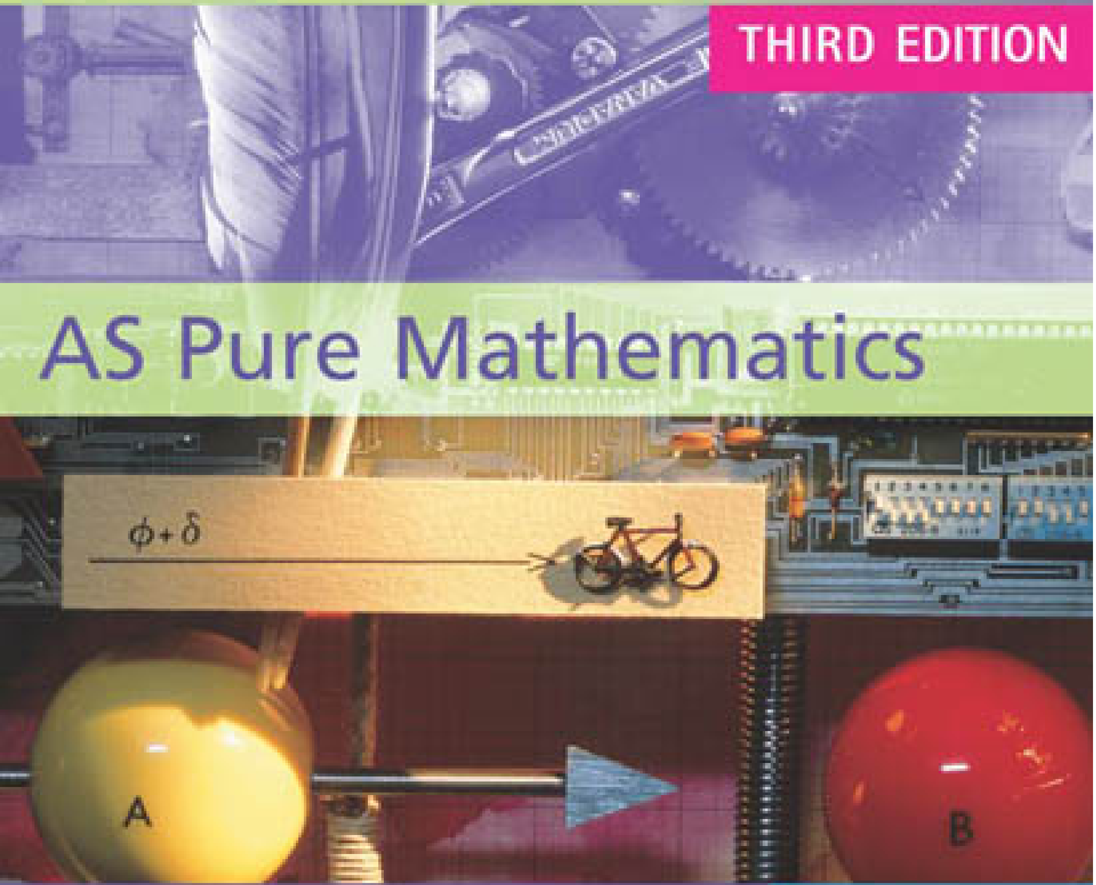
MEI STRUCTURED MATHEMATICS

THIRD EDITION

AS Pure Mathematics



Val Hanrahan
Jean Matthews
Roger Porkess
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C1
C2

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Series Editor: Roger Porkess

