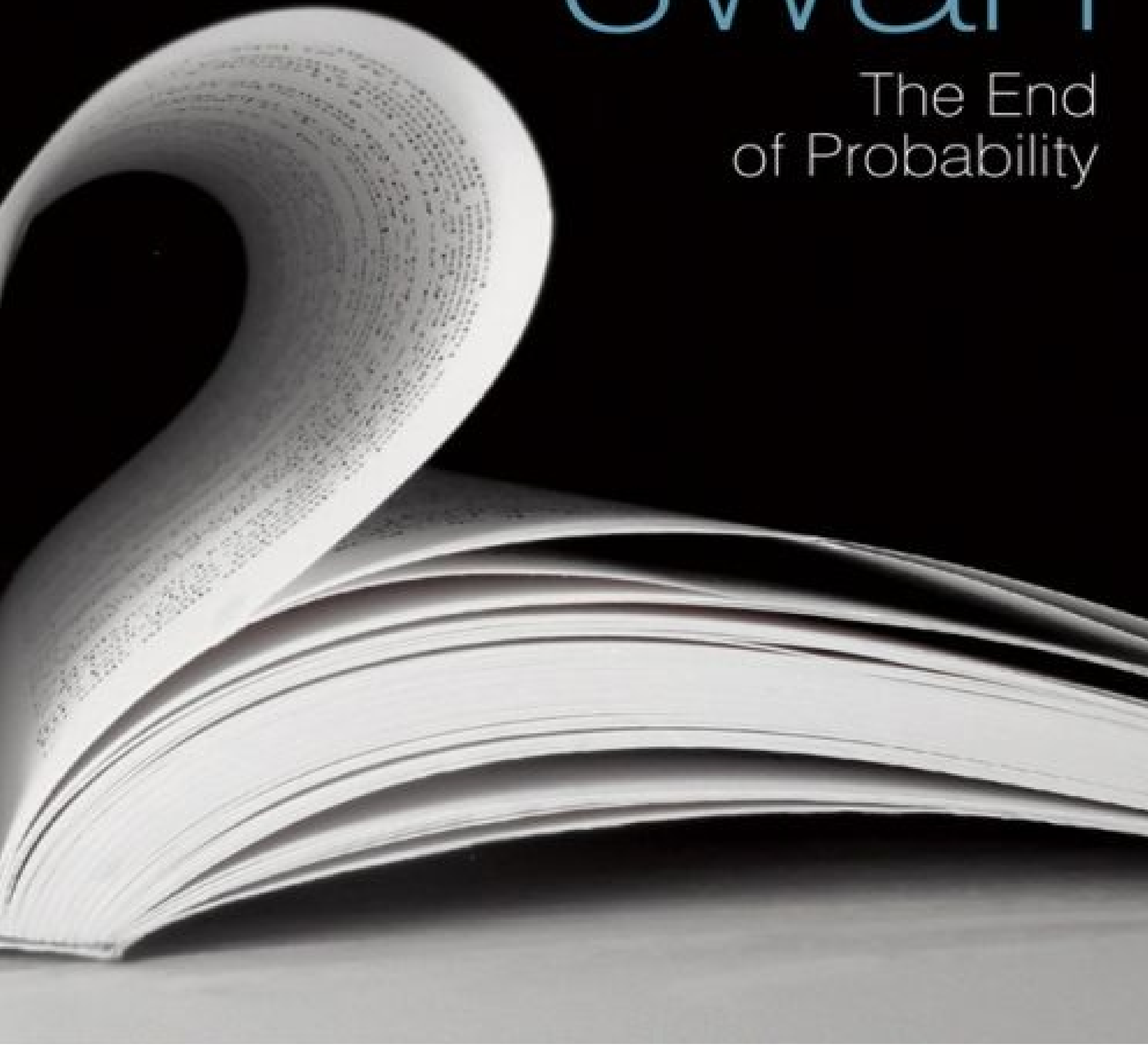
