
5.1.3	Gravity assist for Mars return missions . . . . .	355
5.1.4	Reaching Jupiter . . . . .	356



---

5.1.5	Gravity-assisted tours of Jupiter's moons . . . . .	362
5.1.6	Transfers to the outer planets . . . . .	368
5.1.7	Missions to minor bodies . . . . .	379
5.1.8	Escaping the Solar System . . . . .	382
5.2	Low-thrust missions . . . . .	389
5.2.1	Analysis of a low-thrust, multi-gravity assist mission to Mercury . . . . .	389
5.2.2	Analysis of missions to Jupiter and Saturn using low thrust . . . . .	397
5.2.3	Missions to Pluto with low-thrust . . . . .	407
5.3	Missions using gravity escape and capture . . . . .	409
5.3.1	Transfer from Jupiter to Saturn . . . . .	410
5.3.2	Transfer from Jupiter to Uranus . . . . .	417
5.3.3	Analysis of a mission to Venus using capture via the Lagrange points . . . . .	421
<b>Appendix 1: Keplerian orbits . . . . .</b>		<b>429</b>
<b>Appendix 2: Frames of reference. . . . .</b>		<b>443</b>
<b>Appendix 3: The planets . . . . .</b>		<b>449</b>
<b>Appendix 4: Optimising launcher injection. . . . .</b>		<b>459</b>
<b>Index . . . . .</b>		<b>477</b>

---

## Preface

One of the enduring legacies of the twentieth century will be the advent of space travel. This achievement has changed the way we think about our presence in the Universe and now offers the possibility to explore beyond our own world. Nobody experiencing the Apollo missions of the 1960s and 70s could be unaffected by the magnitude of those achievements. Now, nearly forty years later, space travel is almost commonplace, although man's personal presence in space is still limited.

The pioneering edge of space exploration now lies beyond Earth and its moon, as numerous spacecraft over the last thirty years have visited most of the planets of the Solar System. Mars remains the focus of many such missions and is likely to be the first planet that man visits personally, not just through robotic craft. Perhaps the most fascinating interplanetary missions to date have been the Voyagers. Launched in the late 1970s, these spacecraft were flung out of the Solar System after flying by the outer planets. They have now passed far beyond Pluto as they leave the Sun's domain behind.

These technical achievements have inspired numerous science fiction stories, which in turn have themselves perhaps influenced the drive for new space missions and exploration. Although 'warp drives' remain for the present in the realm of science fiction, more adventurous missions to explore our Solar System are being planned. These include an initiative to place a man on Mars and also for a detailed robotic exploration of Jupiter and its family of moons. This latter system has already been inspected by the Galileo spacecraft and revealed a fascinating 'micro solar system' with a rich variety of features. Also, both current (Messenger) and planned future missions (Bepi-Colombo) to Mercury will undertake the difficult route to the innermost planet of the Solar System. In addition to the planets, the minor bodies of the Solar System are being explored. A challenging example is ESA's Rosetta mission, following a complex route to achieve a rendez-vous with a comet.

## MISSION ANALYSIS AND DESIGN

This diverse range of missions require numerous techniques for their analysis and design. These aspects will be considered in this book, including the key issues of escaping from a planet, interplanetary transfer, and capture at a target planet. Certain ‘classical’ methods for the design of such trajectories have been employed for many years. As missions became more demanding, then new techniques were developed to enable more efficient designs to be realised. These allow the efficient utilisation of newly evolving propulsion technologies. This theme is continuing, as both new mathematical and computational ideas are considered for the solution of these problems.

The objective of this book is to describe a selection of techniques that may be applied to the analysis and design of interplanetary missions. The focus is on methods that enable the efficient solution of the problems considered. Details of the methods are given. However, this text is not intended as a reference on astrodynamics. Summaries of key derivations are included.

The terms ‘mission analysis’ and ‘mission design’ can have several meanings. The one taken here is that of the analysis and design of spacecraft transfers. Therefore, the focus is on techniques in orbital mechanics and trajectory optimisation that may be applied to the objective. The aspects of mission analysis particularly relevant to interplanetary missions are considered here.

This book is divided into five major chapters. The first chapter focuses on ‘conventional’ analysis and design techniques for interplanetary missions. This includes the basic ideas of Hohmann transfers, the solution of Lambert’s problem and the fundamentals of planetary escape and capture. These basic ideas of interplanetary transfers are then extended to consider return missions to the planets. The issue of escape from Earth is also considered in more detail, in the context of the efficient utilisation of launch vehicle capabilities. This subject is also further expanded in Appendix 4.

The second chapter briefly considers aspects of spacecraft propulsion systems. These systems are a fundamental factor in the nature of interplanetary mission designs and therefore warrant some consideration in a book such as this.

The third chapter focuses on optimisation. In particular, methods for obtaining solutions to local optimisation problems are considered. These generally require gradient evaluations and are often called ‘gradient based’ methods. These methods are essential for the efficient design of interplanetary missions. As more complex propulsion system types are considered, so the complexity of the optimisation problems increases. Many developments are taking place in this area. Only gradient-based methods are considered here. However, it should be noted that alternative techniques such as evolutionary computing offer very interesting prospects for the identification of globally optimal solutions. In addition to trajectory optimisation problems, the nature of spacecraft optimisation is discussed, where the design of the propulsion installation may be optimised together with the transfer trajectory.

