Geometry

includes plane, analytic, and
transformational geometries

Fifth Edition

Barnett Rich, PhD
Former Chairman, Department of Mathematics
Brooklyn Technical High School, New York City

Christopher Thomas, PhD
Assistant Professor, Department of Mathematics
Massachusetts College of Liberal Arts, North Adams, MA

Schaum's Outline Series

New York Chicago San Francisco
Lisbon London Madrid Mexico City
Milan New Delhi San Juan
Seoul Singapore Sydney Toronto

BARNETT RICH held a doctor of philosophy degree (PhD) from Columbia University and a doctor o jurisprudence (JD) from New York University. He began his professional career at Townsend Harris Hall High School of New York City and was one of the prominent organizers of the High School of Music and Art where he served as the Administrative Assistant. Later he taught at CUNY and Columbia University and held the post of chairman of mathematics at Brooklyn Technical High School for 14 years. Among his many achievements are the 6 degrees that he earned and the 23 books that he wrote, among them Schaum's Outlines of Elementary Algebra, Modern Elementary Algebra, and Review of Elementary Algebra.

CHRISTOPHER THOMAS has a BS from University of Massachusetts at Amherst and a PhD from Tufts University, both in mathematics. He first taught as a Peace Corps volunteer at the Mozano Senior Secondary School in Ghana. Since then he has taught at Tufts University, Texas A&M University, and the Massachusetts College of Liberal Arts. He has written Schaum's Outline of Math for the Liberal Arts as well as other books on calculus and trigonometry.

The McGraw-Hill Companies

Copyright © 2013 by The McGraw-Hill Companies, Inc. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of the publisher.

ISBN: 978-0-07-179541-8
MHID: 0-07-179541-3

The material in this eBook also appears in the print version of this title: ISBN: 978-0-07-179540-1, MHID: 0-07-179540-5.

All trademarks are trademarks of their respective owners. Rather than put a trademark symbol after every occurrence of a trademarked name, we use names in an editorial fashion only, and to the benefi of the trademark owner, with no intention of infringement of the trademark. Where such designations appear in this book, they have been printed with initial caps.

McGraw-Hill eBooks are available at special quantity discounts to use as premiums and sales promotions, or for use in corporate training programs. To contact a representative please e-mail us at bulksales@mcgraw-hill.com.

McGraw-Hill, the McGraw-Hill Publishing logo, Schaum's, and related trade dress are trademarks or registered trademarks of The McGraw-Hill Companies and/or its affiliates in the United States and other countries and may not be used without written permission. All other trademarks are the propert of their respective owners. The McGraw-Hill Companies is not associated with any product or vendo mentioned in this book.

TERMS OF USE

This is a copyrighted work and The McGraw-Hill Companies, Inc. ("McGraw-Hill") and its licensors reserve all rights in and to the work. Use of this work is subject to these terms. Except as permitted under the Copyright Act of 1976 and the right to store and retrieve one copy of the work, you may no decompile, disassemble, reverse engineer, reproduce, modify, create derivative works based upon, transmit, distribute, disseminate, sell, publish or sublicense the work or any part of it without Mc~~Graw-Hill's prior consent. You may use the work for your own noncommercial and personal~~ use; any other use of the work is strictly prohibited. Your right to use the work may be terminated if you fail to comply with these terms.

THE WORK IS PROVIDED "AS IS." McGRAW-HILL AND ITS LICENSORS MAKE NO GUARANTEES OR WARRANTIES AS TO THE ACCURACY, ADEQUACY OR COMPLETENESS OF OR RESULTS TO BE OBTAINED FROM USING THE WORK, INCLUDING ANY INFORMATION THAT CAN BE ACCESSED THROUGH THE WORK VIA HYPERLINK OR OTHERWISE, AND EXPRESSLY DISCLAIM ANY WARRANTY, EXPRESS OR IMPLIED, INCLUDING BUT NOT LIMITED TO IMPLIED WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. McGraw-Hill and its licensors do not warrant or guarantee that the functions contained in the work will meet your requirements or that its operation will be uninterrupted or error free. Neither McGraw-Hill nor its licensors shall be liable to you or anyone else for any inaccuracy, error or omission, regardless of cause, in the work or for any damage resulting there from. McGraw-Hill has no responsibility for the content of any information accessed through the work. Under no circumstances shall McGraw-Hill and/or its licensors be liable for any indirect, incidental, special, punitive, consequential or similar damages that result from the use of or inability to use the work, even if any of them has been advised of the possibility of such damages. This limitation of liability shall apply to any claim or cause whatsoever whether such claim or cause arises in contract, tort or otherwise.

Preface to the First Editio

The central purpose of this book is to provide maximum help for the student and maximum service for the teacher.

