Gerald R. Hintz

Orbital Mechanics and Astrodynamics

Techniques and Tools for Space Missions

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To: My wife
Mary Louise Hintz

Preface

This book is based on my work as an engineer and functional area manager for 37 years at NASA's Jet Propulsion Laboratory (JPL) and my teaching experience with graduate-level courses in Astronautical Engineering at the University of Southern California (USC).

At JPL, I worked on the development and flight operations of space missions, including Viking I and II (two orbiters and two landers to Mars), Mariner 9 (orbiter to Mars), Seasat (an earth orbiter), Voyager (for the Neptune encounter), Pioneer Venus Orbiter, Galileo (probe and orbiter to Jupiter), Ulysses (solar polar mission), Cassini-Huygens (orbiter to Saturn and lander to Titan), and Aquarius (an earth orbiter). I provided mission development or operation services to space missions that traveled to all the eight planets, except Mercury. These missions furnish many of the examples of mission design and analysis and navigation activities that are described in this text. The engineering experience at JPL has furnished the set of techniques and tools for space missions that are the core of this textbook.

I am an adjunct professor at USC, where I have taught a graduate course in Orbital Mechanics since 1979, plus three other graduate courses that I have initiated and developed. This teaching experience has enabled me to show that the techniques and tools for space missions have been developed from the basic principles of Newton and Kepler. The book has been written from my class notes. So, in a sense, I have been writing it for 35 years and I am very proud to see it in print.

The reason for writing this book is to put the results from these experiences together in one presentation, which I will continue to use at USC and share with my students and colleagues. The reader can expect to find an organized and detailed study of the controlled flight paths of spacecraft, including especially the techniques and tools used in analyzing, designing, and navigating space missions.

In academia, this book will be used by graduate students to study Orbital Mechanics or to do research in challenging endeavors such as the safe return of humans to the moon. (See Chaps. 6 and 7.) It will also serve well as a textbook for an Orbital Mechanics course for upper-division undergraduate and other advanced undergraduate students. Professional engineers working on space missions and people who are interested in learning how space missions are designed and navigated will also use the book as a reference.

This presentation benefits significantly from the many references listed in the back of the book. The list includes excellent textbooks by Marshall H. Kaplan, John E. Prussing and Bruce A. Conway, Richard H. Battin, and others and a technical report by Paul A. Penzo for the Apollo missions. Papers include those by Leon Blitzer, John E. Prussing, and Roger Broucke. Finally, there is the contribution of online sources, such as Eric Weisstein's "World of Scientific Biography," JPL's Near-Earth Objects and Solar System Dynamics, and the Rocket & Space Technology websites. To all these sources and the many others cited in the text, I express my gratitude.

My gratitude is also extended to my wife, Mary Louise Hintz, and our three children, JJ, Tana, and Kristin, for their support.

Los Angeles, CA, USA Gerald R. Hintz

Contents

Introduction

Our objective is to study the controlled flight paths of spacecraft, especially the techniques and tools used in this process. The study starts from basic principles derived empirically by Isaac Newton, that is, Newton's Laws of Motion, which were derived from experience or observation. Thus, we develop the relative 2-body model consisting of two particles, where one particle is more massive (the central body) and the other (the spacecraft) moves about the first, and the only forces acting on this system are the mutual gravitational forces. Kepler's Laws of Motion are proved from Newton's Laws. Solving the resulting equations of motion shows that the less-massive particle moves in a conic section orbit, i.e., a circle, ellipse, parabola, or hyperbola, while satisfying Kepler's Equation. Geometric properties of conic section orbits, orbit classification, and types of orbits are considered with examples. Astronomical constants needed in this study are supplied, together with several tables of geometric formulas for elliptic orbits.