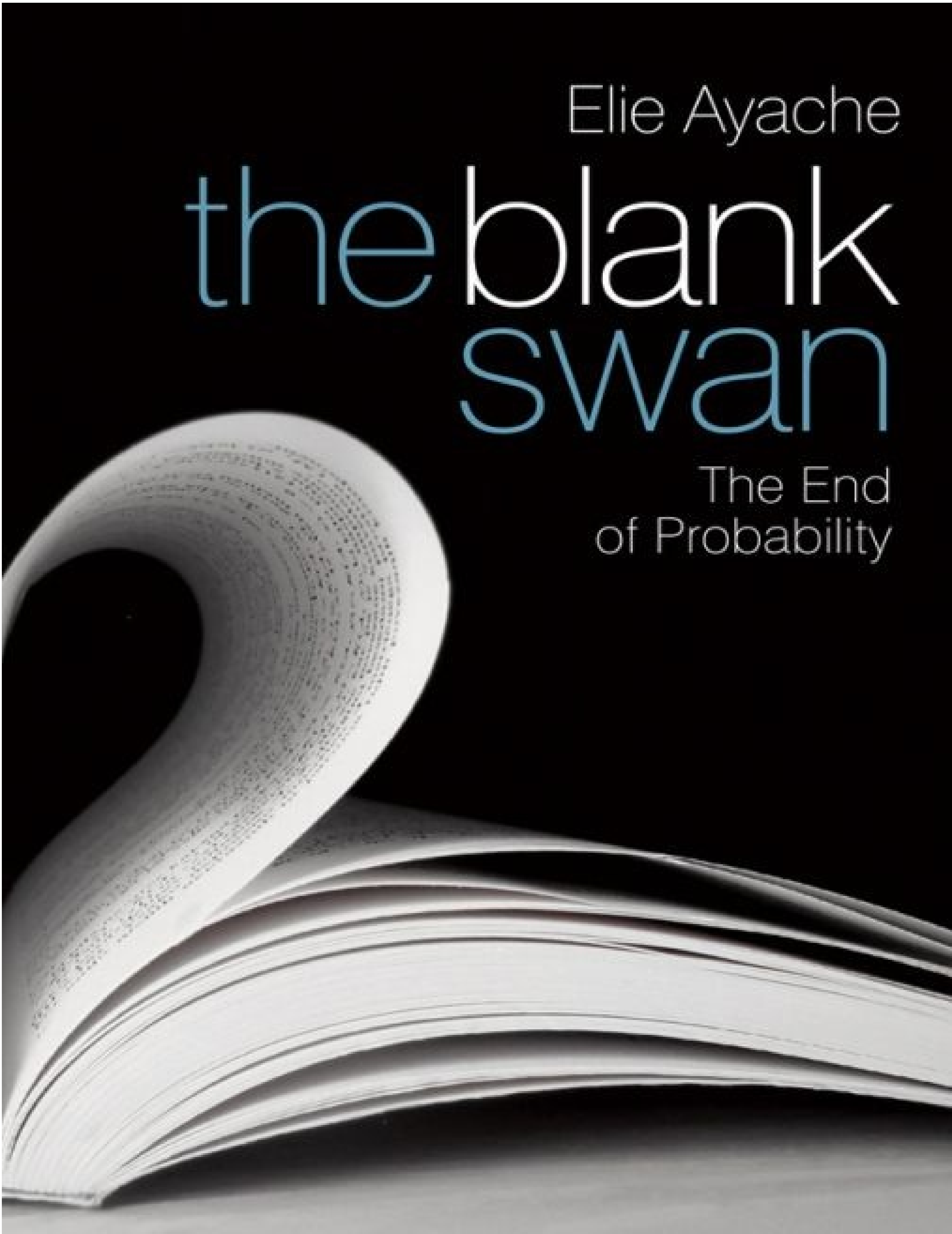
Elie Ayache