Providing Help for the Student

This book has been designed to improve the learning of geometry far beyond that of the typical and traditional book in the subject. Students will find this text useful for these reasons:

(1) ***Learning Each Rule, Formula, and Principle***

Each rule, formula, and principle is stated in simple language, is made to stand out in distinctive typ is kept together with those related to it, and is clearly illustrated by examples.

(2) ***Learning Each Set of Solved Problems***

Each set of solved problems is used to clarify and apply the more important rules and principles. Th character of each set is indicated by a title.

(3) ***Learning Each Set of Supplementary Problems***

Each set of supplementary problems provides further application of rules and principles. A guide number for each set refers the student to the set of related solved problems. There are more than 200 additional related supplementary problems. Answers for the supplementary problems have been placed in the back of the book.

(4) ***Integrating the Learning of Plane Geometry***

The book integrates plane geometry with arithmetic, algebra, numerical trigonometry, analytic geometry, and simple logic. To carry out this integration:

(a) A separate chapter is devoted to analytic geometry.

(b) A separate chapter includes the complete proofs of the most important theorems together with th plan for each.

(c) A separate chapter fully explains 23 basic geometric constructions. Underlying geometric principles are provided for the constructions, as needed.

(d) Two separate chapters on methods of proof and improvement of reasoning present the simple an basic ideas of formal logic suitable for students at this stage.

(e) Throughout the book, algebra is emphasized as the major means of solving geometric problems through algebraic symbolism, algebraic equations, and algebraic proof.

(5) ***Learning Geometry Through Self-study***

The method of presentation in the book makes it ideal as a means of self-study. For able students, th book ~~will enable then to accomplish the work of the standard course of study in much less time.~~ For the less able, the presentation of numerous illustrations and solutions provides the help needed to remedy weaknesses and overcome difficulties, and in this way keep up with the class and at the same time gain a measure of confidence and security.

(6) ***Extending Plane Geometry into Solid Geometry***

A separate chapter is devoted to the extension of two-dimensional plane geometry into three-dimensional solid geometry. It is especially important in this day and age that the student understand how the basic ideas of space are outgrowths of principles learned in plane geometry.

Providing Service for the Teacher

Teachers of geometry will find this text useful for these reasons:

(1) ***Teaching Each Chapter***

Each chapter has a central unifying theme. Each chapter is divided into two to ten major subdivision which support its central theme. In turn, these chapter subdivisions are arranged in graded sequence for greater teaching effectiveness.

(2) ***Teaching Each Chapter Subdivision***

Each of the chapter subdivisions contains the problems and materials needed for a complete lesson developing the related principles.

(3) ***Making Teaching More Effective Through Solved Problems***

Through proper use of the solved problems, students gain greater understanding of the way in which principles are applied in varied situations. By solving problems, mathematics is learned as it should l learned–by doing mathematics. To ensure effective learning, solutions should be reproduced on pape Students should seek the why as well as the how of each step. Once students sees how a principle is applied to a solved problem, they are then ready to extend the principle to a related supplementary problem. Geometry is not learned through the reading of a textbook and the memorizing of a set of formulas. Until an adequate variety of suitable problems has been solved, a student will gain little more than a vague impression of plane geometry.

(4) ***Making Teaching More Effective Through Problem Assignment***

The preparation of homework assignments and class assignments of problems is facilitated because the supplementary problems in this book are related to the sets of solved problems. Greatest attentio should be given to the underlying principle and the major steps in the solution of the solved problem After this, the student can reproduce the solved problems and then proceed to do those supplementar problems which are related to the solved ones.

Others Who will Find this Text Advantageous

This book can be used profitably by others besides students and teachers. In this group we include: (the parents of geometry students who wish to help their children through the use of the book's self-study materials, or who may wish to refresh their own memory of geometry in order to properly help their children; (2) the supervisor who wishes to provide enrichment materials in geometry, or who seeks to improve teaching effectiveness in geometry; (3) the person who seeks to review geometry or to learn it through independent self-study.

BARNETT RI
Brooklyn Technical High Scho
April, 19

Requirements

To fully appreciate this geometry book, you must have a basic understanding of algebra. If that is what you have really come to learn, then may I suggest you get a copy of Schaum's Outline of *Colleg Algebra*. You will learn everything you need and more (things you don't need to know!)

If you have come to learn geometry, it begins at Chapter one.

As for algebra, you must understand that we can talk about numbers we do not know by assigning them variables like x, y, and A.

You must understand that variables can be combined when they are exactly the same, like $x + x = 2x$ and $3x^2 + 11x^2 = 14x^2$, but not when there is any difference, like $3x^2y–9xy = 3x^2y–9xy$.

You should understand the deep importance of the equals sign, which indicates that two things that appear different are actually exactly the same. If $3x = 15$, then this means that $3x$ is just another name for 15. If we do the same thing to both sides of an equation (add the same thing, divide both sides by something, take a square root, etc.), then the result will still be equal.