After the orbit determination analyst estimates the spacecraft's orbit, trajectory correction maneuvers (TCMs) are designed to correct that estimated orbit to the baseline that satisfies mission and operational requirements and constraints. Such TCMs correct statistical (usually small) errors, while other maneuvers make adjustments (usually large) such as insertion of the spacecraft into an orbit about a planet from a heliocentric trajectory. Maneuver strategies considered include the optimal 2-maneuver Hohmann transfer and the optimal 3-maneuver bi-elliptic transfer with examples for comparison purposes. The design of TCMs determines the amount of velocity change required to correct the trajectory. The Rocket Equation is then used to determine the amount of propellant required to achieve the required change in velocity. Various fuel and oxidizer combinations are considered that generate the specific impulse, the measure of a propellant's capability, required to implement the orbit correction.

Gravity assists obtained when flying by planets in flight to the target body (another planet, comet or asteroid, or the sun) can produce a large velocity change with no expenditure of onboard propellant. Types and examples of interplanetary missions and the targeting space used in designing the required trajectories are described.

Techniques of Astrodynamics include algorithms for propagating the spacecraft's trajectory, Keplerian orbit elements which describe the orbit's size, shape, and orientation in space and the spacecraft's location in the orbit, and Lambert's Problem, which is used to generate mission design curves called "pork chop plots". Other models advance our study to treat n bodies and distributed masses instead of just two point masses, and measure time, which is fundamental to our equations.

Non-Keplerian motion takes into account perturbations to the Keplerian model, such as oblateness of the central body, gravitational forces of other bodies ("3rd body effects"), solar wind and pressure, and attitude correction maneuvers. The study identifies the primary perturbations for an earth-orbiting vehicle, resolves a satellite orbit paradox, and considers "zero G" (or is it "zero W"?).

A strategy for rendezvousing a spacecraft with other vehicles such as the International Space Station is described with examples. One example is rendezvous of the Apollo 11 Lunar Excursion Module with the Command Module. One strategy is intended to avoid an unintentional rendezvous by placing the spacecraft in a standoff position with respect to another vehicle to, for example, allow the astronauts to sleep in safety.

Navigation techniques and tools include a TCM design tool and two methods for designing free-return circumlunar trajectories for use in returning humans to the moon safely. Launching into a free-return trajectory will ensure that the spacecraft will return to a landing site on the earth without the use of any propulsive maneuvers in case of an accident such as the one experienced by Apollo 13. After the spacecraft is determined to be in good working condition, it can be transferred from the free-return trajectory into one favorable for injection into a lunar orbit.

Chapter 8 discusses opportunities for further study in navigation, mission design and analysis, and related topics. Appendix A gives a brief review of vector analysis, which is especially important for students who are returning to academia after a long absence. Appendix B defines projects the students can perform to test and strengthen their knowledge of Astrodynamics and the techniques and tools for space missions. Appendix C provides additional parameters for use in designing free-return circumlunar trajectories.

There are exercises at the end of each of Chaps. 1–7 for use in strengthening and testing the students' grasp of the technical material. To aid the students in this process, numerical answers with units are supplied in the back of the book for selected exercises.

References for the material covered are listed at the end of the book. References are also listed at the end of Chaps. 1–7 and at selected points within these chapters, where they are identified by the names of the authors (or editors). In the case of authors who provided multiple sources, the year or year and month of the reference is (are) given. The students can then check the list in the back of the book to obtain the complete bibliographic information for identifying the particular source. Chapter 8 gives complete bibliographic information for the references it cites as sources of material for further study. The references in Chapter 8 are not repeated in the list at the end of the book.

Many terms are used in discussing Orbital Mechanics and Astronautics. The definitions of terminology used in this textbook are called out as "Def.:" followed by the definition with the term being defined underlined for clarity. Acronyms and abbreviations are defined at their first use and included in a list in the back of the book. Finally, an index is also provided to aid the reader in finding the various terms and topics covered in this text.

Fundamentals of Astrodynamics 1

1.1 Introduction

One of the most important uses of vector analysis (cf. Appendix A) is in the concise formulation of physical laws and the derivation of other results from these laws. We will develop and use the differential equations of motion for a body moving under the influence of a gravitational force only. In Chap. 5, we will add other (perturbing) forces to our model.