Hodder & Stoughton
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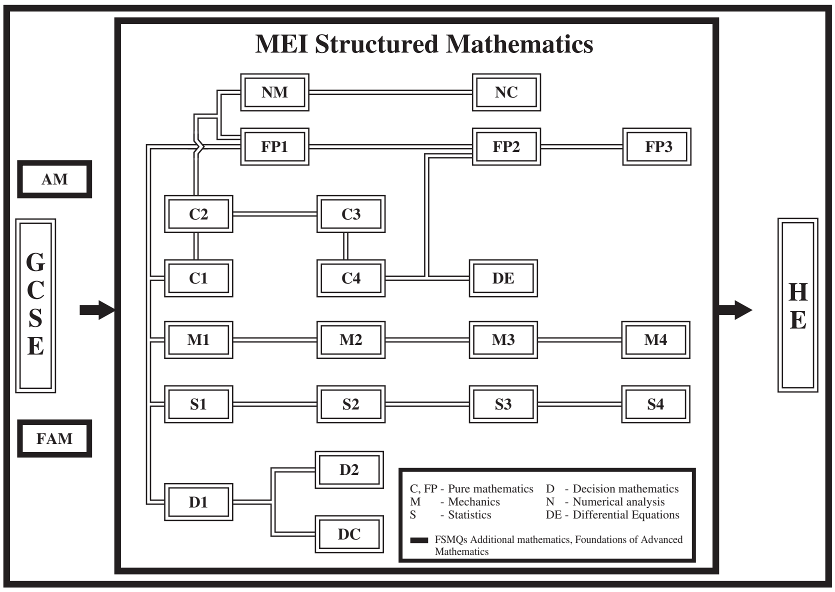
MEI Structured Mathematics

Mathematics is not only a beautiful and exciting subject in its own right but also one that underpins many other branches of learning. It is consequently fundamental to the success of a modern economy.

MEI Structured Mathematics is designed to increase substantially the number of people taking the subject post-GCSE, by making it accessible, interesting and relevant to a wide range of students.

It is a credit accumulation scheme based on 45 hour units which may be taken individually or aggregated to give Advanced Subsidiary (AS) and Advanced GCE (A Level) qualifications in Mathematics and Further Mathematics. The units may also be used to obtain credit towards other types of qualification.

The course is examined by OCR (previously the Oxford and Cambridge Schools Examination Board) with examinations held in January and June each year.



This is one of the series of books written to support the course. Its position within the whole scheme can be seen in the diagram above.

Mathematics in Education and Industry is a curriculum development body which aims to promote the links between Education and Industry in Mathematics at secondary school level, and to produce relevant examination and teaching syllabuses and support material. Since its foundation in the 1960s, MEI has provided syllabuses for GCSE (or O Level), Additional Mathematics and A Level.

For more information about MEI Structured Mathematics or other syllabuses and materials, write to MEI Office, Albion House, Market Place, Westbury, Wiltshire, BA13 3DE or visit www.mei.org.uk.

Introduction

The twelve chapters of this book cover the pure mathematics required for the AS subject criteria. The material is divided into the two units (or modules) for MEI Structured Mathematics: C1, *Introduction to Advanced Mathematics* and C2, *Concepts for Advanced Mathematics*. It is the first in a series of pure mathematics books for AS and A Levels in Mathematics and Further Mathematics.

Since their total content is the same, this book also covers the requirements of all the other specifications for AS Mathematics, and it is also suitable for other courses at this level.

Throughout the series the emphasis is on understanding rather than mere routine calculations, but the various exercises do nonetheless provide plenty of scope for practising basic techniques. Extensive on-line support is available via the MEI site, www.mei.org.uk.

This book is written on the assumption that you have at least grade C in GCSE Mathematics. You may, of course, know more mathematics than that; if so some of the early chapters will be partly revision for you but it is still important to check that you understand the basic work, particularly the algebra, and can handle it fluently and accurately.

This is the third edition of this series. Much of the content in this book was previously in *Pure Mathematics 1* and *2* but it has now been reorganised to meet the requirements of the new specification being first taught in September 2004. The original authors would like to thank Jean Matthews for her work in updating this book, and for her original contributions. We would also like to thank the various examination boards who have given permission for their past questions to be included in the exercises.

Roger Porkess
Series Editor

Key to symbols in this book

This symbol means that you want to discuss a point with your teacher. If you are working on your own there are answers in the back of the book. It is important, however, that you have a go at answering the questions before looking up the answers if you are to understand the mathematics fully.

