McGraw Hill
The Most Trusted Name in Education

PRACTICE MAKES PERFECT™

MASTER MATH at your own pace

Trigonometry

- Make confident first steps in learning trigonometry
- Get a thorough overview of basic trigonometry
- Practice with more than 700 exercises

Carolyn Wheater

PRACTICE MAKES PERFECT™

Trigonometry

This page intentionally left blank

PRACTICE MAKES PERFECT™

Trigonometry

Carolyn Wheater

McGraw Hill

New York Chicago San Francisco Lisbon London Madrid Mexico City
Milan New Delhi San Juan Seoul Singapore Sydney Toronto

ISBN: 978-0-07-178509-9

MHID: 0-07-178509-4

The material in this eBook also appears in the print version of this title: ISBN: 978-0-07-176179-6,
MHID: 0-07-176179-9.

Contents

Preface

Math has a lot in common with music, dance, and sports. There are skills to be learned and a sequence of activities you need to go through if you want to be good at it. You don't just read math, or just listen to math, or even just understand math. You do math, and to learn to do it well, you have to practice. That's why homework exists, but most people need more practice than homework provides. That's where *Practice Makes Perfect: Trigonometry* comes in.

The study of trigonometry starts with material you've learned in geometry and expands upon it, giving you both more powerful tools for very practical applications and more analytical approaches to the concepts. It merges all this with the skills you acquired in algebra, asking you to think about solving equations and graphing functions. The exercises in this book are designed to help you acquire the skills you need, practice each one individually until you have confidence in it, and then combine various skills to solve more complicated problems.

You can use *Practice Makes Perfect: Trigonometry* as a companion to your classroom study, for that extra experience that helps you solidify your skills. You can use it as a review of concepts you've learned previously, whether you're preparing for an exam or you're taking an advanced course and feel you need a refresher.

With patience and practice, you'll find that you've assembled an impressive set of tools and that you're confident about your ability to use them properly. The skills you acquire in trigonometry will serve you well in other math courses, like calculus, and in other disciplines, like physics. Be persistent. You must keep working at it, bit by bit. Be patient. You will make mistakes, but mistakes are one of the ways we learn, so welcome your mistakes. They'll decrease as you practice, because practice makes perfect.

PRACTICE MAKES PERFECT™

Trigonometry

This page intentionally left blank

·1· Right triangle trigonometry

Trigonometry, or "triangle measurement," developed as a means to calculate the lengths of the sides of right triangles and was based on similar triangle relationships. The fundamental ideas of trigonometry will be extended well beyond right triangles, but for now, we'll look at right triangles and measure the angles in them in degrees.

Angle measurement: degrees

The traditional system for measuring angles in geometry is degree measure. The measurement of an angle involves the amount of rotation between the two sides of the angle. A full rotation, or full circle, is 360°. A half rotation is 180°, the measure of a straight angle. Because the three angles of any triangle total 180°, each angle in the triangle must be less than 180°. Angles that measure less than 90° are acute angles, angles of exactly 90° are right angles, and angles greater than 90° and less than 180° are obtuse angles.

Two acute angles are complementary if the sum of their measures is 90°. Each angle in the pair is the complement of the other. Two angles are supplementary if their measures total 180°. Each angle is the supplement of the other.

EXERCISE
1·1

Classify each angle as acute or obtuse.

1. 42°	4. 83°	7. 174°	10. 11.6°
2. 110°	5. 96°	8. 39°	
3. 17°	6. 108°	9. 7°	

Give the complement (if possible) and the supplement of each angle.

11. 47°	14. 89°	17. 151°	20. 13.8°
12. 130°	15. 92°	18. 40.5°	
13. 19°	16. 123°	19. 9.2°	

Degrees, minutes, seconds

In geometry, it's unusual to talk about fractions of a degree, and when you do, it's usually by a simple fraction or a decimal. In trigonometry, you'll sometimes need a level of precision that can only be accomplished by measuring fractions of a degree. While this is sometimes done by fractions or decimals, it's also common to break a degree into 60 parts called minutes ($1° = 60'$), and a minute into 60 parts called seconds ($1' = 60''$ so $1° = 3{,}600''$).