The fourth chapter considers a range of ‘special’ methods that may be employed

for mission analysis and design. These methods allow the planning of more efficient transfers. A consequence of the improved efficiency is the increased complexity of the transfer routes. Some of these methods take advantage of interesting features of astrodynamics. A good example of this is the phenomenon of gravitational escape or capture at a planet, which has been observed for comets in the Solar System. Consequently, this chapter contains an outline description of the three-body problem and mission designs that can utilise three-body effects. Many of the techniques considered in this chapter allow the efficient utilisation of advanced propulsion system concepts. This is particularly true for low-thrust systems. The effect of low thrust systems on orbit evolution is considered via the application of perturbation equations. Considerable attention is paid to ‘gravity assist’. This technique allows the utilisation of combined gravity fields to enable a spacecraft to significantly modify its orbital energy, without the need for manoeuvre.

The final chapter describes a series of mission examples that utilise the methods described in the previous sections. These include missions using gravity assist, low-thrust propulsion, gravitational escape and capture. Many of the examples are generic, in that they consider typical transfers between planets. However, certain examples are relevant to actual missions, either past, current, or future.

The appendices describe the basics of orbital mechanics, orbital reference frames, and also the properties of the planets. The data is intended as a source of reference for the material in the book.

A CD is included to give some examples of interplanetary missions. A simulation tool is included. This is used to generate animated sequences that show interplanetary transfers. The missions illustrated include both transfers to the inner and outer planets. Instructions for use are contained on the CD. The software runs on Windows based PC systems.

Although every effort has been made to eliminate mathematical and factual errors in the material in this book, complete accuracy cannot be guaranteed. Please send any errors found and suggested corrections to the author.



---

## Acknowledgements

I would like to thank numerous groups and individuals. First, project teams within the European Space Agency (ESA), with whom I have worked. This work has provided much of the background for the material in this text, particularly through the diverse series of projects undertaken. Specifically, the Bepi-Colombo project led by J. Van Casteran with lead mission analyst R. Jehn, the Science Payloads and Advanced Concepts, planetary exploration studies team led by P. Falkner, the Lisa-Pathfinder project led by G. Racca with lead mission analyst M. Landgraf, the SOLO project led by N. Rando with lead mission analyst G. Janin and also M. Hechler, J. Rodriguez Canabal, and W. Flury on a wide range of projects. I would like to thank the many colleagues at EADS Astrium with whom I work, particularly those with whom I regularly work (or have worked) directly on mission analysis topics, including C. Warren, M.J. Taylor, A. Povoleri, and G. Hughes.

I would also like to specifically thank R.C. Parkinson, for support and always asking the most interesting questions!

Also many thanks to Clive Horwood at Praxis and Neil Shuttlewood at Originator.



---

## Figures

1.1.1	Planetary orbital energy with respect to the Sun relative to Earth's energy with respect to the Sun . . . . .	3
1.1.2	Hohmann transfer between circular planetary orbits . . . . .	3
1.1.3	Lambert's problem . . . . .	5
1.1.4	Error function versus sma . . . . .	8
1.1.5	Lambert's problem for a transfer between two planetary orbits . . . . .	9
1.1.6	The velocity change required when departing the first planet . . . . .	10
1.1.7	Transfer $V_\infty$ contours for a transfer from an idealised circular Earth orbit to an idealised Mars circular orbit at zero inclinations . . . . .	11
1.1.8	Transfer $V_\infty$ contours for a transfer from Earth orbit to an idealised Mars orbit possessing zero inclination, but with normal eccentricity . . . . .	12
1.1.9	Transfer $V_\infty$ contours for a transfer from Earth to Mars . . . . .	13
1.1.10	Total Vinfinity versus transfer duration for a launch on 23 October 2011 . . . . .	14
1.1.11	Transfers from Earth to Mars for a launch on 23 October 2011, for a range of transfer durations spanning the two minima and the central maximum in $V_\infty$ . . . . .	15
1.1.12	Transfer types between planets . . . . .	16
1.1.13	Optimum transfer between planets and accelerated and delayed arrival . . . . .	17
1.1.14	2011 transfers from Earth to Mars . . . . .	18
1.1.15	A set of transfers occurring with a given synodic period and making up a global repeat period after three synodic repeat periods . . . . .	20
1.1.16	A 1.5-revolution transfer for a launch in 2011 . . . . .	21
1.1.17	Extended-duration transfers in terms of Vinfinity contours, for Earth–Mars transfers starting 2011 . . . . .	22
1.2.1	Departure orbit geometry . . . . .	24
1.2.2	Apogee altitude vs pericentre $\Delta V$ to reach a given Vinfinity with perigee altitude at 200 km . . . . .	26
1.2.3	Apogee altitude vs pericentre $\Delta V$ to reach a given Vinfinity with perigee altitude at 2000 km . . . . .	26
1.2.4	$\Delta V$ required to reach required excess hyperbolic speed for conjunction-class transfers from Earth . . . . .	27