This is a warning sign. It is used where a common mistake, misunderstanding or tricky point is being described.

This is the ICT icon. It indicates where you should use a graphic calculator or a computer.

This symbol and a dotted line down the right-hand side of the page indicates material which is beyond the criteria for the unit but which is included for completeness.

☆☆ Harder questions are indicated with stars. Many of these go beyond the usual examination standard.

Contents

Introduction to Advanced Mathematics

C1

1 Basic algebra

Sherlock Holmes: "Now the skillful workman is very careful indeed … He will have nothing but the tools which may help him in doing his work, but of these he has a large assortment, and all in the most perfect order."

A. Conan Doyle

Manipulating algebraic expressions

You will often wish to tidy up an expression, or to rearrange it so that it is easier to read its meaning. The following examples show you how to do this. You should practise the techniques for yourself on the questions in Exercise 1A.

Collecting terms

Very often you just need to collect like terms together, in this example those in x, those in y and those in z.

? What are 'like' and 'unlike' terms?

EXAMPLE 1.1

Simplify the expression $2x + 4y - 5z - 5x - 9y + 2z + 4x - 7y + 8z$.

SOLUTION

$$\begin{aligned} \text{Expression} &= 2x + 4x - 5x + 4y - 9y - 7y + 2z + 8z - 5z && \leftarrow \text{Collect like terms} \\ &= 6x - 5x + 4y - 16y + 10z - 5z && \leftarrow \text{Tidy up} \\ &= x - 12y + 5z && \leftarrow \text{This cannot be simplified further and so it is the answer.} \end{aligned}$$

Removing brackets

Sometimes you need to remove brackets before collecting like terms together.

EXAMPLE 1.2 Simplify the expression $3(2x - 4y) - 4(x - 5y)$.

SOLUTION

Open the brackets

Notice $(-4) \times (-5y) = +20y$

$$\begin{aligned} \text{Expression} &= 6x - 12y - 4x + 20y \\ &= 6x - 4x + 20y - 12y \\ &= 2x + 8y \end{aligned}$$

Collect like terms

Answer

EXAMPLE 1.3 Simplify $x(x + 2) - (x - 4)$.

SOLUTION

$$\begin{aligned} \text{Expression} &= x^2 + 2x - x + 4 \\ &= x^2 + x + 4 \end{aligned}$$

Open the brackets

Answer

EXAMPLE 1.4 Simplify $a(b + c) - ac$.

SOLUTION

$$\begin{aligned} \text{Expression} &= ab + ac - ac \\ &= ab \end{aligned}$$

Open the brackets

Answer

Factorisation

It is often possible to rewrite an expression as the product of two or more numbers or expressions, its *factors.* This usually involves using brackets and is called *factorisation.* Factorisation may make an expression easier to use and neater to write, or it may help you to interpret its meaning.

EXAMPLE 1.5 Factorise $12x - 18y$.

SOLUTION

6 is a factor of both 12 and 18.

$$\text{Expression} = 6(2x - 3y)$$

EXAMPLE 1.6 Factorise $x^2 - 2xy + 3xz$.

SOLUTION

x is a factor of all three terms.

$$\text{Expression} = x(x - 2y + 3z)$$

Multiplication

Several of the previous examples have involved multiplication of variables: cases like

$$a \times b = ab \quad \text{and} \quad x \times x = x^2.$$

In the next example the principles are the same but the expressions are not quite so simple.

EXAMPLE 1.7

Multiply $3p^2qr \times 4pq^3 \times 5qr^2$.

SOLUTION

You might well do this line in your head.

$$\begin{aligned}\text{Expression} &= 3 \times 4 \times 5 \times p^2 \times p \times q \times q^3 \times q \times r \times r^2 \\ &= 60 \times p^3 \times q^5 \times r^3 \\ &= 60p^3q^5r^3\end{aligned}$$

Fractions

The rules for working with fractions in algebra are exactly the same as those used in arithmetic.

EXAMPLE 1.8

Simplify $\dfrac{x}{2} - \dfrac{2y}{10} + \dfrac{z}{4}$.