EXERCISE

1·2

Convert each measurement to decimal form.

1. 22°45′
2. 18°12′
3. 39°48
4. 137°27′
5. 96°51′
6. 81°6′45″
7. 1°43′12″
8. 178°22′30″
9. 11°7′30″
10. 78°22′36″

Convert each measurement to degree-minute-second form.

11. 25.3°
12. 18.75°
13. 37.1°
14. 135.545°
15. 94.735°
16. 86.9°
17. 3.25°
18. 167.6°
19. 19.25°
20. 74.3°

Bearings

Because trigonometry is frequently used in navigation, information about an angle is often given in terms of bearings. A bearing first specifies a starting direction, usually north or south, then gives a number of degrees (and possibly minutes and seconds) to rotate, followed by the direction of rotation. A bearing of N 30° W tells you to start facing north and turn 30° toward the west.

EXERCISE

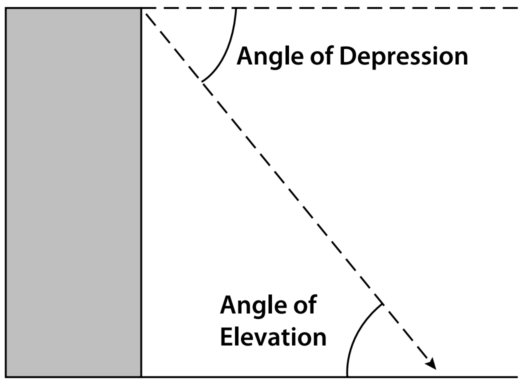
1·3

Point A *is 100 meters due west of point* B. *The bearings of point* C *from point* A *and from point* B *are given. Find the measures of the angles of* $\triangle ABC$.

1. From *A*: N 23° E From *B*: N 36° W
2. From *A*: N 14° E From *B*: N 45° W
3. From *A*: N 62° W From *B*: N 81° W
4. From *A*: S 4° W From *B*: S 72° W
5. From *A*: S 55° E From *B*: S 12° E
6. From *A*: N 22° E From *B*: N 17° W
7. From *A*: N 78° E From *B*: N 29° W
8. From *A*: S 64° E From *B*: S 63° E
9. From *A*: S 64° E From *B*: S 63° E
10. From *A*: N 19° W From *B*: N 89° W

Angle of elevation and angle of depression

In many trigonometry problems, you'll hear about the angle of elevation or the angle of depression. If you imagine standing looking straight ahead and then raising your eyes to look up at an object, the angle between your original horizontal gaze and your line of sight to the object above is the angle of elevation. On the other hand, if you're in an elevated position, looking straight ahead, and shift your gaze down to an object below, the angle between your original horizontal gaze and your line of sight to the object below is the angle of depression. Since the horizontal lines are parallel, a little basic geometry shows that the angle of elevation is equal to the angle of depression.



EXERCISE 1·4

Find the specified angle(s).

1. From a point 200 yards from the foot of a building, the angle of elevation to the top of the building is 37°. Find the measures of the angles of the triangle formed by the building, the ground, and the line of sight to the top of the building.
2. The angle of depression from the top of a lighthouse to a ship at sea is 12°48′. Find the measures of the angles of the triangle formed by the lighthouse, the sea, and the line of sight to the ship.
3. The angle of depression from the top of a tower to an observer on the ground is 37°. Find the angle of elevation from the observer to the top of the tower.
4. The ground, a 90 foot tower, and the line of sight to the top of the tower from a point 25 yards away from the base of the tower form a right triangle. If the acute angles of the triangle are 50° and 40°, what is the angle of depression from the top of the tower to an observer on the ground?
5. From point F, 500 meters from the foot of a cliff, B, the angle of elevation to the top of the cliff, T, is 54°27′. From the top of the cliff, T, the angle of depression to a point N, 200 meters from the foot of the cliff, is 74°3′. Find the measure of $\angle FTN$.
6. From a plane P at an altitude of 2,500 feet, a pilot can see a tower 5 miles ahead. The angle of depression to the top of the tower, T, is 4°58′ and the angle of depression to the bottom of the tower, B, is 5°24′. Find the measure of $\angle PBT$.