the blank swan

The End of Probability

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To serge, without whom not

The subject is immense,
requiring every order of knowledge and endless information.
Besides, when such a complex whole is in question,
the difficulty of reconstructing the past,
even the recent past,
is altogether comparable to that of constructing the future,
even the near future;
or rather, they are the same difficulty.
The prophet is in the same boat as the historian.
Let us leave them there.

Paul Valéry

The necessary book is subtracted from chance.

Maurice Blanchot

Introduction

This is a book about writing, pricing and contingent claims. I hold that the writing process and t pricing process are two special kinds of processes that do not take place in possibility or probability, like the traditional stochastic processes. They fall completely outside prediction. A processes, they keep re-creating themselves and differentiating themselves, yet they do not unfold chronological time. For this reason, their Swan bird, or the event that gives them wings, is BLANK. is neither Black nor White; it is neither loaded with improbability nor with probability. It can only filled with writing, as when we say ‘to fill in the blanks’.

Likewise, the medium of contingent claims - that which transmits them and translates them in prices - is something I call the *market*. This cannot be covered by probability theory or derivati valuation theory. The medium is of the nature of writing, and the trader of contingent claims is writer and a creator. He doesn’t predict the market (or its generalization, which is history) or try anticipate it. The market is his *work*, as when we say that the book is the work of the writer or t poem is the work of the poet.

It was during the period in which I was an option market-maker on the floors of LIFFE (Lond International Financial Futures Exchange), in 1990-95, and MATIF (Marché à Terme International d France), in 1987-90, that I learnt what it really meant to be a market-maker, or a ‘local’ (as we say and how far the open outcry market (which is the only true form of market to my eyes and the on place where contingency, as a meta-probability, can truly strike) can be from the masquerade and tot degeneration of probability to which we were recently led by the Collateralized Debt Obligatio (CDO) blunder and the subsequent rout, and will be led by any other form of complex derivative th we cannot calibrate to the market. I believe this whole branch is going to drop off the tree, and t only way to reconstruct a market and return to the market is to go back to the place (and probably t times) where individuals (locals) *make* the market, as opposed to big institutional players who nev understood how to make money in the market in the first place.

Probability theory is bad for the market as it eventually leads to the metaphysical extremity of t CDO and to the blanket rejection of all probability models by someone like Nassim Taleb, author *The Black Swan*. I think the real intellectual scandal that the whole ‘culture’ of the market is faci today is that traditional financial theory can only understand profit in the market through imperfection of some kind or other: either by an arbitrage opportunity, or asymmetric information, monopoly, or Knightian essential uncertainty, etc. (not to mention sheer luck). What is lacking is account of how to make money in the market as a *perfect medium of contingency*, not as an imperfe case of probability theory.

Apart from rebuilding a market that may emerge as a true vehicle of speculation (in the goo almost philosophical sense of the world), i.e. as *technology of the future*, I believe that a critique li the one I undertake in this book can revolutionize the philosophy of probability. Indeed, it is not probability (or worse, by luck) that money shall be made in the market, but *outside* probability, in medium and continuum that I can find no better way to describe than to compare it with the mediu of the creation of literary work.

The reason why my book is called *The BLANK Swan* is that *it is not* the opposite of *The Black Swa* as a ~~hasty critic of Taleb's book would tend to think. *The White Swan* would be the opposite~~ of *Th* *Black Swan* and Nassim himself, in a characteristically witty passage of his own book, considers th possibility that somebody some day would write a diatribe against him with the title *The White Swan*

My book is not a direct criticism of Nassim's; therefore it is not opposed to it in colour. Rather, criticizes the whole background or framework of thought in which one would assume that Black Swa and White Swan are opposed to each other.

The Black Swan is the perfectly unexpected event; the White Swan is the perfectly expected even Underlying both is the category of prediction and prevision, which is the real object of my criticism The Black Swan refers to something we cannot see or foresee (it is black) and the White Swan something evident and clear. Although opposed, the two are predicated on the idea of *content* vision, or *content* of mind, or *content* of expectation.

My observation is that Black Swans are said to be *unpredictable* only insofar as the framework an whole register of prediction is maintained. For instance, Nassim's book is crammed with th framework of prevision, only he argues that there is something - the Black Swan - that typicall evades that framework. My criticism consists in suggesting that if Black Swans constantly evad prevision, as Nassim warns us, then we should be able to define them and deal with them *complete* *independently of prevision*. Why even keep the framework of prevision when we keep talking something that constantly evades it?