SOLUTION

As in arithmetic you start by finding the common denominator. For 2, 10 and 4 this is 20.

Then you write each part as the equivalent fraction with 20 as its denominator, as follows.

$$\text{Expression} = \frac{10x}{20} - \frac{4y}{20} + \frac{5z}{20}$$

This line would often be left out.

$$= \frac{10x - 4y + 5z}{20}$$

EXAMPLE 1.9

Simplify $\dfrac{x^2}{y} - \dfrac{y^2}{x}$.

SOLUTION

$$\text{Expression} = \frac{x^3}{xy} - \frac{y^3}{xy}$$

The common denominator is xy.

$$= \frac{x^3 - y^3}{xy}$$

EXAMPLE 1.10

Simplify $\dfrac{3x^2}{5y} \times \dfrac{5yz}{6x}$.

SOLUTION

Since the two parts of the expression are multiplied, terms may be cancelled top and bottom as in arithmetic. In this case 3, 5, x and y may all be cancelled.

Expression $= \dfrac{\cancel{3}x^{\cancel{2}}}{\cancel{5}\cancel{y}} \times \dfrac{\cancel{5}\cancel{y}z}{\underset{2}{\cancel{6}\cancel{x}}}$

$= \dfrac{xz}{2}$

EXAMPLE 1.11

Simplify $\dfrac{(x-1)^3}{4x(x-1)}$.

SOLUTION

$(x-1)$ is a common factor of both top and bottom, so may be cancelled. However, x is not a factor of the top (the numerator), so may not be cancelled.

Expression $= \dfrac{(x-1)^2}{4x}$

EXAMPLE 1.12

Simplify $\dfrac{24x+6}{3(4x+1)}$.

SOLUTION

When the numerator (top) and/or the denominator (bottom) are not factorised, first factorise them as much as possible. Then you can see whether there are any common factors which can be cancelled.

Expression $= \dfrac{6(4x+1)}{3(4x+1)}$

$= 2$

EXERCISE 1A

The work in this exercise is almost completely routine. If you have access to a computer algebra system, use it to check your answers. By doing so you will also be learning how to use the system.

1 Simplify the following expressions by collecting like terms.

(i) $8x + 3x + 4x - 6x$
(ii) $3p + 3 + 5p - 7 - 7p - 9$
(iii) $2k + 3m + 8n - 3k - 6m - 5n + 2k - m + n$
(iv) $2a + 3b - 4c + 4a - 5b - 8c - 6a + 2b + 12c$
(v) $r - 2s - t + 2r - 5t - 6r - 7t - s + 5s - 2t + 4r$

2 Factorise the following expressions.

(i) $4x + 8y$ **(ii)** $12a + 15b - 18c$
(iii) $72f - 36g - 48h$ **(iv)** $p^2 - pq + pr$
(v) $12k^2 + 144km - 72kn$

3 Simplify the following expressions, factorising the answers where possible.

(i) $8(3x + 2y) + 4(x + 3y)$
(ii) $2(3a - 4b + 5c) - 3(2a - 5b - c)$
(iii) $6(2p - 3q + 4r) - 5(2p - 6q - 3r) - 3(p - 4q + 2r)$
(iv) $4(l + w + h) + 3(2l - w - 2h) + 5w$
(v) $5u - 6(w - v) + 2(3u + 4w - v) - 11u$

4 Simplify the following expressions, factorising the answers where possible.

(i) $a(b + c) + a(b - c)$ **(ii)** $k(m + n) - m(k + n)$
(iii) $p(2q + r + 3s) - pr - s(3p + q)$ **(iv)** $x(x - 2) - x(x - 6) + 8$
(v) $x(x - 1) + 2(x - 1) - x(x + 1)$

5 Perform the following multiplications, simplifying your answers.

(i) $2xy \times 3x^2y$ **(ii)** $5a^2bc^3 \times 2ab^2 \times 3c$
(iii) $km \times mn \times nk$ **(iv)** $3pq^2r \times 6p^2qr \times 9pqr^2$
(v) $rs \times 2st \times 3tu \times 4ur$