7. From the Top of the Rock, the observation deck at Rockefeller Center (call it *T*), 850 feet above street level, a visitor can look north and see Central Park. If the visitor looks down at the northern edge of the park, *N*, the angle of depression is $2°58'$. If the visitor looks to the southern edge of the park, *S*, the angle of depression is $12°57'$. Find $\angle NST$.

8. When ready for launch, the space shuttle, with its fuel tanks, stands 184 feet high. From a point level with the launch pad, 1 mile away, the angle of elevation to the highest point of the shuttle assembly is 2°. From another point, on the beach south of the launch site, the angle of elevation to the highest point of the shuttle assembly is about $0°10'$. Find the measure of the angle formed by connecting the beach viewing site to the highest point on the shuttle to the 1 mile viewing site.

9. In ideal weather, from the top of the Eiffel Tower, which stands 324 meters high, it is possible to see a point on the horizon 67.5 kilometers away. A tourist at the top of the Eiffel Tower on such a perfect day looks out at that distant horizon and the angle of depression is $0°16'30''$. The tourist then shifts his gaze $3°50'30''$ to look down at the Jardin du Luxembourg, 4.5 kilometers away. What is the angle of depression from the top of the Eiffel Tower to the Jardin du Luxembourg?

10. The London Eye reaches a height of 135 meters, and from the top, it is possible for a rider to see Buckingham Palace, 1.9 kilometers away, at an angle of depression of $4°3'$. If a rider at the top shifts her gaze up $2°54'$, she will be able to see Windsor Castle, 6.6 kilometers away. What is the angle of depression from the top of the London Eye to Windsor Castle?

Similar triangles

In geometry, you studied similar triangles. Triangles with corresponding angles congruent and corresponding sides in proportion have the same shape but different sizes. They appear as enlargements or reductions of one another.

Similar right triangles

When you begin to consider similarity in right triangles, you immediately know that the right angles are congruent. If you also know that an acute angle of one right triangle is congruent to an acute angle of the other, you can be certain that the third angles are congruent as well, and the triangles are similar. If the triangles are similar, the corresponding sides are in proportion.

If you look at two right triangles, each with an acute angle of 25°, you can quickly prove that the two triangles are similar. In fact, all right triangles containing an angle of 25° are similar, and you might think of them as a family. Throughout the family of 25° right triangles, the corresponding sides are in proportion. If you call the legs *a* and *b* and the hypotenuse *c*,

$$\frac{a \text{ in one triangle}}{a \text{ in the second triangle}} = \frac{b \text{ in one triangle}}{b \text{ in the second triangle}} = \frac{c \text{ in one triangle}}{c \text{ in the second triangle}}$$

If you focus on any two of those ratios, so that you have a proportion, and apply a property of proportions that you learned in algebra, you can say

$$\frac{a \text{ in one triangle}}{b \text{ in one triangle}} = \frac{a \text{ in the second triangle}}{b \text{ in the second triangle}}$$

$$\frac{a \text{ in one triangle}}{c \text{ in one triangle}} = \frac{a \text{ in the second triangle}}{c \text{ in the second triangle}}$$

$$\frac{b \text{ in one triangle}}{c \text{ in one triangle}} = \frac{b \text{ in the second triangle}}{c \text{ in the second triangle}}$$

In any right triangle in this family, the ratio of the side opposite the 25° angle to the hypotenuse will always be the same; likewise, the ratios of other pairs of sides will remain constant throughout the family. Trigonometry takes advantage of that fact and assigns a name to each of the possible ratios.