Like I said, Nassim has to maintain this framework (of prevision) because this is the stuff his boo is made of, or the stuffing his bird is filled with, only negatively so. In other words, they are full prevision, only Nassim constantly argues against it. What would become of Nassim's book and th whole concept if prevision was eliminated altogether? What if we completely eliminated the conte of vision, i.e. both the colours White and Black? What if we eliminated the whole context of colo and of content? Hence the BLANK Swan.

It is not with prevision (either of Black or White) or prediction that we should deal with the Blac Swan or define it, but with something else, which, in the course of my book, turns out to be *writin* and whose quantitative avatar is the market. Writing is something we produce without previou knowledge or prevision or prediction. Writing is quicker than thought and vision. It is a materi process, when vision and prevision are only conceptual. Writing transpierces the page it is marked o It is real. It is a process that eventually transports us to the future, making us attain and fulfil o promises (think of the derivative payoff as a promise that the seller of the derivative has *written* to th buyer), yet at no point is it based on prediction or forecast.

The market, whose other name is the 'pricing process', is such a process. Hence my idea is that th market (or writing) should be the way we deal with Black Swans, not the place where we compla helplessly against them (as Taleb does).

Taleb famously declares that he now belongs to the exclusive 'club of single-idea people'.[1] The are people who realize one day that they have been carrying a single idea all along in their heads, an this is when they start writing books about it. Taleb's single idea is that our idealization of the worl what he calls the 'Platonic' (theoretical representations, models, schemas, classified patterns, or mo generally well-organized knowldege that we see with the eye of our mind), keeps rising to the surfac systematically hiding from our sight the underlying intractable and nonrepresentable reality. 'It is wh we do not see Black Swans,' he writes, 'we worry about those that happened, not those that ma happen but did not.'[2] The consequence for the reality known as the 'market' is that no Platonic forr no formalization of the unpredictable, or, technically, no probability theory or model, shall ever appl to it.

Single and simple as this idea may be, I say it is still composed of two parts: probability and th market. One can further simplify the idea by eliminating one of the parts. The idea I defend in th book is that the market should *replace* probability. Thus I end up with the market as the only part an only idea.

The reason why probability or prediction doesn't apply to the market isn't that the market is to complex or too human or too chaotic. The reason is simply that the market should be the *alternative* prediction in matters relating to the future. It remains to say, of course, how we can relate to the futu outside prediction. Trying to answer this question is the stuff my own swan is made of.

Part of the difficulty is that if the market remains single, with only itself to apply to itself, then th apparent tautology will dazzle the thinker and will prevent him from discerning the difference and th differential meaning of the market. Saying that the market is the only way to predict the market, that immersion in the market is the only way to relate to the future of the market, hardly seem informational. By contrast, my argument would have been more compelling and much riskier if I ha contended that the market is the technology, not only of its own future but of the future at large, an that the mathematics of price - which I hold should replace the mathematics of chance and probabili - consequently apply to all kinds of historical events, those belonging to the future of the market well as to the future of humanity. After all, Taleb has written about the role of chance in the marke *and* in life, as the subtitle of his pre-*Black Swan* book indicates.

I will not be nearly as ambitious in the present book. I will limit myself to the market and to th analysis of contingency as it takes place there (the contingency of market prices - what else? - whic fittingly enough, are the prices of contingent claims). When I take up topics of seemingly broad scope than the market (such as philosophical speculation in Part II and literary creation in Part III), will always refer them back to the market or to my personal experience as a market-maker or mark technologist.

I will stick with the uniqueness and the singularity of the market (which constitutes the singulari of my subject), but will extract my differential meaning from sharp distinctions I will have to mal within the strict field of contingent claims and of their pricing technology - distinctions n necessarily visible to the naked eye, such as between possibility and contingency, prediction an writing, derivatives and contingent claims, virtual and possible, etc.

Contrary to history or life, the market possesses a single metric: the up or down movements market prices. This can be a source of confusion and oversimplification, and is probably the reaso why we import numbers from a foreign field, probability theory, in order to account for the numbe in the market. However, this coincidence in numbers can also be a blessing, as it eventually allows u to frame our criticism of probability and prediction with all the greater precision, provided th distinctions I listed above are made.