<b>1.2.5</b>	Orbital energy per unit mass of a 100,000-km semi-major-axis orbit with respect to the planet escape condition . . . . .	27
<b>1.2.6</b>	Fuel tank limited performance for spacecraft injection to high apogee and elliptical orbits . . . . .	31
<b>1.2.7</b>	Fuel tank limited performance for spacecraft injection to high apogee and hyperbolic orbits . . . . .	32
<b>1.2.8</b>	Mass components versus intermediate injection apogee radius . . . . .	34
<b>1.2.9</b>	A case of fixed upper stage mass showing spacecraft mass versus intermediate injection apogee radius, for a target apogee of 1.5 million km for two spacecraft propulsion mass fractions . . . . .	34
<b>1.2.10</b>	A case of fixed upper stage mass showing spacecraft useful mass versus intermediate injection apogee radius, for a range of target escape orbit Vinfinity, and for a spacecraft propulsion mass fraction of 0.15 . . . . .	35
<b>1.2.11</b>	A second case of fixed upper stage mass showing spacecraft useful mass versus intermediate injection apogee radius, for a range of target escape orbit Vinfinity, for a spacecraft propulsion mass fraction of 0.15 . . . . .	36
<b>1.2.12</b>	$\Delta V$ from injection orbit to 1.5-million km apogee target orbit . . . . .	36
<b>1.2.13</b>	Example of apogee raising from a 42,000 km apogee injection orbit to reach a target orbit with apogee of 1.5 million km . . . . .	37
<b>1.2.14</b>	$\Delta V$ from injection orbit to 5 km/sec excess speed . . . . .	38
<b>1.2.15</b>	The departing hyperbolic orbit, showing the asymptotic true anomaly . . . . .	39
<b>1.2.16</b>	The departure vector seen within the orbit plane of the escape hyperbola, showing two alternative solutions attaining the same departure direction . . . . .	40
<b>1.2.17</b>	Simulation of two hyperbolic orbits with ascending nodes separated by 180 degrees (inclinations 60 and 120 degrees) achieving a 30 degree asymptotic departure declination . . . . .	41
<b>1.2.18</b>	The effect of declination requirement on the hyperbolic orbit argument of perigee for a 3 km/sec excess hyperbolic speed orbit leaving Earth . . . . .	42
<b>1.3.1</b>	Definition of the B plane . . . . .	44
<b>1.3.2</b>	The relationship between Beta angle and inclination, for a range of different declinations . . . . .	45
<b>1.3.3</b>	Arrival geometry seen in the orbit plane about the planet . . . . .	46
<b>1.3.4</b>	The effect of excess hyperbolic speed between 3,000 and 5,000 m/s on possible pericentre locations for approaches to Mars and Venus at zero declination . . . . .	47
<b>1.3.5</b>	The effect of approach declination between 0° and 30° on possible pericentre locations for approaches to Mars with an excess hyperbolic speed of 3 km/sec . . . . .	48
<b>1.3.6</b>	A hyperbolic approach to Venus with a pericentre retro-burn to reach a 5-day transfer orbit about Venus . . . . .	48
<b>1.4.1</b>	$V_\infty$ sums for conjunction type two-impulse transfers to Mercury over the launch period 2007 . . . . .	50
<b>1.4.2</b>	$V_\infty$ sums for conjunction type 2 impulse transfers to Venus over the launch period 2010–2013 . . . . .	52
<b>1.4.3</b>	$V_\infty$ sums for conjunction type, two-impulse transfers to Jupiter over launch period 2009 . . . . .	54
<b>1.4.4</b>	Vinfinity sums for conjunction-type two-impulse transfers to Saturn over launch period 2009–2010 . . . . .	57
<b>1.5.1</b>	Return mission geometry . . . . .	59
<b>1.5.2</b>	The effect of stay time on $V_\infty$ total for different conjunction-type transfers for launch in 2011 to Mars . . . . .	62