Trigonometric ratios

If the three sides of the right triangle are labeled as the hypotenuse, the side opposite a particular acute angle, A, and the side adjacent to the acute angle A, six different ratios are possible. The six ratios are called the sine, cosine, tangent, cosecant, secant, and cotangent, and are defined as

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \quad \csc A = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \sec A = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} \quad \cot A = \frac{\text{adjacent}}{\text{opposite}}$$

Notice that three of the ratios are reciprocals of the other three. The cosecant is the reciprocal of the sine, the secant and the cosine are reciprocals, and the cotangent is the reciprocal of the tangent. It's also true that the tangent is equal to the sine divided by the cosine:

$$\frac{\sin A}{\cos A} = \frac{\text{opposite}}{\text{hypotenuse}} \,\Big|\, \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{opposite}}{\cancel{\text{hypotenuse}}} \cdot \frac{\cancel{\text{hypotenuse}}}{\text{adjacent}} = \frac{\text{opposite}}{\text{adjacent}} = \tan A$$

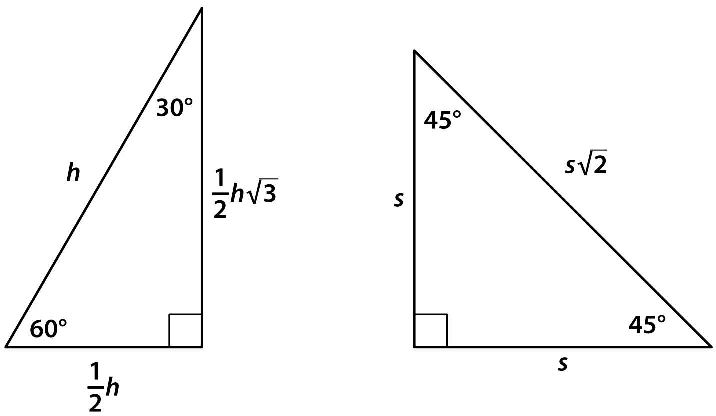
A similar argument shows that the cotangent is equal to the cosine divided by the sine.

If $\angle A$ and $\angle B$ are the acute angles of right ΔABC, the side opposite $\angle A$ is adjacent to $\angle B$ and the side opposite $\angle B$ is adjacent to $\angle A$, but the hypotenuse is always the hypotenuse. This means that $\sin\angle A = \dfrac{BC}{AB} = \cos\angle B$, $\cos\angle A = \dfrac{AC}{AB} = \sin\angle B$, and $\tan\angle A = \dfrac{BC}{AC} = \cot\angle B$.

In more general terms, because the two acute angles of a right triangle are complementary, the sine of an angle is the cosine of its complement, and the tangent of an angle is the cotangent of its complement. In fact, the "co" in cosine (and cotangent and cosecant) comes from the "co" in complementary. The cosine is the complementary sine, or the sine of the complement. The cosecant is the secant of the complement, and the cotangent is the tangent of the complement.

Special right triangles

Isosceles right triangles and 30°-60°-90° right triangles pop up in enough different circumstances that you probably learned the relationships of the sides by memory. The isosceles, or 45°-45°-90°, right triangle has legs of equal length and a hypotenuse equal to that length times the square root of 2. If the legs of the isosceles right triangle measure 8 centimeters, the hypotenuse will be $8\sqrt{2}$ centimeters. If the hypotenuse is 14 inches, the legs will be $\frac{14}{\sqrt{2}} = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$.