It is those distinctions, rather than the singular occasion that made me realize them (the market that will stand ready for generalization to fields that may be strictly superior to the market, either scope or importance or degree of human concern. In this way, it will truly appear from inside, fro the *internal difference* that the market is in itself quite independently of the things of different scop and ~~different order to which it may later be externally compared, that the market is indeed a gener~~ *category of thought* that surpasses the strict acceptation of the financial market. It is no wonder, fro this perspective, that I should draw heavily on the work of philosophers of event and difference suc as Bergson, Deleuze and Badiou.

Perhaps the main distinction on which the whole book hinges is the one between possibility ar contingency, or, correlatively, between the (theoretical) value of a derivative and the (market) price c a contingent claim. This distinction becomes clearest in the last part of the book, thanks to th *conversion*, and the three preceding parts can be read (or listened to) as different voices or differe variations on the same theme, leading to the final movement, like in a symphony.

In a sense, the three parts say the same thing; they are three voices of the market that do n equivocate on its notion. On the contrary, they say the market in the same sense. *Univocity* of th market and of contingency is indeed the main theme of this book.

The three preparatory parts are different approaches and travel towards the culmination of the boo which appears, in the final part, to be the *genesis* of the market. Think of the three parts as occurrin in the context of discovery and of the concluding part as taking place in the context of justification.

In Part I, the totality of themes is introduced. In Part II, contingency is given a firm philosophic basis (even an absolute status), which will secure the way to think of it independently of possibilit even independently of thought itself.

Part III is literally a travel to the ‘land of the market’. It traverses at once all the relevant space geographical, literary and the space of the book itself. In fact, it is a narrative adventure or a kind reverie, which the strictly mathematically or philosophically inclined reader may wish to skip at first reading. Nonetheless, it is the part I personally prefer and in which I have travelled the farthest could. There are outer reaches in this exploration that even I do not yet quite understand.

Part IV is the promised land, or the new world finally reached. It illuminates the whole book like beacon. If the reader isn’t afraid of travelling at infinite speed, as Deleuze would say, and discarding, right at the entrance, any prejudice or preconceived idea he may have about the pricing contingent claims and their market, he or she is welcome to engage in Part IV directly and to read independently of the rest. From a talk I have recently given, I have discovered that the reader (listener) may be all the more prepared to assimilate a given idea that the idea is truly revolutiona and radically new and that it has not the slightest resemblance or even the remotest relation wi anything he or she may be familiar with. Of course, the idea has nevertheless to be consistent.

Perhaps I can introduce this idea along the following lines. I will start with a question. Imagine yo are writing a foundational book on options (or on derivatives generally - let us not yet call the ‘contingent claims’), a script that begins from scratch, literally from genesis, as if you were writin the history of that world. Can you say what options are, what meaning they have, that is to say *wh difference they make*, or what they are meant for, without using the notion of possibility probability?

Imagine you are a speculative philosopher, and by this I mean that you wish to build your system philosophy and your ontology from scratch, and that the notion of possibility is not available to yo Possibility and possible worlds are metaphysical constructs. As such, they don’t exist. Perhaps we wi make them exist. Perhaps it will turn out we cannot talk of markets or of options without talking possibility. Still, my question is for the moment: 'Can we conceive of options, of their meaning and o their destination, without thinking of possibility?'

Options are contingent claims. The vanilla call option pays $(S - K)$ if the stock price S is above th strike price K at the expiry of the option, and 0 otherwise. The formula 'If . . . then, else if . . . then' the formula of contingency. It points to a dissymmetry in the otherwise indifferent space; it points to difference. It also points to an event: something that can be, but that can be otherwise. This 'making difference', or contingency, is the essence of options. This is the creative act of options, th productive difference they are made of.

Possibility has nothing to do with this. Contingency is of this world; possibility is of another worl There are even philosophers of the modern speculative brand (e.g. Quentin Meillassoux) who abh metaphysics and the language of possibility and necessity as the totalization of possibilities, yet wh believe that the world is only made of contingency and that the only necessity there is in the materi world is the necessity of contingency. (I examine Meillassoux's work at length in Part II.)