You must know how to solve an equation like $3x + 8 = 23$ by subtracting eight from both sides, $3x + – 8 = 23 – 8 = 15$, and then dividing both sides by 3 to get $3x/3 = 15/3 = 5$. In this case, the variable was *constrained;* there was only one possible value and so x would have to be 5.

You must know how to add these sorts of things together, such as $(3x + 8) + (9–x) = (3x–x) + (8 + 9)$ $2x + 17$. You don't need to know that the ability to rearrange the parentheses is called *associativity* and the ability to change the order is called *commutativity*.

You must also know how to multiply them: $(3x + 8)\cdot(9–x) = 27x–3x^2 + 72–8x =–3x^2 + 19x + 72$

Actually, you might not even need to know that.

You must also be comfortable using more than one variable at a time, such as taking an equation in terms of y like $y = x2 + 3$ and rearranging the equation to put it in terms of x like $y–3 = x^2$. so $\sqrt{y-3} = \sqrt{x^2}$ and thus $\sqrt{y-3} = \pm x$, so $x = \pm \sqrt{y-3}$.

You should know about square roots, how $\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = 2\sqrt{5}$. It is useful to keep in mind that there are many *irrational numbers*, like $\sqrt{2}$, which could never be written as a neat ratio or fraction, but only approximated with a number of decimals.

You shouldn't be scared when there are lots of variables, either, such as $F = \frac{gM_1M_2}{r^2}$; thus, $Fr^2 = gM_1M_2$ by cross-multiplication, so $r = \pm\sqrt{\frac{gM_1M_2}{F}}$.

Most important of all, you should know how to take a formula like $V = \frac{1}{3}\pi r^2 h$ and replace values and simplify. If r = 5 cm and h = 8 cm, then

$V = \frac{1}{3}\pi$ (5 cm)^2 (8 cm) = $\frac{200\pi}{3}$ cm^3.

Content

CHAPTER

Lines, Angles, and Triangle

1.1 Historical Background of Geometry

The word *geometry* is derived from the Greek words *geos* (meaning *earth)* and *metron* (meaning *measure)*.

The ancient Egyptians, Chinese, Babylonians, Romans, and Greeks used geometry for surveying, navigation, astronomy, and other practical occupations.

The Greeks sought to systematize the geometric facts they knew by establishing logical reasons fo them and relationships among them. The work of men such as Thales (600 B.C.), Pythagoras (540 B.C. Plato (390 B.C.), and Aristotle (350 B.C.) in systematizing geometric facts and principles culminated i the geometry text *Elements*, written in approximately 325 B.C. by Euclid. This most remarkable text has been in use for over 2000 years.

1.2 Undefined Terms of Geometry: Point, Line, and Plane

1.2A Point, Line, and Plane are Undefined Terms

These undefined terms underlie the definitions of all geometric terms. They can be given meanings b way of descriptions. However, these descriptions, which follow, are not to be thought of as definition

1.2B Point

A *point* has position only. It has no length, width, or thickness.

A point is represented by a dot. Keep in mind, however, that the dot *represents* a point but *is not* a point, just as a dot on a map may represent a locality but is not the locality. A dot, unlike a point, has size.

A point is designated by a capital letter next to the dot, thus point A is represented: *A*.

1.2C Line

A line has length but has no width or thickness.

A line may be represented by the path of a piece of chalk on the blackboard or by a stretched rubbe band.

A line is designated by the capital letters of any two of its points or by a small letter, thus:

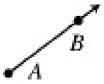
, or $\overleftrightarrow{AB}$.

A *line* may be straight, curved, or a combination of these. To understand how lines differ, think of line as being generated by a moving point. A *straight line*, such as $\longleftrightarrow$, is generated by a point moving always in the same direction. A *curved line*, such as , is generated by a point moving i a continuously changing direction.

Two lines intersect in a point.

A straight line is unlimited in extent. It may be extended in either direction indefinitely.

A *ray* is the part of a straight line beginning at a given point and extending limitlessly in one direction:

$\overleftrightarrow{AB}$ and 

designate rays.

In this book, the word *line* will mean “straight line” unless otherwise stated.

1.2D Surface

A *surface* has length and width but no thickness. It may be represented by a blackboard, a side of a box, or the outside of a sphere; remember, however, that these are representations of a surface but are not surfaces.

A plane surface (or *plane)* is a surface such that a straight line connecting any two of its points lies entirely in it. A plane is a flat surface.

Plane geometry is the geometry of plane figures–those that may be drawn on a plane. Unless otherwise stated, the word *figure* will mean “plane figure” in this book.