In the 30°-60°-90° right triangle, the shorter leg, opposite the 30° angle, is half as long as the hypotenuse, and the longer leg, opposite the 60° angle, is half the hypotenuse times the square root of 3. If the hypotenuse is 10 centimeters, the shorter leg is 5 centimeters and the longer leg is $5\sqrt{3}$ centimeters. If you know the shorter side, tack on a $\sqrt{3}$ to get the longer side but double the short leg to get the hypotenuse. If the short leg is 3 centimeters, the longer leg is $3\sqrt{3}$ centimeters and the hypotenuse is 6 centimeters. To find the other sides when you're given the longer leg, divide by $\sqrt{3}$ to get the shorter leg and then double the shorter leg to get the hypotenuse. In a 30°-60°-90° right triangle with a longer leg of 18 centimeters, the shorter leg is $\frac{18}{\sqrt{3}} = \frac{18\sqrt{3}}{3} = 6\sqrt{3}$ centimeters and the hypotenuse is $12\sqrt{3}$ centimeters.

Because you know the relationships of the sides of those triangles, you can easily determine the values of the trigonometric ratios for angles of 30°, 45°, and 60°:

	sin	**cos**	**tan**
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

EXERCISE
1·5

Find the missing sides of each 45°-45°-90° right triangle.

1. ΔABC with hypotenuse AC measuring $7\sqrt{2}$ inches
2. ΔXYZ with leg XY measuring 4 centimeters
3. ΔARM with hypotenuse AM measuring 12 feet
4. ΔLEG with leg EG measuring $5\sqrt{6}$ meters
5. ΔRST with hypotenuse RT measuring $8\sqrt{14}$ centimeters

Find the missing sides of each 30°-60°-90° right triangle.

6. ΔXYZ with hypotenuse XZ measuring 50 meters
7. ΔABC with shorter leg AB measuring 9 centimeters
8. ΔRST with longer leg ST measuring $7\sqrt{3}$ inches
9. ΔCAT with shorter leg CA measuring $14\sqrt{6}$ feet
10. ΔDOG with hypotenuse DG measuring $4\sqrt{21}$ centimeters

Find the indicated ratios from memory.

11. sin 45°
12. tan 30°
13. cos 60°
14. tan 45°
15. sec 60°
16. csc 30°
17. cot 60°
18. cos 45°
19. sin 30°
20. sec 45°

Finding sides

With these six ratios, it is possible to solve for any unknown side of the right triangle, if another side and an acute angle are known, or to find the angle if two sides are known. You just need to choose a ratio that incorporates the side you know and the side that you want to find, and substitute the values you know. Then you'll have to look up the sine, cosine, or tangent of the angle. Once upon a time, students had to rely on tables to look up the values of the ratios for each family of right triangles, but now the sine, cosine, and tangent of an angle can be found with a few keystrokes on your calculator.

In right ΔABC, hypotenuse AC is 6 centimeters long and $\angle A$ measures 32°. To find the length of the shorter leg, first make a sketch to help you visualize the triangle. The shorter leg will be opposite the smaller angle. If one of the acute angles is 32°, the other is 58°, so you need to find the side opposite the 32° angle, or side BC. If you use the 32° angle, you need a ratio that includes the opposite side and the hypotenuse. You can choose between sine (sin) and cosecant (csc), but since your calculator has a key for sin but not for csc, sine is more convenient:

$$\sin 32° = \frac{BC}{AC} = \frac{x}{6}$$

From your calculator, you can find that

$$\sin 32° \approx 0.53$$

so

$$0.53 = \frac{x}{6}$$

and

$$x \approx 3.2$$

EXERCISE

1·6

Solve the following.

1. In right ΔRST, $\angle S$ is a right angle and $RT = 24$. If $\angle T$ measures 30°, find the length of RS.
2. Given ΔXYZ with $\angle Y$ a right angle and hypotenuse XZ equal to 42. If $\angle X = 56°$, find the length of side YZ to the nearest tenth.
3. In right ΔABC with right angle at C, $\angle A = 46°36'$ and side AC is 42 feet. Find the lengths of the other two sides.
4. In right ΔABC with right angle at C, $\angle B = 76°30'$ and side BC is 80 feet. Find the lengths of the other two sides.
5. In right ΔXYZ with right angle at Y, $\angle X = 32°$ and side YZ is 58 meters. Find the lengths of the other two sides.
6. From onboard a ship at sea, the angle of elevation to the top of a lighthouse is 41°. If the lighthouse is known to be 50 feet high, how far from shore is the ship?
7. A ladder 28 feet long makes an angle of 15° with the wall of a building. How far from the wall is the foot of the ladder?
8. From the top of the ski slope, Elise sees the lodge at an angle of depression of 18°30′. If the slope is known to have an elevation of 1,500 feet, how far does Elise have to ski to reach the lodge?
9. From a point 85 feet from the base of the schoolhouse, the angle of elevation to the bottom of a flagpole on the roof of the schoolhouse is 38°30′. Find the height of the schoolhouse.
10. If the angle of elevation to the top of the flagpole in question 9 is 54°36′, how tall is the flagpole?