6 Simplify the following fractions as much as possible.

(i) $\dfrac{ab}{ac}$ **(ii)** $\dfrac{2e}{4f}$ **(iii)** $\dfrac{x^2}{5x}$ **(iv)** $\dfrac{4a^2b}{2ab}$ **(v)** $\dfrac{6p^2q^3r}{3p^3q^3r^2}$

7 Simplify the following as much as possible.

(i) $\dfrac{a}{b} \times \dfrac{b}{c} \times \dfrac{c}{a}$ **(ii)** $\dfrac{3x}{2y} \times \dfrac{8y}{3z} \times \dfrac{5z}{4x}$ **(iii)** $\dfrac{p^2}{q} \times \dfrac{q^2}{p}$

(iv) $\dfrac{2fg}{16h} \times \dfrac{4gh^2}{4fh} \times \dfrac{32fh^3}{12f^3}$ **(v)** $\dfrac{kmn}{3n^3} \times \dfrac{6k^2m^3}{2k^3m}$

8 Write the following as single fractions.

(i) $\dfrac{x}{2} + \dfrac{x}{3}$ **(ii)** $\dfrac{2x}{5} - \dfrac{x}{3} + \dfrac{3x}{4}$ **(iii)** $\dfrac{3z}{8} + \dfrac{2z}{12} - \dfrac{5z}{24}$

(iv) $\dfrac{2x}{3} - \dfrac{x}{4}$ **(v)** $\dfrac{y}{2} - \dfrac{5y}{8} + \dfrac{4y}{5}$

9 Write the following as single fractions.

(i) $\dfrac{3}{x} + \dfrac{5}{x}$ **(ii)** $\dfrac{1}{x} + \dfrac{1}{y}$ **(iii)** $\dfrac{4}{x} + \dfrac{x}{y}$

(iv) $\dfrac{p}{q} + \dfrac{q}{p}$ **(v)** $\dfrac{1}{a} - \dfrac{1}{b} + \dfrac{1}{c}$

10 Write the following as single fractions.

(i) $\dfrac{x+1}{4} + \dfrac{x-1}{2}$ **(ii)** $\dfrac{2x}{3} - \dfrac{x-1}{5}$ **(iii)** $\dfrac{3x-5}{4} + \dfrac{x-7}{6}$

(iv) $\dfrac{3(2x+1)}{5} - \dfrac{7(x-2)}{2}$ **(v)** $\dfrac{4x+1}{8} + \dfrac{7x-3}{12}$

11 Simplify the following expressions.

(i) $\dfrac{x+3}{2x+6}$ **(ii)** $\dfrac{6(2x+1)^2}{3(2x+1)^5}$ **(iii)** $\dfrac{2x(y-3)^4}{8x^2(y-3)}$

(iv) $\dfrac{6x-12}{x-2}$ **(v)** $\dfrac{(3x+2)^2}{6x} \times \dfrac{x^4}{6x+4}$

Equations

? What is a variable?

You will often need to find the value of the variable in an expression in a particular case, as in the following example.

EXAMPLE 1.13

A polygon is a closed figure whose sides are straight lines. Figure 1.1 shows a seven-sided polygon (a heptagon).

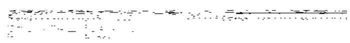


Figure 1.1

An expression for $S°$, the sum of the angles of a polygon with n sides, is

$$S = 180(n-2).$$

? How is this expression obtained?
Try dividing a polygon into triangles, starting from one vertex.

Find the number of sides in a polygon with an angle sum of **(i)** 180° **(ii)** 1080°.

SOLUTION

(i) Substituting 180 for S gives $180 = 180(n-2)$

This is an equation which can be solved to find n.

Dividing both sides by 180 $\Rightarrow 1 = n-2$

Adding 2 to both sides $\Rightarrow 3 = n$

The polygon has three sides: it is a triangle.