SOLVED PROBLEMS

1.1 Illustrating undefined terms

Point, line, and plane are undefined terms. State which of these terms is illustrated by (a) the top of a desk; (b) a projection screen; (c) a ruler’s edge; (d) a stretched thread; (e) the tip of a pin.

Solutions

(a) surface; (b) surface; (c) line; (d) line; (e) point.

1.3 Line Segments

A straight line segment is the part of a straight line between two of its points, including the two points called *endpoints*. It is designated by the capital letters of these points with a bar over them or by a small letter. Thus, $\overline{AB}$ or *r* represents the straight line segment $A \overset{r}{\text{\_\_\_}} B$ between *A* and *B*.

The expression *straight line segment* may be shortened to *line segment* or to *segment*, if the meaning is clear. Thus, $\overline{AB}$ and *segment AB* both mean “the straight line segment AB.”

1.3A Dividing a Line Segment into Parts

If a line segment is divided into parts:

1. The length of the whole line segment equals the sum of the lengths of its parts. Note that the length of $\overline{AB}$ is designated *AB*. A number written beside a line segment designates its length.

2. The length of the whole line segment is greater than the length of any part.

Suppose $\overline{AB}$ is divided into three parts of lengths a, b, and c; thus  . Then $AB =$ + b + c. Also, AB is greater than a; this may be written as $AB > a$.

If a line segment is divided into two equal parts:

1. The point of division is the *midpoint* of the line segment.

2. A line that crosses at the midpoint is said to *bisect* the segment.

Because $AM = MB$ in Fig. 1-1, M is the midpoint of $\overline{AB}$, and $\overline{CD}$ bisects $\overline{AB}$. Equal line segments may be shown by crossing them with the same number of strokes. Note that $\overline{AM}$ and $\overline{MB}$ are crossed with a single stroke.

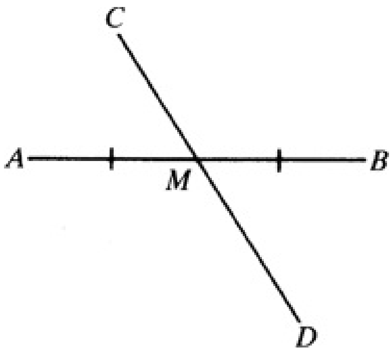


Fig. 1-1

3. If three points A, B, and C lie on a line, then we say they are *collinear*. If A, B, and C are collinear and $AB + BC = AC$, then B is between A and C (see Fig. 1-2).

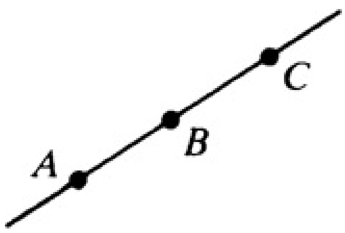


Fig. 1-2

1.3B Congruent Segments

Two line segments having the same length are said to be *congruent*. Thus, if $AB = CD$, then $\overline{AB}$ is congruent to $\overline{CD}$, written $\overline{AB} \cong \overline{CD}$.

SOLVED PROBLEMS

1.2 Naming line segments and points

See Fig. 1-3.

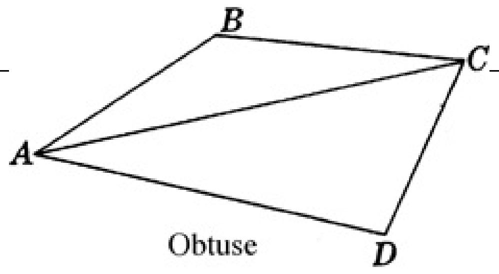


Fig. 1-3

(a) Name each line segment shown.

(b) Name the line segments that intersect at *A*.

(c) What other line segment can be drawn using points *A*, *B*, *C*, and *D?*

(d) Name the point of intersection of $\overline{CD}$ and $\overline{AD}$.

(e) Name the point of intersection of $\overline{BC}$, $\overline{AC}$, and $\overline{CD}$.

Solutions

(a) $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AC}$, and $\overline{AB}$. These segments may also be named by interchanging the letters; thus, $\overline{BA}$, $\overline{CB}$, $\overline{DC}$, $\overline{CA}$,, and $\overline{DA}$ are also correct.

(b) $\overline{AB}$, $\overline{AC}$, and $\overline{AD}$

(c) $\overline{BD}$

(d) *D*

(e) *C*

1.3 Finding lengths and points of line segments

See Fig. 1-4.

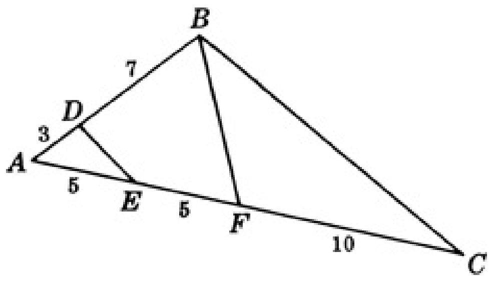


Fig. 1-4

(a) State the lengths of $\overline{AB}$, $\overline{AC}$, and $\overline{AF}$.