There are related disciplines, which are part of Flight Dynamics.

Def.: Celestial Mechanics is the study of the natural motion of celestial bodies.

Def.: Astrodynamics is the study of the controlled flight paths of spacecraft.

Def.: Orbital Mechanics is the study of the principles governing the motion of bodies around other bodies under the influence of gravity and other forces.

These subjects consider translational motion in a gravity field.

Attitude Dynamics and Attitude Control consider the spacecraft's rotational motion about its center of mass.

Def.: Spacecraft attitude dynamics is the applied science whose aim is to understand and predict how the spacecraft's orientation evolves.

In spacecraft mission activities, there is a coupling between satellite translation (the orbital variables) and spacecraft rotation (the attitude variables). In spite of the coupling effects, much of orbital mechanics proceeds by largely ignoring the effects of spacecraft attitude dynamics and vice versa. The field of Flight Dynamics, however, considers 6 degrees of freedom (DOF), consisting of 3 DOF from Orbital Mechanics and 3 DOF from Attitude Dynamics.

An example of an essentially 6DOF problem is: EDL (entry, descent, and landing), e.g., the landing of the Phoenix spacecraft on Mars 5/25/08. More information on the Phoenix mission can be found at the Phoenix Mars Mission Website at http://phoenix.lpl.arizona.edu/.

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Parallel disciplines that must be part of spacecraft mission analyses include:

Orbital Mechanics	Attitude Dynamics
Orbit Determination	Attitude Determination
Flight Path Control	Attitude Control

Of these six disciplines, we consider primarily Orbital Mechanics plus related issues in Flight Path Control. Hence, our objective is to study the controlled flight paths of spacecraft, viz., Astrodynamics.

1.2 Mathematical Models

Use of Mathematical Models to Solve Physical Problems

Figure 1.1 describes the procedure for using a mathematical model to solve a physical problem.

In engineering, we make simplifying assumptions in our mathematical model to:

1. Get a good approximation to a solution
2. Gain insight into the problem
3. Get a good starting point for a more accurate numerical solution
4. Reduce computing time and costs.

Example: Archimedes

The king told Archimedes that he had given the goldsmith gold to make a crown for him. However, he suspected that the goldsmith had kept some of the gold and added a baser metal in its place. So his task for Archimedes was to determine whether or not his new crown was made of pure gold. Archimedes thought about this problem until one day when he was in the public bath and he saw water splashing out of a bathtub. Then, he yelled "Eureka" and ran to his working area to demonstrate the answer to the problem.

He placed the crown in a vat filled with water with a basin below the vat to catch the overflow. He obtained the amount of gold that equaled the volume of water that overflowed the vat. Then he placed this amount of gold on one side of a lever and the crown on the other side as shown in Fig. 1.2. The end of the lever with the gold descended, indicating that the crown was not pure gold. After Archimedes reported his findings to the king, the goldsmith did not cheat any more kings.

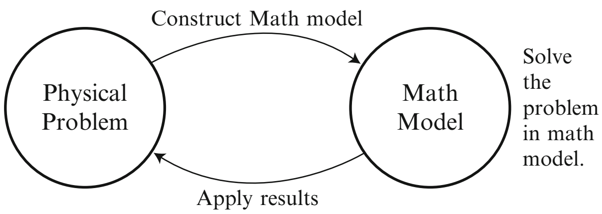

Fig. 1.1 Using a mathematical model to solve physical problems

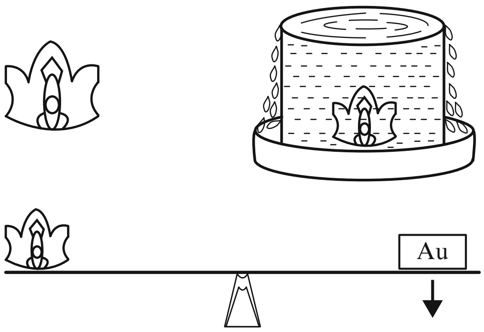

Fig. 1.2 Archimedes' Gold Experiment

Dynamics, including Astrodynamics, is a deductive discipline, which enables us:

1. To describe in quantitative terms how mechanical systems move when acted on by given forces or
2. To determine which forces must be applied to a system to cause it to move in a specified manner.