(ii) Substituting 1080 for S gives $1080 = 180(n-2)$

Dividing both sides by 180 $\Rightarrow 6 = n-2$

Adding 2 to both sides $\Rightarrow 8 = n$

The polygon has eight sides: it is an octagon.

Example 1.13 illustrates the process of solving an equation. An *equation* is formed when an expression, in this case $180(n-2)$, is set equal to a value, in this case 180 or 1080, or to another expression. *Solving* means finding the value(s) of the variable(s) in the equation.

Since both sides of an equation are equal, you may do what you wish to an equation provided that you do exactly the same thing to both sides. If there is only one variable involved (like n in the above examples), you aim to get that on one side of the equation, and everything else on the other. The two examples which follow illustrate this.

In both of these examples the working is given in full, step by step. In practice you would expect to omit some of these lines by tidying up as you went along.

Look at the statement $5(x-1) = 5x-5$.
What happens when you try to solve it as an equation?

This is an *identity* and not an equation. It is true for *all* values of x.

For example, try $x = 11$: $5(x-1) = 5 \times (11-1) = 50$; $5x-5 = 55-5 = 50$ ✓,
or try $x = 46$: $5(x-1) = 5 \times (46-1) = 225$; $5x-5 = 230-5 = 225$ ✓,
or try $x =$ anything else and it will still be true.

To distinguish an identity from an equation, the symbol $\equiv$ is sometimes used.

Thus $5(x-1) \equiv 5x-5$.

EXAMPLE 1.14

Solve the equation $5(x-3) = 2(x+6)$.

SOLUTION

Open the brackets $\Rightarrow 5x-15 = 2x+12$

Subtract $2x$ from both sides $\Rightarrow 5x-2x-15 = 2x-2x+12$

Tidy up $\Rightarrow 3x-15 = 12$

Add 15 to both sides $\Rightarrow 3x-15+15 = 12+15$

Tidy up $\Rightarrow 3x = 27$

Divide both sides by 3 $\Rightarrow \dfrac{3x}{3} = \dfrac{27}{3}$

$\Rightarrow x = 9$

CHECK

When the answer is substituted in the original equation both sides should come out to be equal. If they are different, you have made a mistake.

Left-hand side	**Right-hand side**
$5(x - 3)$	$2(x + 6)$
$5(9 - 3)$	$2(9 + 6)$
5×6	2×15
30	30 (as required).

EXAMPLE 1.15

Solve the equation $\frac{1}{2}(x + 6) = x + \frac{1}{3}(2x - 5)$.

SOLUTION

Start by clearing the fractions. Since the numbers 2 and 3 appear on the bottom line, multiply through by 6 which cancels both of them.

Multiply both sides by 6	$\Rightarrow$	$6 \times \frac{1}{2}(x + 6) = 6 \times x + 6 \times \frac{1}{3}(2x - 5)$
Tidy up	$\Rightarrow$	$3(x + 6) = 6x + 2(2x - 5)$
Open the brackets	$\Rightarrow$	$3x + 18 = 6x + 4x - 10$
Subtract $6x$, $4x$, and 18 from both sides	$\Rightarrow$	$3x - 6x - 4x = -10 - 18$
Tidy up	$\Rightarrow$	$-7x = -28$
Divide both sides by (-7)	$\Rightarrow$	$\frac{-7x}{-7} = \frac{-28}{-7}$
	$\Rightarrow$	$x = 4$

CHECK

Substituting $x = 4$ in $\frac{1}{2}(x + 6) = x + \frac{1}{3}(2x - 5)$ gives:

Left-hand side	**Right-hand side**
$\frac{1}{2}(4 + 6)$	$4 - \frac{1}{3}(8 - 5)$
$\frac{10}{2}$	$4 + \frac{3}{3}$
5	5 (as required).

EXERCISE 1B

The first question in this exercise involves routine solution of equations. Good calculators and computer algebra systems will do this work for you. If you have either of these facilities to hand, use them to check your answers.