(b) Name two midpoints.

(c) Name two bisectors.

(d) Name all congruent segments.

Solutions

(a) $AB = 3 + 7 = 10$; $AC = 5 + 5 + 10 = 20$; $AF = 5 + 5 = 10$.

(b) E is midpoint of $\overline{AF}$; F is midpoint of $\overline{AC}$.

(c) $\overline{DE}$ is bisector of $\overline{AF}$; $\overline{DF}$ is bisector of $\overline{AC}$.

(d) $\overline{AB}, \overline{AF}$, and $\overline{FC}$ (all have length 10); $\overline{AE}$ and $\overline{EF}$ (both have length 5).

1.4 Circles

A *circle* is the set of all points in a plane that are the same distance from the *center*. The symbol for circle is ⊙; for circles, ⓢ. Thus, ⊙O stands for the circle whose center is O.

The *circumference* of a circle is the distance around the circle. It contains 360 *degrees* (360°).

A *radius* is a segment joining the center of a circle to a point on the circle (see Fig. 1-5). From the definition of a circle, it follows that the radii of a circle are congruent. Thus, $\overline{OA}$, $\overline{OB}$, and OC of Fig 1-5 are radii of ⊙O and $\overline{OA} \propto \overline{OB} \propto \overline{OC}$.

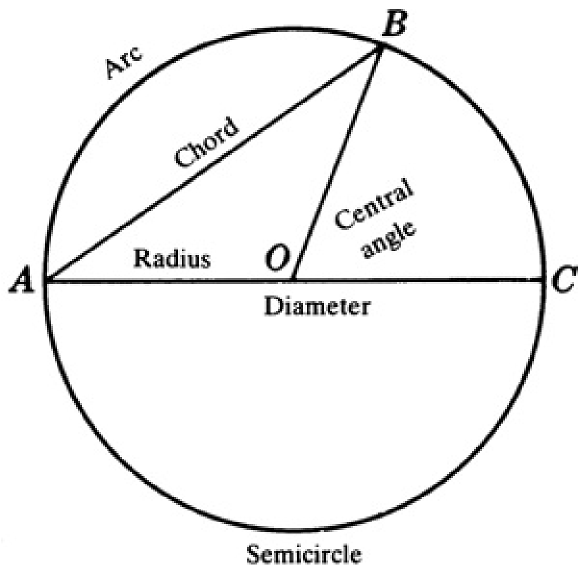


Fig. 1-5

A *chord* is a segment joining any two points on a circle. Thus, $\overline{AB}$ and $\overline{AC}$ are chords of ⊙O.

A diameter is a chord through the center of the circle; it is the longest chord and is twice the length of a radius. $\overline{AC}$ is a diameter of ⊙O.

An *arc* is a continuous part of a circle. The symbol for arc is ⌒, so that $\widehat{AB}$ stands for arc AB. An arc of measure 1° is 1/360th of a circumference.

A *semicircle* is an arc measuring one-half of the circumference of a circle and thus contains 180°. diameter divides a circle into two semicircles. For example, diameter $\overline{AC}$ cuts ⊙O of Fig. 1-5 into two semicircles.

A *central angle* is an angle formed by two radii. Thus, the angle between radii $\overline{OB}$ and $\overline{OC}$ is a central angle. A central angle measuring 1° cuts off an arc of 1°; thus, if the central angle between $\overline{O}$ and $\overline{OF}$ in Fig. 1-6 is 1°, then $\widehat{EF}$ measures 1°.

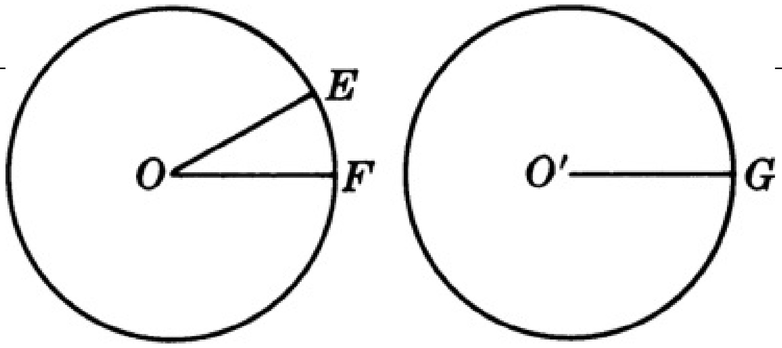


Fig. 1-6

Congruent circles are circles having congruent radii. Thus, if $\overline{OE} \cong \overline{O'G}$, then $\odot O = \odot O'$.