1.5.3	The effect of stay time on $V_\infty$ total for different transfer types for launch in 2020	64
1.5.4	Transfer options from Mars to Earth leaving in 2021, with two local minima in $V_\infty$ totals	65
1.5.5	A return mission to Mars with optimal conjunction-type transfer (launch in 2020) on the outward leg, 60-day stay-time, and minimum $\Delta V$ trajectory on the return leg	66
1.5.6	The effect of stay-time on return $\Delta V$ and $V_\infty$ after a short conjunction-class transfer from Earth, departing in mid-2020	68
1.5.7	The effect of stay time on total mission duration after a short conjunction-class transfer from Earth, departing in mid-2020	69
1.5.8	A fast return transfer from Mars to Earth with a short stay at Mars after a conjunction-class outward transfer	69
1.5.9	Examples of short transfers from Earth to Mars with short stay-time and return	70
2.1.1	Illustration of a nozzle	76
2.3.1	A gridded ion thruster in operation	81
2.3.2	A conceptual illustration of a Hall-effect thruster	82
2.3.3	The principle of the solar sail	84
2.3.4	A solar sail concept proposed by NASA	86
3.2.1	An example of a parameterisation of a continuously variable steering angle	96
3.2.2	Trajectory segmentation used by multiple shooting algorithms	98
3.2.3	Initial trajectory segmentation for multiple shooting, for the case of a transfer from Earth escape orbit to Jupiter approach using two Earth GAs	99
3.4.1	Thrust arc dependence on thrust/mass for an apogee raising manoeuvre of 100 km	132
3.4.2	The relationship between $\Delta V$ loss and thrust for a low-thrust apocentre raise manoeuvre of 100 km	132
3.4.3	The relationship between fuel mass, specific impulse and thrust for a low-thrust apocentre raise manoeuvre of 100 km with an initial mass of 1,000 kg	133
4.1.1	Illustration of gravitational equality ‘sphere’	138
4.1.2	Motion in the presence of two gravity fields	139
4.1.3	Orbital geometry in the circular, restricted three-body problem	145
4.2.1	Relative velocity geometry	148
4.2.2	Idealised planetary escape geometry by vector addition of an instantaneous excess hyperbolic departure vector	149
4.2.3	Motion relative to the planet after departure	150
4.2.4	Transfer from Earth to raised aphelion orbit with Earth $V_\infty$ at 2.3 km/sec	156
4.2.5	Semi-major axis and energy with respect to Earth, evolution over distance from circular orbiting Earth	157
4.2.6	Osculating excess hyperbolic speed evolution with respect to Earth, evolution over distance from circular orbiting Earth	157
4.2.7	Energy with respect to Sun, evolution over distance from a circular orbiting Earth	158
4.2.8	Angular momentum with respect to Sun, evolution over distance from circular orbiting Earth	159

<b>4.2.9</b>	Semi-major axis and energy with respect to Earth, evolution over distance from circular orbiting Earth, for a radial Earth relative departure case . . . . .	160
<b>4.2.10</b>	Transfer from Earth to increased eccentricity orbit with Earth relative radial V <sub>infinity</sub> at 2.3 km/sec . . . . .	161
<b>4.2.11</b>	Energy with respect to Sun, evolution over distance from circular orbiting Earth, for a radial Earth departing case . . . . .	161
<b>4.2.12</b>	Comparison of Earth relative semi-major axis evolution for two-body and three-body departure cases . . . . .	162
<b>4.2.13</b>	Comparison of Earth relative range evolution for two-body and three-body departure cases . . . . .	163
<b>4.2.14</b>	Comparison of Earth relative angular momentum evolution for two-body and three-body departure cases . . . . .	164
<b>4.2.15</b>	Transfer from Jupiter to reduced perihelion orbit with Jupiter osculating V <sub>infinity</sub> at 5.86 km/sec . . . . .	164
<b>4.2.16</b>	Semi-major axis with respect to Jupiter, evolution over distance from Jupiter . . . . .	166
<b>4.2.17</b>	Excess hyperbolic speed with respect to Jupiter, evolution over distance from Jupiter . . . . .	166
<b>4.2.18</b>	Semi-major axis and energy with respect to Sun, evolution over distance from Jupiter . . . . .	167
<b>4.3.1</b>	The principle of patched conics . . . . .	170
<b>4.3.2</b>	Deflection of the asymptote velocity vector . . . . .	172
<b>4.3.3</b>	Geometry of the 2D fly-by with coplanar planet and spacecraft orbits . . . . .	172
<b>4.3.4</b>	Alternative deflection possibilities for 2D fly-by . . . . .	173
<b>4.3.5</b>	Velocity vector addition at the fly-by for a 2D case . . . . .	173
<b>4.3.6</b>	Two possible locations for a fly-by . . . . .	175
<b>4.3.7</b>	The rotation of the line of apses after a gravity assist at location 1 . . . . .	176
<b>4.3.8</b>	The rotation of the line of apses after a gravity assist at location 2 . . . . .	176
<b>4.3.9</b>	The effect of fly-by altitude on the final orbit . . . . .	177
<b>4.3.10</b>	The effect of initial orbit perihelion and eccentricity on gravity assist effectiveness at Venus . . . . .	178
<b>4.3.11</b>	The effect of initial orbit excess hyperbolic speed on gravity assist effectiveness at Venus for different initial aphelions . . . . .	179
<b>4.3.12</b>	The effect of initial orbit perihelion and eccentricity on gravity assist effectiveness at Earth . . . . .	180
<b>4.3.13</b>	The effect of gravity assists at different Jovian moons for the same spacecraft orbits . . . . .	182
<b>4.3.14</b>	The effect of gravity assists on orbital energy at different Jovian moons for the same spacecraft orbits . . . . .	183
<b>4.3.15</b>	Velocity vector addition at the $\Delta V$ assisted fly-by . . . . .	184
<b>4.3.16</b>	The effectiveness of $\Delta V$ at pericentre of Jupiter fly-by . . . . .	185
<b>4.3.17</b>	Definition of the B plane and approach plane . . . . .	187
<b>4.3.18</b>	The fly-by plane and the relationship between Beta angle, deflection angle, deflection angle components and the approach plane . . . . .	188
<b>4.3.19</b>	Definition of deflection angles and axis set . . . . .	189
<b>4.3.20</b>	Definition of relationships between the initial approach plane and major body orbit plane . . . . .	191
<b>4.3.21</b>	Post-Ganymede gravity-assist Jupiter-centred speed versus fly-by Beta angle, evaluated for a range of approach orbit pericentres from 500,000 to 900,000 km . . . . .	194
<b>4.3.22</b>	Post-Ganymede gravity-assist Jupiter-centred inclination, apocentre and	