In the final eight questions, you are asked to set up the equations as well as solve them, and this is something which a machine will not do for you.

1 Solve the following equations.

(i) $5a - 32 = 68$
(ii) $4b - 6 = 3b + 2$
(iii) $2c + 12 = 5c + 12$

(iv) $5(2d + 8) = 2(3d + 24)$

(v) $3(2e - 1) = 6(e + 2) + 3e$

(vi) $7(2 - f) - 3(f - 4) = 10f - 4$

(vii) $5g + 2(g - 9) = 3(2g - 5) + 11$

(viii) $3(2h - 6) - 6(h + 5) = 2(4h - 4) - 10(h + 4)$

(ix) $\frac{1}{2}k + \frac{1}{4}k = 36$

(x) $\frac{1}{2}(\ell - 5) + \ell = 11$

(xi) $\frac{1}{2}(3m + 5) + 1\frac{1}{2}(2m - 1) = 5\frac{1}{2}$

(xii) $n + \frac{1}{3}(n + 1) + \frac{1}{4}(n + 2) = \frac{5}{6}$

2 The largest angle of a triangle is six times as big as the smallest. The third angle is 75°.

(i) Write this information in the form of an equation for a, the size in degrees of the smallest angle.

(ii) Solve the equation and so find the sizes of the three angles.

3 Beth and Polly are twins and their sister Louise is 2 years older than them. The total of their ages is 32 years.

(i) Write this information in the form of an equation for ℓ, Louise's age in years.

(ii) What are the ages of the three girls?

4 The length, d m, of a rectangular field is 40 m greater than the width. The perimeter of the field is 400 m.

(i) Write this information in the form of an equation for d.

(ii) Solve the equation and so find the area of the field.

5 Simon can buy three pencils and have 49p change, or he can buy five pencils and have 15p change.

(i) Write this information as an equation for x, the cost in pence of one pencil.

(ii) How much money did Simon have to start with?

6 A train has 8 coaches, f of which are first class and the rest standard class. A first class coach seats 48 passengers, a standard class 64.

(i) Write down an expression in terms of f for the seating capacity of the train.

(ii) The seating capacity of the train is 480. Form an equation for f and solve it. How many standard class coaches does the train have?

7 In a multiple-choice examination of 25 questions, four marks are given for each correct answer and two marks are deducted for each wrong answer. One mark is deducted for any question which is not attempted. A candidate attempts q questions and gets c correct.

(i) Write down an expression for the candidate's total mark in terms of q and c.

(ii) James attempts 22 questions and scores 55 marks. Write down and solve an equation for the number of questions which James gets right.

8 Joe buys 18 kg of potatoes. Some of these are old potatoes at 22 p per kilogram, the rest are new ones at 36p per kilogram.

(i) Denoting the weight of old potatoes he buys by w kg, write down an expression for the total cost of Joe's potatoes.

(ii) Joe pays with a £5 note and receives 20p change. What weight of new potatoes does he buy?

9 In 18 years' time Halley will be five times as old as he was 2 years ago.

(i) Write this information in the form of an equation involving Halley's present age, a years.

(ii) How old is Halley now?

Changing the subject of an equation

The area of a trapezium is given by

$$A = \tfrac{1}{2}(a + b)h$$

where a and b are the lengths of the parallel sides and h is the distance between them (see figure 1.2). An equation like this is often called a *formula*.

Figure 1.2

The variable A is called the subject of this formula because it only appears once on its own on the left-hand side. You often need to make one of the other variables the subject of a formula. In that case, the steps involved are just the same as those in solving an equation, as the following examples show.

EXAMPLE 1.16

Make a the subject in $A = \tfrac{1}{2}(a + b)h$.

SOLUTION

It is usually easiest if you start by arranging the equation so that the variable you want to be its subject is on the left-hand side.

$$\tfrac{1}{2}(a + b)h = A$$

Multiply both sides by 2 $\Rightarrow$ $(a + b)h = 2A$

Divide both sides by h $\Rightarrow$ $a + b = \dfrac{2A}{h}$

Subtract b from both sides $\Rightarrow$ $a = \dfrac{2A}{h} - b$

EXAMPLE 1.17

Make T the subject in the simple interest formula $I = \dfrac{PRT}{100}$.