SOLVED PROBLEMS

1.4 Finding lines and arcs in a circle

In Fig. 1-7 find (a) *OC* and *AB;* (b) the number of degrees in $\widehat{AD}$; (c) the number of degrees in $\widehat{BC}$.

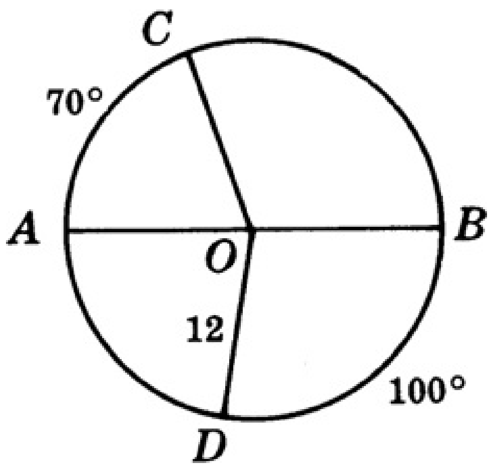


Fig. 1-7

Solutions

(a) Radius *OC* = radius *OD* = 12. Diameter *AB* = 24.

(b) Since semicircle *ADB* contains 180°, $\widehat{AD}$ contains 180° – 100° = 80°.

(c) Since semicircle *ACB* contains 180°, $\widehat{BC}$ contains 180° – 70° = 110°.

1.5 Angles

An *angle* is the figure formed by two rays with a common end point. The rays are the *sides* of the angle, while the end point is its *vertex*. The symbol for angle is ∠. or ∡; the plural is ∡s.

Thus, $\overleftrightarrow{AB}$ and $\overrightarrow{AC}$ are the sides of the angle shown in Fig. 1-8(a), and *A* is its vertex.

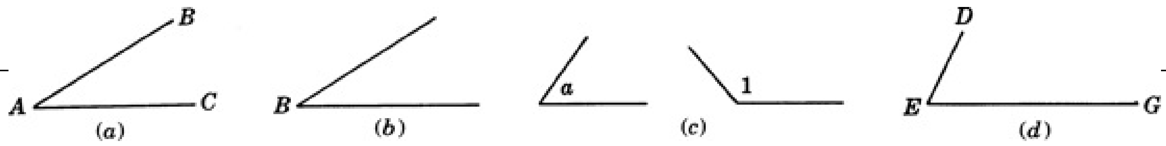


Fig. 1-8

1.5A Naming an Angle

An angle may be named in any of the following ways:

1. With the vertex letter, if there is only one angle having this vertex, as $\angle B$ in Fig. 1-8(b).
2. With a small letter or a number placed between the sides of the angle and near the vertex, as $\angle A$ o $\angle 1$ in Fig. 1-8(c).
3. With three capital letters, such that the vertex letter is between two others, one from each side of the angle. In Fig. 1-8(d), $\angle E$ may be named $\angle DEG$ or $\angle GED$.

1.5B Measuring the Size of an Angle

The size of an angle depends on the extent to which one side of the angle must be rotated, or turned about the vertex, until it meets the other side. We choose degrees to be the unit of measure for angles The measure of an angle is the number of degrees it contains. We will write $m\angle A = 60°$ to denote tha "angle *A* measures 60°."

The protractor in Fig. 1-9 shows that $\angle A$ measures of 60°. If $\overrightarrow{AC}$ were rotated about the vertex *A* until it met *AB*, the amount of turn would be 60°.

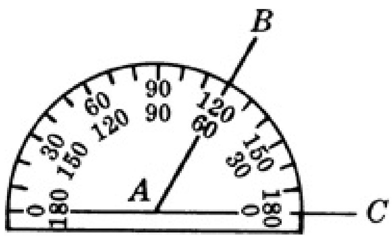


Fig. 1-9

In using a protractor, be sure that the vertex of the angle is at the center and that one side is along the 0°–180° diameter.

The size of an angle *does not* depend on the lengths of the sides of the angle.

The size of $\angle B$ in Fig. 1-10 would not be changed if its sides $\overleftrightarrow{AB}$ and $\overrightarrow{BC}$ were made larger or smaller.

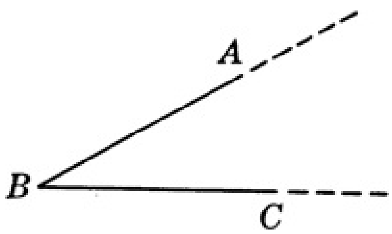


Fig. 1-10

No matter how large or small a clock is, the angle formed by its hands at 3 o’clock measures 90°, a shown in Figs. 1-11 and 1-12.

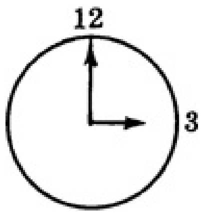


Fig. 1-11

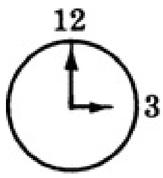


Fig. 1-12

Angles that measure less than 1° are usually represented as fractions or decimals. For example, one-thousandth of the way around a circle is either $\frac{360^\circ}{1000}$ or 0.36°.