	pericentre change, versus fly-by Beta angle, evaluated for a range of approach orbit pericentres from 500,000 to 900,000 km . . . . .	195
4.3.23	Post-Ganymede gravity-assist Jupiter-centred inclination and apocentre change, polar plot versus fly-by Beta angle, evaluated for a range of approach orbit pericentres from 600,000 to 900,000 km . . . . .	196
4.3.24	Post-Ganymede gravity-assist Jupiter-centred inclination and apocentre change loci, evaluated for a range of approach orbit pericentres, between 700,000 and 900,000 km . . . . .	197
4.3.25	Relative velocity in a resonant orbit . . . . .	197
4.3.26	Limiting fly-by geometry . . . . .	198
4.3.27	Apocentre and pericentre evolution for a Ganymede gravity assist sequence in the Jovian system . . . . .	199
4.3.28	Examples for four resonant gravity assists at Ganymede. . . . .	200
4.3.29	Orbital period–pericentre relationship for gravity assist at Earth for a range of excess hyperbolic speeds between 3,000 and 5,000 m/s. . . . .	203
4.3.30	Apocentre–pericentre relationship for gravity assist at Earth for a range of excess hyperbolic speeds between 3,000 and 5,000 m/s. . . . .	203
4.3.31	Apocentre–pericentre relationship for gravity assist at Venus for a range of excess hyperbolic speeds from 3,000 to 7,000 m/s . . . . .	204
4.3.32	Apocentre–pericentre relationship for gravity assists at Earth and Venus for a range of excess hyperbolic speeds. Speeds of 3,000 to 5,000 m/s at Earth are considered, and 3,000 to 7,000 m/s at Venus . . . . .	205
4.3.33	Apocentre–pericentre relationship for gravity assist at Europa, Ganymede and Callisto for a range of excess hyperbolic speeds . . . . .	205
4.3.34	Period–pericentre relationship for gravity assist at Europa, Ganymede and Callisto for a range of excess hyperbolic speeds . . . . .	206
4.3.35	The definition of deflection angles in a multi-gravity assist case . . . . .	207
4.3.36	Two gravity assists maintaining resonance, showing both approach planes . . . . .	209
4.3.37	Effect of progressive deflection of the fly-by plane for resonant orbits targeted at inclination increase . . . . .	210
4.3.38	Limiting case defining maximum inclination relative to the major body orbit plane when in a resonant orbit . . . . .	211
4.3.39	A maximum inclination case when central body relative velocity may be varied at subsequent fly-bys from $V_1$ to $V_{limit}$ . . . . .	212
4.3.40	Example of multiple resonant gravity assists at Ganymede, with excess hyperbolic speed at Ganymede of 9.5 km/sec . . . . .	212
4.3.41	Inclination, aphelion and perihelion evolution for a sequence of gravity assist at Venus with $V_{infinity}$ at 22.4 km/sec. . . . .	214
4.3.42	Orbit evolution for a sequence of gravity assist at Venus . . . . .	215
4.3.43	The effect of apogee altitude on Earth excess hyperbolic speed after lunar fly-by for a fly-by altitude of 260 km . . . . .	216
4.3.44	The effect of $\Delta V$ for apogee raising on Earth excess hyperbolic speed . . . . .	216
4.3.45	The principle of planetary capture using gravity assist at a moon . . . . .	217
4.3.46	Approach excess hyperbolic speed versus target Jupiter pericentre altitude after Ganymede fly-by for a range of post-gravity assisted captured apocentre altitudes . . . . .	218
4.3.47	Approach excess hyperbolic speed versus target Saturn pericentre altitude after Tital fly-by for a range of post-gravity assist captured apocentre altitudes . . . . .	218