11. The angle of depression from the top of a security tower to the entrance to a plaza is 42°. If the tower is 20 feet high, how far is it from the entrance to the base of the tower?

12. From the seats in the top deck of a baseball stadium, the angle of depression to home plate is 22°. If the diagonal distance from home plate to the top deck is 320 feet, how high is the top deck?

13. When Claire is lying in bed watching television, the angle of elevation from her pillow to the TV is 23°. If Claire's TV is mounted on the wall 7.5 feet above the level of the bed, how far is her pillow from the wall?

14. If an observer notes that the angle of elevation to the top of a 162 meter tower is 38°24′, how far is the observer from the tower?

15. In ΔABC, $AB = 13$ feet and $\angle A$ measures 29.5°. If ΔABC is not a right triangle, find the altitude BD from B to AC.

16. In ΔABC, as noted in question 15, $AB = 13$ feet, $\angle A$ measures 29.5°, and ΔABC is not a right triangle. If AC measures 22 feet and the altitude from B meets AC at D, find the length of AD.

17. In ΔABC, as noted in questions 15 and 16, $AB = 13$ feet, $\angle A$ measures 29.5°, and ΔABC is not a right triangle. If $\angle C = 18°$ and the altitude from B meets AC at D, find the length of BC.

18. ΔPQR has $\angle Q = 32°$ and $\angle R = 38°$. If $PR = 369$ feet, find the length of the altitude PT from P to QR.

19. In ΔPQR, as noted in question 18, $\angle Q = 32°$, $\angle R = 38°$, and $PR = 369$ feet. Find the length of PQ.

20. Use the information in questions 18 and 19 to find the length of QR.

Finding angles

In addition to finding the other sides of a right triangle when you know one side and the angle measures, trigonometric ratios can be used to find the measures of the acute angles of the right triangle if you know the lengths of the sides. Knowing two sides is adequate, because you know that the Pythagorean theorem applies to the sides, so you could find the third side if necessary. You also know that one of the angles is 90°, so you only need to find one of the acute angles and subtract from 90° to find the other.

If you know the lengths of two sides of the right triangle, you can calculate one of the ratios. Which ratio will be determined by which sides you know. Once you know the ratio, you'll work backward to the angle. If you're working with tables of trigonometric ratios, that means poring through the tables, looking for the value of the sin, cos, or tan closest to the value you have to see what angle it belongs to. If you're using a calculator, it's a little easier.

Arcsin, arccos, arctan

Working backward means you know the sine (or cosine or tangent) of the angle and want to find the angle that has that sine. The common way to say "the angle whose sine is *N*" is arcsin *N*. The "arc" in arcsin comes from the fact that the measure of a central angle is equal to the measure of its intercepted arc. The name is saying "the arc (and therefore the angle) that has this sine." In the next chapter, when talking about trigonometric functions, you'll use the inverse function notation $\sin^{-1} N$ to denote the number whose sine is *N*. You'll see these two notations used interchangeably, although there actually is a subtle difference in their meaning.