In some fields, such as navigation and astronomy, small angles are measured in *minutes* and *seconds*. One degree is comprised of 60 minutes, written 1° = 60’. A minute is 60 seconds, written 1′ 60″. In this notation, one-thousandth of a circle is 21′ 36’ because $\frac{21}{60} + \frac{36}{3600} = \frac{1296}{3600} = \frac{360}{1000}$.

1.5C Kinds of Angles

1. *Acute angle:* An acute angle is an angle whose measure is less than 90°.

Thus, in Fig. 1-13 *a*° is less than 90°; this is symbolized as *a*° 90°.

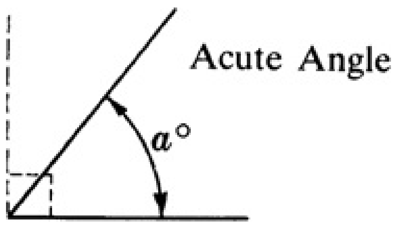


Fig. 1-13

2. *Right angle: A* right angle is an angle that measures 90°.

Thus, in Fig. 1-14, m(rt. ∠*A)* = 90°. The square corner denotes a right angle.

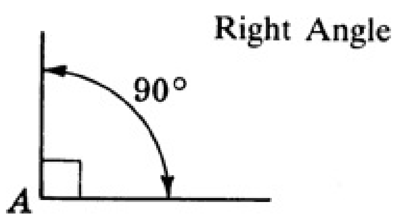


Fig. 1-14

3. *Obtuse angle:* An obtuse angle is an angle whose measure is more than 90° and less than 180°.

Thus, in Fig. 1-15, 90° is less than $b°$ and $b°$is less than 180°; this is denoted by 90° ° 180°.

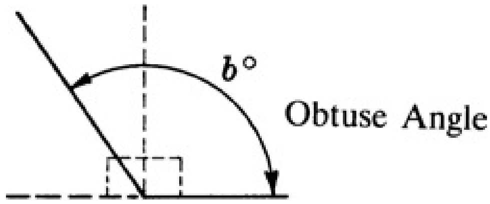


Fig. 1-15

4. *Straight angle: A* straight angle is an angle that measures 180°.

Thus, in Fig. 1-16, m(st. $\angle B$) = 180°. Note that the sides of a straight angle lie in the same straight line. But do not confuse a straight angle with a straight line!

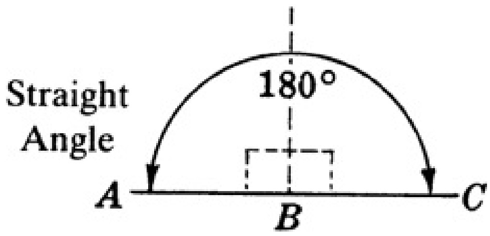


Fig. 1-16

5. *Reflex angle:* A reflex angle is an angle whose measure is more than 180° and less than 360°.

Thus, in Fig. 1-17, 180° is less than $c°$ and $c°$ is less than 360°; this is symbolized as 180° < $c°$ 360°.

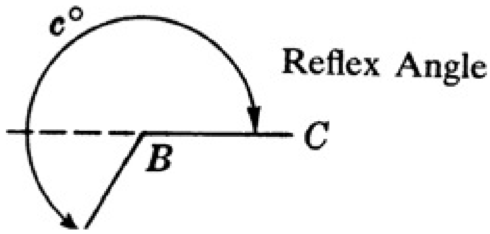


Fig. 1-17

1.5D Additional Angle Facts

1. *Congruent angles* are angles that have the same number of degrees. In other words, if $m\angle A = m\angle$ then $\angle A \cong \angle B$.

Thus, in Fig. 1-18, rt. $\angle A \cong$ rt.$\angle B$ since each measures 90°.

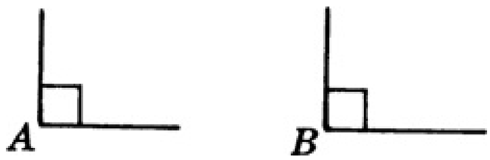


Fig. 1-18

2. A line that *bisects* an angle divides it into two congruent parts.

Thus, in Fig. 1-19, if $\overline{AD}$ bisects $\angle A$, then $\angle 1 = \angle 2$. (Congruent angles may be shown by crossing their arcs with the same number of strokes. Here the arcs of $\angle$s 1 and 2 are crossed by a single stroke.

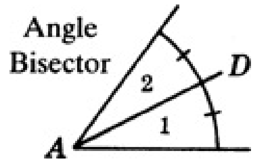


Fig. 1-19

3. *Perpendiculars* are lines or rays or segments that meet at right angles.