Possibility is only the metaphysical channel that we have invented in order to guide us contingency. While contingency is all difference and shift and dislocation, possibility is but the (ver understandable) move to fold that difference back on identity and on presence. The proposition possibility is the following: Why don't we imagine a process, a sequence of possibilities, that takes u to that contingency?

Possibility is a representation and, as such, it re-presents, it pins back on the presence of mind, th pure difference that contingency is supposed to make. Therefore, before I have even started talkin about market completeness and the threat of redundancy of derivatives, you can tell that possibility, a a metaphysical notion, in itself already contains the germs of completeness or the death of the marke Possibility is already an attempt to bring back within the confines of representation, and domesticate, the utter otherness of contingency.

In a world that is only made of contingency, it is only natural that we should invent options derivative contracts. It is only natural that we should circulate, today, things that we know will *make difference in the future*. This is why I have always thought of derivatives and derivatives markets the *technology of the future*. Thus we write those derivatives contracts, and in their writing (the 'If . then, else if . . . then' formula) there is nothing even remotely related to possibility. It is the pur material writing of contingency. It is pure difference. Writing is difference, as Derrida would say.

However, writing, as we all know and as Roland Barthes suggests, is only the prelude to a *exchange*.[3] Typically, we write contracts when we are about to exchange something for fear that, whe we no longer hold what we used to possess, in the interval of the exchange, but have let go of it order that the counterparty may receive it, the counterparty might disappear, thus alienating us of ou property.

Therefore, my next statement, as I am sure one can guess already, is that we write derivatives, thos pure formulas of contingency, *only insofar as we wish that the difference they will make in the futu may make a difference today.* Instead of wondering at their present value, we immediately go and se what difference they make in front of somebody *else*. This is the most direct statement that I cou think of, to the effect that we write derivatives for the sole purpose that they may be exchanged an admit of a price. Price is a differential, not a splendid present value as general equilibrium theo holds. To price something is to trade something in order *to earn a difference*. It is to make difference.

In the very essence of derivative writing, in its very meaning, the fact is embedded that they shou admit of a price. The link is immanent and material. *We don't need possibility or metaphysics t travel from the derivative payoff to its present price*. To be a derivative is to admit of a price. That to say, *to add a price*, a new state of the market-world, to the existing prices.

Possibility and probability and stochastic processes, all these constructs, are themselves derivati concepts, that is to say secondary. They came later. Contingency and price (or market) came firs Market (both of the underlying asset and of any stratum of overlying derivative you care to write o it) is the original entity (note that 'underlying' will henceforth be used as a noun). Absent possibilit and stochastic process, there is no way to talk of replication other than as static or intrinsic or mode independent. Creating a new derivative payoff - and by 'new' I mean that this payoff is not statical replicable with the existing ones - is only meaningful to the extent that it will bring to the table, i.e. the market, a new price: that is to say a new 'making a difference today' by something that 'will mak a difference in the future in a new way' (say an innovative, path-dependent or knock-out payoff).

It is fine that we should devise algorithms, as a matter of fact devise the whole metaphysical notio of possibility and probability, in order to compute present values of derivatives. I even think that too that allow us to value and dynamically replicate derivatives are a constitutive part of the creation an growth of derivative markets (see chapter 3). However, given how I have straightened up the genes of markets, going directly from contingency to price without the mediation of possibility, it should b clear that derivatives can have as their only meaning and destination that they may trade in a mark and never end up perfectly replicable and redundant.

Therefore, when the Black-Scholes-Merton framework of option valuation - to only cite th paradigmatic case - shows that the option value is perfectly reducible to a number called volatility an to a deterministic replication strategy, this doesn't mean the end of the game. On the contrary, this just one way of slicing the price, and if this designates anything and puts pressure on anything, designates that the thing called 'volatility' as the thing that you should expect, by virtue of the fir principle that options must trade, to become stochastic.