A dynamics problem is solved in two major steps:

1. Formulation of the equation of motion (EOM), the math model, and
2. Extraction of information from the EOM.

Optimization of rocket trajectories is usually accomplished by analytical and numerical approaches in a complimentary fashion. Dereck Lawden (cf. reference for Lawden) says; "... by making suitable simplifying assumptions, the actual problem can be transformed into an idealized problem whose solution is analytically tractable, then this latter solution will often provide an excellent substitute for the optimal motor thrust programme in the actual situation. All that then remains to be done is to recompute the trajectory employing this programme and taking account of the real circumstances. Further, it is only by adopting the analytical approach in any field of research, that those general principles, which lead to a real understanding of the nature of the solutions, are discovered. Lacking such an appreciation, our sense of direction for the numerical attack will be defective and, as a consequence, computations will become unnecessarily lengthy or even quite ineffective."

The analytical solution provides insight into how to approach a problem. It also enables you to verify that your solution is plausible and correct. You do not want to put yourself in the position of having your boss tell you that the results you have presented violate a basic principle and then be forced to say, "But the computer said . . . ." Another reason for looking at an idealized problem is that the insight gained can be used for mission planning and design purposes or feasibility studies for which exact values are not available.

High-precision software run in land-based computers or powerful real-time onboard computing provide precision numerical results and ultimately the commands to be executed by the spacecraft's onboard subsystems.

Coordinate Systems

To study motion, we need to set up a reference frame because we need to know "motion with respect to what?"

Inertial frames are "fixed with respect to the fixed stars," i.e., non-rotating and non-accelerating with respect to the fixed (from our perspective) stars, which is an imaginary situation. Practically speaking, an inertial system is moving with essentially constant velocity.

Example Geocentric equatorial system or Earth-centered Inertial (ECI) coordinate system

Use: To study orbital motion about the Earth

Definition:

- Origin at the center of the earth
- X-axis pointing to the first point of Aries, i.e., the vernal equinox. The vernal equinox direction is a directed line from the earth to the sun at the instant the sun passes through the earth's equatorial plane at the beginning of spring.
- Z-axis—normal to the instantaneous equatorial plane
- Y satisfies $\mathbf{Y} = \mathbf{Z} \times \mathbf{X}$, which completes the right-handed coordinate system

Example Heliocentric-ecliptic system

Use: for example to study orbital motion in interplanetary (I/P) flight

Definition:

- Origin at the center of mass of the sun
- The fundamental (XY) plane is the mean plane of the earth's orbit, called the ecliptic plane.
- The reference (X) direction is again the vernal equinox, where the X axis is the intersection of these two fundamental planes and points to the sun when it crosses the equator from south to north in its apparent annual motion along the ecliptic.

The directions of the vernal equinox and the earth's axis of rotation shift slowly in ways to be discussed later (Chap. 5). Therefore, we will refer to X, Y, Z coordinates for the equator and equinox of a particular year or date, e.g., equator and equinox of 2000.0 or "of date." We will consider measures of time in Chap. 4. For now, we will not consider this level of precision, ignoring perturbations such as the precession of the earth. We consider an inertial system that is fixed with respect to the fixed stars as Newton did. For more information on coordinate systems, see for example Sect. 2.2 of the reference by Bate, Mueller, and White (BMW).

Non-inertial systems are rotating or accelerating. For example, a system that is fixed to the earth is rotating and, therefore, non-inertial. Such a coordinate system is chosen as the one that is natural for a particular type of problem.

1.3 Physical Principles

Tycho Brahe (1546–1601), a Danish astronomer, took accurate observations of the position of Mars before the telescope was invented. Brahe used a quadrant circle to sight the planets and stars. His large, accurate instruments yielded measurements that were accurate to within 4 min of arc. Brahe hired Kepler as an assistant to analyze the vast bulk of data that he had collected.