<b>4.3.48</b>	Gravity assisted transfer at Earth to reach raised aphelion with Earth $V_\infty$ at 6 km/sec . . . . .	223
<b>4.3.49</b>	Semi-major axis and energy with respect to Earth during an Earth gravity assist, evolution over distance from Earth . . . . .	223
<b>4.3.50</b>	Excess hyperbolic speed with respect to Earth during an Earth gravity assist, evolution over distance from Earth . . . . .	224
<b>4.3.51</b>	Semi-major axis with respect to Sun during an Earth gravity assist, evolution over distance from Earth . . . . .	224
<b>4.3.52</b>	Energy with respect to Sun, evolution over distance from Earth . . . . .	225
<b>4.5.1</b>	Evolution of a near circular orbit with constant perturbing acceleration perpendicular to the radius vector. . . . .	235
<b>4.5.2</b>	A comparison of optimum and zero steering-angle options for increasing the semi-major axis for an initial elliptical orbit with an eccentricity of 0.73 (GTO) . . . . .	237
<b>4.5.3</b>	A low-thrust trajectory with maximum rate of change of semi-major axis steering over a period of nearly 9 days . . . . .	237
<b>4.5.4</b>	Low-thrust semi-major axis evolution with maximum rate of change of semi-major axis and zero $\alpha$ steering over a period of nearly 9 days . . . . .	238
<b>4.5.5</b>	A comparison of optimum and zero steering-angle options for increasing the semi-major axis for an initial elliptical orbit with an eccentricity of 0.37. . . . .	238
<b>4.5.6</b>	A comparison of optimum apogee raising and optimum semi-major axis raising steering-angle options for increasing apogee for an initial elliptical orbit with an eccentricity of 0.73 (GTO) . . . . .	239
<b>4.5.7</b>	A comparison of steering angles for optimum apogee raising and optimum semi-major axis raising steering-angle options for increasing apogee for an initial elliptical orbit with an eccentricity of 0.73 (GTO) . . . . .	240
<b>4.5.8</b>	A low-thrust trajectory with maximum rate of change apogee altitude steering over a period of nearly 9 days . . . . .	240
<b>4.5.9</b>	A comparison of optimum apogee raising and optimum semi-major axis raising steering angle options for increasing apogee for an initial elliptical orbit with an eccentricity of 0.37 . . . . .	241
<b>4.5.10</b>	Low-thrust GTO trajectory evolution with thrust/mass at 0.0004 and initial inclination at $1^\circ$ . . . . .	242
<b>4.5.11</b>	Low-thrust GTO trajectory evolution with thrust/mass at 0.0004 and initial inclination at $1^\circ$ . . . . .	243
<b>4.5.12</b>	Low-thrust heliocentric 1-AU orbit ephemeris evolution with thrust/mass at 0.0004 m/s/s and initial inclination at $1^\circ$ . . . . .	243
<b>4.5.13</b>	Low-thrust heliocentric 1-AU trajectory evolution with thrust/mass at 0.0004 m/s/s and initial inclination at $1^\circ$ , over a period of two years . . . . .	244
<b>4.6.1</b>	The change in semi-major axis per unit applied $\Delta V$ versus true anomaly . . . . .	245
<b>4.6.2</b>	The change in semi-major axis per unit applied $\Delta V$ (time averaged over an applied true anomaly range) versus true anomaly range from perigee. . . . .	246
<b>4.6.3</b>	The change in semi-major axis per unit applied $\Delta V$ (time averaged over an applied true anomaly range) versus true anomaly range from perigee. . . . .	246
<b>4.6.4</b>	Transfer duration and $\Delta V$ dependence on initial circular orbit altitude in reaching Earth escape . . . . .	248
<b>4.6.5</b>	Transfer from GTO to earth escape . . . . .	249
<b>4.6.6</b>	Transfer from GTO to Earth escape with 60-degree coast phase at apogee. . . . .	250
<b>4.6.7</b>	The effects of coast arcs on $\Delta V$ and the time to escape for the transfer to Earth escape using lunar gravity assist and direct thrust to Earth escape. . . . .	251

<b>4.6.8</b>	$\Delta V$ versus thrust and transfer duration for low-thrust Earth escape from GTO	252
<b>4.6.9</b>	$\Delta V$ needed to reach a given Vinfinity from an initial GTO orbit about Earth.	254
<b>4.6.10</b>	The effect of Earth departure $V_\infty$ on total transfer fuel usage for a Mars approach $V_\infty$ of 500 m/s	256
<b>4.6.11</b>	The effect of Mars approach $V_\infty$ on total transfer fuel usage for an Earth departure $V_\infty$ of 1,000 m/s	256
<b>4.7.1</b>	Example of a scheme for utilisation of Earth gravity assist	258
<b>4.7.2</b>	$\Delta V$ implications to reach alternative targets by varying initial orbit aphelion, the initial $V_\infty$ is 1,500 m/s and inwards departure is used	260
<b>4.7.3</b>	$\Delta V/V_\infty$ change ratio implications to reach alternative targets by varying the initial orbit aphelion	261
<b>4.7.4</b>	$\Delta V$ implications to reach alternative targets by varying initial orbit aphelion	261
<b>4.7.5</b>	Thrust-mass of 0.2 and 0.1 N/t for transfer to a 2.2-AU orbit	263
<b>4.7.6</b>	The $\Delta V$ penalty for reducing thrust/mass for transfer to a 2.2-AU target	264
<b>4.7.7</b>	Comparison of observed and predicted $\Delta V$ s for reaching target aphelion or equivalently targetting Earth relative $V_\infty$	264
<b>4.7.8</b>	Comparison of observed $\Delta V$ using final manoeuvre after EGA and predicted $\Delta V$ s for reaching target aphelion or equivalently targetting Earth-relative $V_\infty$	265
<b>4.7.9</b>	Comparison of $\Delta V$ s for reaching target aphelion for a two-burn strategy (pre EGA) and the effect of introducing a third burn after Earth gravity assist	265
<b>4.7.10</b>	Transfer to a 2.87-AU orbit using strategy B	266
<b>4.7.11</b>	The effect of a final post-EGA burn for case B	267
<b>4.7.12</b>	Transfer to a 2.87-AU orbit using strategy C	267
<b>4.7.13</b>	The effect of a final, post-EGA burn for case C	268
<b>4.7.14</b>	Comparison of $\Delta V$ s and EGA Vinfinitys for the three alternative cases over a range of target aphelions	269
<b>4.7.15</b>	Effect of manoeuvre on achievable excess hyperbolic speed at Earth and post-GA aphelion in a 2-year Earth resonant orbit	270
<b>4.7.16</b>	The use of a double EGA to raise aphelion to a target at 10 AU, using manoeuvres in the two-year resonant loop and also after the final EGA	271
<b>4.7.17</b>	The use of a double EGA to raise aphelion to a target at 80 AU, using manoeuvres in the two-year resonant loop and also after the final EGA	271
<b>4.7.18</b>	The deep-space $\Delta V$ required to reach a target aphelion after a low-energy escape after lunar gravity assist and 2 EGAs	272
<b>4.8.1</b>	Zero-velocity boundaries for the Earth–Sun system, showing the zone centred on Earth on the rotating system X axis	274
<b>4.8.2</b>	Zero-velocity boundaries for the Jupiter–Sun system, showing a wide area zone centred on the Jupiter and Sun co-linear Lagrange points	275
<b>4.8.3</b>	The positions relative to the two bodies in the rotating reference frame	276
<b>4.8.4</b>	Locations of the Lagrange libration points	277
<b>4.8.5</b>	A manifold at the Earth–Sun L1 Lagrange point exhibiting Lissajous and high Earth elliptical orbit behaviour	287
<b>4.8.6</b>	Energy and angular momentum variation over a manifold at the Earth–Sun L1 Lagrange point exhibiting Lissajous and initial high Earth elliptical orbit behaviours	288
<b>4.8.7</b>	Manifolds at the Earth–Sun L1 Lagrange point exhibiting Lissajous and initial high Earth elliptical orbit behaviours, for a range of initial orbit osculating semi-major axes from 575,000 to 700,000 km	289