SOLUTION

Arrange with T on the left-hand side $\quad \dfrac{PRT}{100} = I$

Multiply both sides by 100 $\quad \Rightarrow \quad PRT = 100I$

Divide both sides by P and R $\quad \Rightarrow \quad T = \dfrac{100I}{PR}$

EXAMPLE 1.18

Make x the subject in the formula $v = \omega\sqrt{a^2 - x^2}$. (This formula gives the speed of an oscillating point.)

SOLUTION

Square both sides $\quad \Rightarrow \quad v^2 = \omega^2(a^2 - x^2)$

Divide both sides by ω^2 $\quad \Rightarrow \quad \dfrac{v^2}{\omega^2} = a^2 - x^2$

Add x^2 to both sides $\quad \Rightarrow \quad \dfrac{v^2}{\omega^2} + x^2 = a^2$

Subtract $\dfrac{v^2}{\omega^2}$ from both sides $\quad \Rightarrow \quad x^2 = a^2 - \dfrac{v^2}{\omega^2}$

Take the square root of both sides $\quad \Rightarrow \quad x = \pm\sqrt{a^2 - \dfrac{v^2}{\omega^2}}$

EXAMPLE 1.19

Make m the subject of the formula $mv = I + mu$. (This formula gives the momentum after an impulse.)

SOLUTION

Collect terms in m on the left-hand side
and terms without m on the other. $\quad \Rightarrow \quad mv - mu = I$

Factorise the left-hand side $\quad \Rightarrow \quad m(v - u) = I$

Divide both sides by $(v - u)$ $\quad \Rightarrow \quad m = \dfrac{I}{v - u}$

EXERCISE 1C

1 Make **(i)** a **(ii)** t the subject in $v = u + at$.

2 Make h the subject in $V = \ell wh$.

3 Make r the subject in $A = \pi r^2$.

4 Make **(i)** s **(ii)** u the subject in $v^2 - u^2 = 2as$.

5 Make h the subject in $A = 2\pi rh + 2\pi r^2$.

6 Make a the subject in $s = ut + \frac{1}{2}at^2$.

7 Make b the subject in $h = \sqrt{a^2 + b^2}$.

8 Make g the subject in $T = 2\pi\sqrt{\frac{\ell}{g}}$.

9 Make m the subject in $E = mgh + \frac{1}{2}mv^2$.

10 Make R the subject in $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

11 Make h the subject in $bh = 2A - ah$.

12 Make u the subject in $f = \dfrac{uv}{u + v}$.

13 Make d the subject in $u^2 - du + fd = 0$.

14 Make V the subject in $p_1VM = mRT + p_2VM$.

? All the equations in Exercise 1C refer to real situations. Can you recognise them?

Quadratic equations

EXAMPLE 1.20

The length of a rectangular field is 40 m greater than its width, and its area is 6000 m^2. Form an equation involving the length, x m, of the field.

SOLUTION

Since the length of the field is 40 m greater than the width,

the width in m must be $x - 40$

and the area in m^2 is $x(x - 40)$.

So the required equation is $x(x - 40) = 6000$

or $x^2 - 40x - 6000 = 0$.

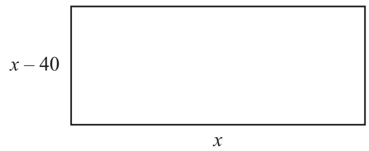


Figure 1.3

This equation, involving terms in x^2 and x as well as a constant term (i.e. a number, in this case 6000), is an example of a *quadratic equation.* This is in contrast to a linear equation. A linear equation in the variable x involves only terms in x and constant terms.

It is usual to write a quadratic equation with the right-hand side equal to zero. To solve it, you first factorise the left-hand side if possible, and this requires a particular technique.