Does this mean that the underlying process has changed and that you must now assume a proce with stochastic volatility? No! Nothing has changed. The market is the eternally contingent an eternally self-differentiating thing it has always been; it is the *single dice throw* that you will never f in a fully settled and developed representation of states of the world. It is your valuation tool th needs to change and evolve and differentiate[4] - unless you combined your valuation tool with yo trading skills and finally came up with the *universal pricing tool* that, hopefully, would never nee changing, namely you the trader.

Part I

Writing and Event

1

Writer of *The BLANK Swan*

1.1 PREDICTION VERSUS PRESCRIPTION

1.1.1 My private *Black Swan*

Taleb has surprised me by giving me *The Black Swan* I expected most.[5] For there to be a gift and surprise, there must be a difference and an exchange. I couldn't have given myself *The Black Swa* that I expected most (even though *The Black Swan* was, in a sense, already given to me, as I w acquainted with it and none of its themes came as a real surprise to me), for the reason that it tak two to make a gift and a giving, let alone an exchange. Not to mention that I couldn't have personal written *The Black Swan* or even imagined that something like it could ever be written. What cou there be to record, indeed, on the subject of the extremely improbable and of its impact, beyond t fact that the event goes down in history and records itself by itself when it occurs, all the deeper a the more indelibly that it was precisely improbable and had such a great impact?

I should thank Taleb for allowing this book - *The Black Swan* - 'to write itself', as he himself sa in the prologue; it is only because *The Black Swan* is out now and because it is written that there cou be material at all for my surprise. Recall that *The Black Swan* that I know best and am mo acquainted with - the private *The Black Swan* I know so well that I could have written it myself couldn't possibly have surprised me *in thought* (not forgetting my intimate belief that it shouldn't written). In order to surprise me, it had to leave me and to differ from me; it had to leave my mind a assume the spaced and very different (almost strange) shape of the written material. In a way, o biggest surprise is when we come upon our own thoughts in a text that is not our own.

The literary work reverses the category of probability. True, a totally unexpected text may thrilling, but what is more thrilling (and perhaps most thrilling) is that the material we are readi should fulfil us and correspond perfectly to what we want and what we expect - almost to the point total coincidence - yet that the writing, this finite thread we are following word after word and spaci after spacing, should make it totally improbable and almost unimaginable, at every juncture, that su a thing should be written. This is why the greatest thinkers can make us the gift of the thoughts expect most and understand best - our *own* thoughts - literally by writing them, by surprising us wi the improbability, not of the thought but of the written text. This is why Wittgenstein is able to writ in the preface of the *Tractatus*, that only the reader who has had similar thoughts can understand h book.[6] This is why he is able to write an *impossible* book, surprising us all the more that what w once impossible for thought becomes possible in writing, even necessary.[7] This is why Pierre Mena is able to progress towards the only and, for this reason, most expected outcome of his writi process, Cervantes' *Quixote*, without giving up the element of surprise and improbability at any poi or any juncture.[8]

1.1.2 Pierre Menard's trading room

Menard did not rewrite the *Quixote*, for then there would have been no risk in his enterprise. He tru wrote it. This tells us that there could be risk in writing without there being probability, or eve possibility. From the point of view of possibilities, there is none facing Menard except to end u writing the *Quixote*, which is another way of saying that he faces no possibility at all.

I express this by saying that *writing is a capacity, not a possibility*. What Menard has done, in th process of his writing, is probe a space of variation of a new kind; he has hollowed out a trading *roor* strictly independent of the space of possible alternatives to the *Quixote*. Only in the writing of futur unknown texts is capacity confused with possibility, for the thought is then that the writer could hav equally written another text. The tradability and the element of improbability in writing (what I hav called the risk of writing) are then confused with the variability of the outcome *within the space alternative outcomes*. In his endeavour of writing an existing text, Menard has managed, by contras to bring risk and probability apart again. Or rather, this is what Borges has managed to do, fo Menard's whole work and whole existence are, of course, totally incredible.