<b>4.8.8</b>	Energy and angular momentum variation over manifolds at the Earth–Sun L1 Lagrange point exhibiting Lissajous and initial high Earth elliptical orbit behaviours, for a range of initial orbit osculating semi-major axes between 575,000 and 700,000 km . . . . .	290
<b>4.8.9</b>	Manifolds at the Earth–Sun L1 Lagrange point exhibiting Lissajous and initial high Earth elliptical orbit behaviours, for a range of initial orbit osculating perigee altitudes from 500 to 36,000 km . . . . .	292
<b>4.8.10</b>	Energy and angular momentum variation over manifolds at the Earth–Sun L1 Lagrange point exhibiting Lissajous and initial high Earth elliptical orbit behaviours, for a range of initial orbit osculating perigee altitudes from 500 to 36,000 km . . . . .	293
<b>4.8.11</b>	State transition elements showing final position sensitivity to initial velocity, for a manifold linking low Earth perigee to a Lissajous orbit at L1 . . . . .	295
<b>4.8.12</b>	Transfer evolution starting from near Earth perigee with variations in argument of perigee of $0.05^\circ$ . . . . .	296
<b>4.8.13</b>	Transfer evolution of a 775000 semi-major axis initial orbit starting from near-Earth perigee, with variations in right ascension of ascending node of an approximately 110 degrees, reaching free injection Lissajous orbits . . . . .	296
<b>4.8.14</b>	Transfers from a 500 km perigee orbit to Lissajous orbits about the Earth–Sun L1 point, showing transfers both north and south of the ecliptic . . . . .	299
<b>4.8.15</b>	Transfers from a 500-km perigee orbit to Lissajous orbits about the Earth–Sun L1 point, showing transfers both north and south of the ecliptic and motion in the rotating $YZ$ plane . . . . .	300
<b>4.8.16</b>	Variation in rotating frame relative velocity with $X$ and $Y$ displacement in the Earth–Sun system for Jacobi: constant at the reference value . . . . .	302
<b>4.8.17</b>	Variation in rotating frame relative velocity with $X$ and $Y$ displacement in the Earth–Sun system for the case of the incremented Jacobi constant . . . . .	303
<b>4.8.18</b>	Planet relative velocity . . . . .	304
<b>4.8.19</b>	Variation in Earth relative energy (using rotating frame radial velocity vector direction assumption and reference Jacobi constant value) with rotating frame $X$ and $Y$ displacement . . . . .	304
<b>4.8.20</b>	Variation in Earth relative energy using radial (top) and prograde (middle) and retrograde (bottom) rotating frame velocity vector directions, with $X$ and $Y$ rotating frame displacement and reference Jacobi constant value . . . . .	306
<b>4.8.21</b>	Escape from an initial Earth-bound orbit with a semi-major axis of 1.8 million km, using gravitational assistance . . . . .	312
<b>4.8.22</b>	Evolution of Earth relative semi-major axis and energy from initial bound orbit to escape . . . . .	313
<b>4.8.23</b>	Evolution of angular momentum and position–velocity angle from the initial bound orbit to escape . . . . .	314
<b>4.8.24</b>	Evolution of Sun-relative semi-major axis and angular momentum from initial Earth bound orbit to escape . . . . .	316
<b>4.8.25</b>	Escape from lower energy initial Earth bound orbit and evolution of Sun-relative semi-major axis . . . . .	317
<b>4.8.26</b>	Transfer to a large-amplitude free injection Lissajous orbit from initial orbit with a semi-major axis of 1.8 million km . . . . .	318
<b>4.8.27</b>	Evolution of energy and semi-major axis with distance from Earth, for a transfer to a large amplitude free injection Lissajous orbit . . . . .	319