Each of the trigonometric ratios has an inverse. Just as the angle whose sine is N can be denoted as arcsin N, the angle whose cosine is N can be indicated by arcos N and the angle whose tangent is N by arctan N. You'll find a way to enter each of the inverses on your calculator, usually as a second function or inverse function on the keys for sin, cos, and tan. The calculator keys may be marked with the inverse function symbols $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

If the legs of a right triangle measure 18 centimeters and 25 centimeters, you can find the measures of the acute angles by using the two known sides to find the tangent of one of the angles. The tangent of the smaller angle will be $\frac{18}{25}$, or the tangent of the larger angle will be $\frac{25}{18}$, but you can work with either one. To find the measure of the angle, use the $\tan^{-1}$ key on your calculator: $\tan^{-1}\frac{18}{25} \approx 35.75°$ or $35°45'$. The two acute angles of a right triangle are complementary, so the larger of the acute angles measures approximately $90° - 35°45' = 54°15'$.

EXERCISE 1·7

Solve the following.

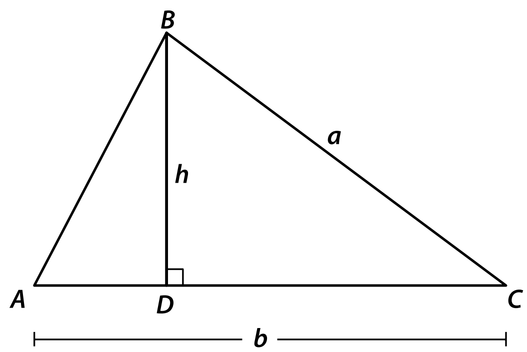
1. Find the measures of the angles of a right triangle with legs of 16 inches and 35 inches.
2. Find the measures of the angles of a right triangle with a hypotenuse of 592.7 meters and a leg of 86.4 meters.
3. If the leg of a right triangle measures 349.2 centimeters and the hypotenuse measures 716.8 centimeters, find the measure of each of the acute angles.
4. Find the measures of the angles of a 3-4-5 right triangle.
5. Find the measures of the angles of a 5-12-13 right triangle.
6. A mountain slope rises 760 feet in a quarter mile on the horizontal. If 1 mile = 5,280 feet, what angle does the slope make with the horizontal?
7. If the sides of a rectangle measure 5 inches and 12 inches, what angle does the diagonal of the rectangle make with the longer side?
8. What is the angle of elevation of the sun at the instant a 68 foot flagpole casts a shadow of 81 feet?
9. In $\triangle ABC$, $AB = 25$ inches and altitude BD measures 16 inches. Find the measure of $\angle A$ to the nearest degree.
10. What angle does a stairway make with the floor if the steps have a tread of 9 inches and a rise of 7.5 inches?
11. In right $\triangle ABC$ with right angle at C, $AC = 22$ meters and $BC = 72$ meters. Find the measure of $\angle B$.
12. In right $\triangle XYZ$, leg $XZ = 35$ inches and leg $YZ = 16$ inches. Find the measure of $\angle X$.
13. If a 20 foot ladder is positioned to reach 15 feet up on the wall, what angle does the foot of the ladder make with the ground?
14. If the legs of a right triangle measure 349.2 meters and 716.8 meters, find the measures of the acute angles of the triangle.

15. Midville is 47.39 miles due north of Smalltown and 96.42 miles from Centerville. If Centerville is due west of Smalltown, find the bearing of Midville from Centerville.
16. Katrina's office building is exactly half a mile from City Hall, and she knows that the office building is 220 feet high. If Katrina stands on the roof of her office building and looks down at city hall, what is the angle of depression? (There are 5,280 feet in 1 mile.)
17. The diagonal of a rectangle measures 19 inches. If the shorter side of the rectangle measures 8 inches, find the measure of the angle between the diagonal and the longer side.
18. In football, the crossbar of the goal post is 10 feet high and is positioned at the end line, 10 yards beyond the goal line, so a field goal kicked from the 30-yard line must travel 40 yards. Find the angle of elevation of the crossbar from the 30-yard line.
19. Using the information in question 18, find the angle of elevation of the crossbar from the 20 yard line.
20. Using the information in question 18, find the angle of elevation of the crossbar from the 5 yard line.