There is more to the *space* of trading/writing (I've called it a 'room') than just the collection 'preconceived' alternative texts. Consider that Menard was absolutely equipped with the algorithm perfect replication of the *Quixote* (since the *Quixote* already existed), yet he managed to trade it won't say 'to change it', for he didn't, or 'to lose it then earn it back', for he didn't either - as I sai he just *wrote* it). The risk in trading/writing extends beyond probability. We can even define risk a that which *sways and trades*, after probability is done with and saturated. The market takes place - starts - after the end of probability. It is the end that *can* start (the 'can' of capacity, not possibility).[9]

1.1.3 Probability, replication, context and beyond

Probability is always defined relative to a given context or collection of states of the world.[10] E replicating the derivative in all possible states of the world, the derivative pricing model exhaus probability and saturates the context. However, this is not yet trading. Trading the derivative precisely what *happens next*. It puts into play the parameter (or parameters) whose fixity was th guarantee of fixity of the context and of the corresponding dynamic replication. For instance, optic trading is a volatility play and volatility is assumed to be fixed by the replication algorithm of Black Scholes-Merton.

Note that derivatives were written *in order to trade* (not in order to conceal payoffs in a seal envelope that would be opened only at expiry). Derivatives are the natural offspring of the market an are the stuff the market is made of. (In a way, therefore, the market is also their produce: in fac derivatives and the market are identical and not just hierarchically, or genetically, related.) Thus w see that trading (the process that is supposed to record a value, as of today and day after day, for th derivative that was once written and sentenced to have no value until a future date and unless fraction of space) will never be the reiteration and the replication of the values that were initial planned for the derivative by the theoretical stochastic process and its prescribed dynamics.

The derivative valuation model is supposed to prescribe a value today, and every day, from th values the derivative is supposed to receive in the intervening states of the world, and in this it ma seem indeed that the model is giving back to the derivative the present value it was once denied o account of its derivative nature and of the instruction to pay off in a future that may never be prese and in a state of the world that may never be realized. The irony, however, is that this making-prese of the deferred value of the derivative has no other present to be imprinted against but the present the market and the actuality of trading. Derivative valuation models are *intended for* derivati trading and pricing, despite what all the academics working and theorizing in the field may have say about that. Yet we saw that the trading of the derivative, or in other words its market, will b necessity take place and extend in this 'improbable' space we have called the *trading room*, and whic is by definition absent from the replication plan and from probability.

I propose that risk, or the market, or contingency, is always posterior to context, possibility an probability. It always comes after probability, which is also a way of saying that context an probability are needed in the *first place*. Indeed, the market-maker wouldn't be pricing the derivativ and trading them were it not for the derivative valuation model and the dynamic replication algorithm which implicate him in the price process of the derivative.[11]

Some have proposed that risk is all *model risk*. This is certainly compatible with my propositio and even derivative on it (where 'derivative on' means 'derives from'; I will maintain this usag throughout the book, even though it may sound odd to the reader, because of the financial connotatio of 'being a derivative written on'.) However, I think this attitude is reductive of risk, in the la instance. It tends to put the model at centre stage and to depict risk only negatively and derivative relative to the model. By contrast, my proposition of contingency exceeds the model and goes beyo it. It doesn't counter the model but requires it. Contingency takes place *after* the model (as it tak place after the context and the probability), and for this reason it cannot limit itself to the model or b called 'model risk'.

1.1.4 Contingency

Contingency is the writing/trading thread that we keep pursuing despite the fact that the context h been saturated by replication. Rather, the thread of derivative *writing* and *trading* (these tw interchangeable sides of the destination of derivatives) is what the market is all about: the market as writing process that is irreducible to a stochastic process; the market as a writing capacity, as *medium of contingency*, that is irreducible to a writing possibility.

This is something I state of the market at large and not just of the derivatives market: what th market means in its *essence*. When you think of it, the story of the derivatives (which it would b more appropriate to call 'contingent claims', as we will definitely do, later) or equivalently the sto of dynamic replication in *Pierre Menard* has only helped us separate the context from what exceeds and helped us distinguish between possibility and contingency. It put this distinction in all the great relief since it was *necessary* both to have the context and to trade through it (i.e. to change it) in ord to ascertain the writing and the risk of writing, both of which are alternative definitions of the marke