The symbol for perpendicular is $\perp$ for perpendiculars, $\perp$s. In Fig. 1-20, $\overline{CD} \perp \overline{AB}$ so right angles 1 and 2 are formed.

4. A *perpendicular bisector* of a given segment is perpendicular to the segment and bisects it.

In Fig. 1-21, $\overleftrightarrow{GH}$ is the bisector of $\overline{EF}$; thus, $\angle 1$ and $\angle 2$ are right angles and M is the midpoint of $\overline{EF}$.

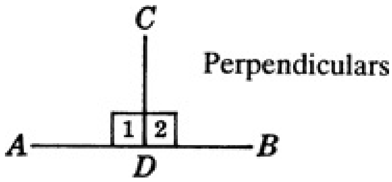


Fig. 1-20

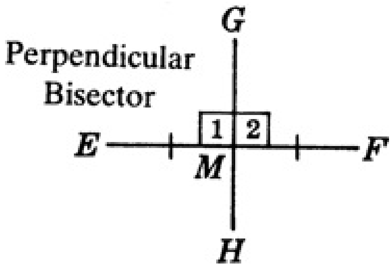


Fig. 1-21

SOLVED PROBLEMS

1.5 Naming an angle

Name the following angles in Fig. 1-22: (a) two obtuse angles; (b) a right angle; (c) a straight angle; (d) an acute angle at D; (e) an acute angle at B.

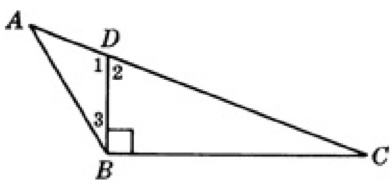


Fig. 1-22

Solutions

(a) $\angle ABC$ and $\angle ADB$ (or $\angle$ 1). The angles may also be named by reversing the order of the letters: $\angle CBA$ and $\angle BDA$.

(b) $\angle DBC$

(c) $\angle ADC$

(d) $\angle 2$ or $\angle BDC$

(e) $\angle 3$ or $\angle ABD$

1.6 Adding and subtracting angles

In Fig. 1-23, find (a) $m\angle AOC$; (b) $m\angle BOE$; (c) the measure of obtuse $\angle AOE$.

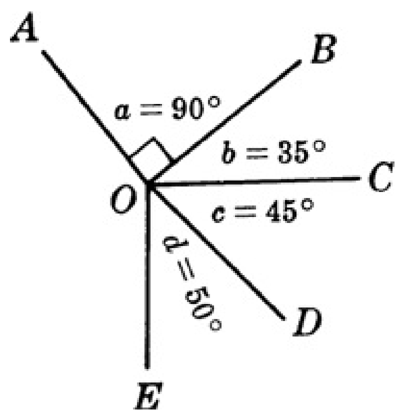


Fig. 1-23

Solutions

(a) $m\angle AOC = m\angle a + m\angle b = 90° + 35° = 125°$

(b) $m\angle BOE = m\angle b + m\angle c + m\angle d = 35° + 45° + 50° = 130°$

(c) $m\angle AOE = 360° – (m\angle a + m\angle b + m\angle c + m\angle d) = 360° – 220° = 140°$

1.7 Finding parts of angles

Find (a) $\frac{2}{5}$ of the measure of a rt $\angle$; (b) $\frac{2}{3}$ of the measure of a st. $\angle$; (c) $\frac{1}{2}$ of 31°; (d) $\frac{1}{10}$–of 70°20'.

Solutions

(a) $\frac{2}{5}(90°) = 36°$

(b) $\frac{2}{3}(180°) = 120°$

(c) $\frac{1}{2}(31°) = 15\frac{1}{2}° = 15°30'$

(d) $\frac{1}{10}(70°20') = \frac{1}{10}(70°) + \frac{1}{10}(20') = 7°2'$

1.8 Finding rotations

sample content of Schaum's Outline of Geometry, 5th Edition: 665 Solved Problems + 25 Videos (Schaum's Outlines)

- read Internet-auftritt
- **read Amazing Fantastic Incredible: A Marvelous Memoir pdf, azw (kindle), epub**
- *The Ultimate Italian Review and Practice (Ultimate Review & Reference Series) pdf, azw (kindle)*
- Crabs: The Human Sacrifice pdf

- http://aneventshop.com/ebooks/Internet-auftritt.pdf
- http://studystrategically.com/freebooks/Amazing-Fantastic-Incredible--A-Marvelous-Memoir.pdf
- http://www.satilik-kopek.com/library/The-God-Tattoo--Untold-Tales-from-the-Twilight-Reign.pdf
- http://www.freightunlocked.co.uk/lib/Crabs--The-Human-Sacrifice.pdf