Finding areas

In geometry, you learned that the area of a parallelogram is the product of its base and its height, $A = bh$, and that the area of a triangle is half the product of its base and its height, $A = \frac{1}{2}bh$. The problem you sometimes encountered in trying to use those formulas was that while you might know the lengths of the sides, you didn't always know the altitude, or height, and didn't have a convenient way to find it.

Thanks to right triangle trigonometry, that problem can sometimes be solved. If you know two sides of a triangle and the angle included between them, or two adjacent sides of a parallelogram and the angle included between them, it's possible to use trig ratios to find the height.



Look first at the triangle. You know the lengths of two sides, a and b, and the measurement of the angle between them, $\angle C$. Drop a perpendicular from vertex B to side b. If you can find the length of that altitude to side b, you can calculate the area. Because the perpendicular creates a right triangle, in which side a is the hypotenuse and the altitude, call it x, is the side opposite $\angle C$, you can find the height by using the trigonometric ratio $\sin C = \frac{x}{a}$ or $x = a\sin C$.

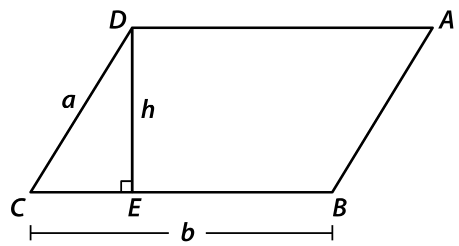
If you incorporate that new information into the area formula for a triangle, $A = \frac{1}{2}bh$, you get a new formula for the area of a triangle:

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}b \cdot a\sin C \\ &= \frac{1}{2}ab\sin C \end{aligned}$$

If you know two sides of a triangle and the angle included between them, the area of the triangle is half the product of the two sides and the sine of the included angle. A triangle with sides of 4 centimeters and 7 centimeters and an included angle of 50° has an area of

$$\begin{aligned} A &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}(4)(7)\sin 50^\circ \\ &= 14(0.766) \\ &\approx 10.725 \text{ cm}^2 \end{aligned}$$

With a similar logic, you can modify the formula for the area of a parallelogram. Drop an altitude from one vertex to the opposite side. If you know the lengths of two adjacent sides, you can find the altitude because one of your sides forms the hypotenuse of the right triangle created by the altitude. If you call the side that forms the hypotenuse a and call the included angle $\angle C$, then the length of the altitude is $a\sin C$ and the area of the parallelogram is $A = ab\sin C$.



To find the area of a parallelogram with sides of 18 inches and 22 inches and an included angle of 60°, use the area formula

$$\begin{aligned} A &= ab\sin C \\ &= 18 \cdot 22\sin 60^\circ \\ &= 396 \cdot \frac{\sqrt{3}}{2} \\ &= 198\sqrt{3} \text{ in.}^2 \end{aligned}$$

sample content of Practice Makes Perfect Trigonometry

- The Splendid Table pdf, azw (kindle), epub
- download Å’uvres complÃ¨tes, tome 5: La Somme athÃ©ologique I: L'ExpÃ©rience intÃ©rieure - MÃ©thode de mÃ©ditation - Post-scriptum 1953 - Le Coupable - L'Alleluiah
- *read online Microsoft SQL Server 2012 High-Performance T-SQL Using Window Functions here*
- download online Bitch, Issue 58: Pulp (Spring 2013)

- http://deltaphenomics.nl/?library/The-Splendid-Table.pdf
- http://crackingscience.org/?library/--uvres-compl--tes--tome-5--La-Somme-ath--ologique-I--L-Exp--rience-int--rieure---M--thode-de-m--ditation---Po
- http://drmurphreesnewsletters.com/library/Microsoft-SQL-Server-2012-High-Performance-T-SQL-Using-Window-Functions.pdf
- http://reseauplatoparis.com/library/Bitch--Issue-58--Pulp--Spring-2013-.pdf