4.8.28	Evolution of Sun relative semi-major axis and angular momentum from initial perigee to free injection orbit . . . . .	320
4.8.29	Escape from an initial Jupiter-bound orbit with semi-major axis of 25 million km, using gravitational assistance . . . . .	321
4.8.30	Evolution of Jupiter relative semi-major axis and energy from initial bound orbit to escape . . . . .	322
4.8.31	Evolution of Sun relative semi-major axis and angular momentum from initial Jupiter bound orbit to escape . . . . .	323
4.8.32	Using low-thrust apogee-raising for gravitational escape from initial GTO . . .	324
4.8.33	Gravitational escape after low-thrust apogee-raising . . . . .	325
4.9.1	The principle of aerocapture. . . . .	329
4.9.2	Aerobraking drag and altitude profile during a typical pericentre passage. . .	331
4.9.3	Aerobraking velocity and altitude profile over a period of 100 days. . . . .	331
4.9.4	Aerobraking velocity and altitude profile over a period of 100–140 days . . .	332
4.9.5	Aerobraking at Mars from a 100,000-km apocentre insertion orbit over a 100-day period . . . . .	333
5.1.1	Transfer to Mercury in 2004: leaving Earth for Venus . . . . .	346
5.1.2	Transfer to Mercury in 2004: Venus to Venus 4:3 resonant orbit . . . . .	346
5.1.3	Transfer to Mercury in 2004: Venus to Mercury rendezvous after 1.5 revolutions . . . . .	347
5.1.4	Transfer to Mercury in 2004: Venus to Mercury and Mercury 2:3 and 3:4 resonant orbits . . . . .	347
5.1.5	Transfer to Mercury in 2012: motion relative to Mercury during the last two 1:1 resonant orbits . . . . .	348
5.1.6	Transfer with VGA/VGA/MGA/MGA/MGA/MGA/MGA for 2004 case . . .	350
5.1.7	Transfer with VGA/VGA/MGA/MGA/MGA/MGA/MGA for 2004 case, showing out-of-ecliptic motion . . . . .	350
5.1.8	Transfer with VGA/VGA/MGA/MGA for 2009 launch case. . . . .	351
5.1.9	Messenger type mission: initial Earth to Venus and Venus 1:1 resonant phases	353
5.1.10	Messenger type mission . . . . .	354
5.1.11	Messenger type mission. Transfer with VGA/VGA/MGA/MGA/MGA for final Earth departure in August 2005, showing motion out of the ecliptic . . . . .	354
5.1.12	Return from Mars via Venus gravity assist . . . . .	356
5.1.13	The V-V-E Cassini mission launched in 1989 and adapted to terminate at Jupiter . . . . .	359
5.1.14	The V-E-E Galileo mission, launched in 1989 . . . . .	360
5.1.15	Multiple gravity assist transfer with a launch in 2012 . . . . .	361
5.1.16	Period–pericentre loci for a range of Ganymede and Europa Vinfinities . . . .	363
5.1.17	Gravity assist and manoeuvre sequence in the Jovian system for orbit insertion strategy 1 at Europa plotted over period–pericentre loci at Ganymede and Europa. . . . .	365
5.1.18	Gravity assist and manoeuvre sequence in the Jovian system for orbit insertion strategy 2 at Europa plotted over period–pericentre loci at Ganymede and Europa. . . . .	366
5.1.19	Sequence of gravity assists at Ganymede and Europa. . . . .	367
5.1.20	Minimum $\Delta V$ transfer to Neptune with a V-E-E-J sequence after launch in 2013	370
5.1.21	Minimum $\Delta V$ transfer to Neptune with a V-E-E-J sequence after launch in 2013 but with an upper limit on transfer duration . . . . .	371



- [Loss of Eden: A Biography of Charles and Anne Morrow Lindbergh book](#)
- [read online Captive Bride](#)
- [click The Farm to Table Cookbook: The Art of Eating Locally](#)
- [click An Experiment in Criticism online](#)
- [read Without Reservations: The Travels of an Independent Woman](#)
- [read Worldviews: An Introduction to the History and Philosophy of Science](#)
  
- <http://toko-gumilar.com/books/A-Master-Plan-for-Rescue--A-Novel.pdf>
- <http://www.shreesaiexport.com/library/Captive-Bride.pdf>
- <http://metromekanik.com/ebooks/Yellowcake.pdf>
- <http://drmurphreesnewsletters.com/library/An-Experiment-in-Criticism.pdf>
- <http://aircon.servicessingaporecompany.com/?lib/Without-Reservations--The-Travels-of-an-Independent-Woman.pdf>
- <http://drmurphreesnewsletters.com/library/Worldviews--An-Introduction-to-the-History-and-Philosophy-of-Science.pdf>