Johannes Kepler (1571–1630), an Austrian mathematician and astronomer, worked briefly with Tycho Brahe and inherited his data books after Brahe's death in 1601. Kepler devoted many years to intense study of these data to determine a mathematical description of the planetary motion described by the data. He derived a set of three empirical laws that describe planetary motion and led to our current understanding of the orbital motion of planets, moons, asteroids, and comets as well as artificial satellites and spacecraft.

Empirical laws are known from experience or observation. We derive results from these laws. In particular, we will derive the equations of motion from Newton's Laws of Motion and his Universal Law of Gravitation.

Kepler's Laws

Kepler's Laws are:

1. The orbit of each planet is an ellipse with the sun at a focus.
2. The line joining the planet to the sun sweeps out equal areas inside the ellipse in equal time intervals.

   Therefore, the velocity at closest approach is greater than the velocity at the furthest distance from the sun. Kepler's Second Law is illustrated in Fig. 1.3.
3. The square of the period of a planet is proportional to the cube of its mean distance from the sun. That is,

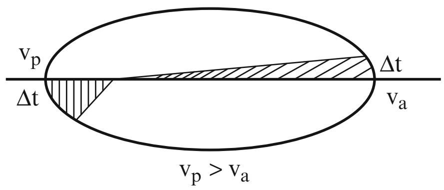

Fig. 1.3 Kepler's Second Law

$$\tau_{\text{planet}}^2 \propto (\text{mean distance from sun})^3$$

where the symbol τ denotes the period (duration) of an orbit.

Johannes Kepler (1571–1630), an Austrian mathematician and astronomer, pursued his scientific career with extraordinary enthusiasm and diligence despite several hardships. His hands were crippled and his eyesight impaired from smallpox as a boy. He suffered from religious persecution for his protestant beliefs. He lost his first wife and several children. Often in desperate financial difficulties, he endured a bare subsistence livelihood. He even had to defend his mother from a charge of witchcraft.

Kepler, as Imperial Mathematician in Prague, published his third law in *Harmonice Mundi* (The Harmony of the World) in 1619, 10 years after the appearance of his first two laws in *Astronomia Nova De Motibus Stellae Martis*, known as *Astronomia Nova*.

Newton's Laws

Sir Isaac Newton[1] (1642–1727) defined the forces at work in *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy), usually called the *Principia,* 1687. Kepler's Laws must follow. Newton determined why the planets move in this manner. Newton's Laws apply only to particles moving in an inertial reference frame.

Newton's Laws of Motion are:

1. Principle of Inertia: Every body is at rest or in uniform motion along a straight line unless it is acted on by a force.
2. Principle of Momentum: The rate of change of linear momentum is equal to the force impressed and is in the same direction as that force. That is,

[1] Isaac Newton (1642–1727) is generally regarded as one of the greatest mathematicians of all time. He entered Trinity College, Cambridge, in 1661 and graduated with a BA degree in 1665. In 1668, he received a master's degree and was appointed Lucasian Professor of Mathematics, one of the most prestigious positions in English academia at the time. In his latter years, Newton served in Parliament and was warden of the mint. In 1703, he was elected president of the Royal Society of London, of which he had been a member since 1672. Two years later, he was knighted by Queen Anne.

Newton is given co-credit, along with the German Wilhelm Gottfried von Leibnitz, for the discovery and development of calculus-work that Newton did in the period 1664–1666 but did not publish until years later, thus laying the groundwork for an ugly argument with Leibnitz over who should get credit for the discovery. In 1687, at the urging of the astronomer Edmund Halley, Newton published his ground-breaking compilation of mathematics and science, *Principia Mathematica,* which is apparently the first place that the root-finding method that bears his name appears, although he probably had used it as early as 1669. This method is called "Newton's Method" or "the Newton–Raphson Method."