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THE EVERYTHING® GUIDE TO Calculus I

A step-by-step guide to the basics of calculus—*in plain English!*

Limits • Continuity • Differentiability • Derivatives • Chain Rule • Inverse Functions • Graph Analysis • Definite Integral • Accumulation • Differential Equations • Antidifferentiation • *and much more!*

Greg Hill
National Council of Teachers of Mathematics

THE EVERYTHING® GUIDE TO CALCULUS I

Dear Reader,

Ever since my initial introduction to calculus as a high school senior in 1974, I have been fascinated by the subject. Looking back over my thirty years as a high school mathematics teacher provides an even more interesting perspective on how the teaching and learning of calculus have evolved even in such a relatively short period of time. I first learned calculus without the benefit of technological supports. The course consisted of a great deal of memorization of definitions, theorems, and rules applied to very algebraically complex problems. It was not uncommon for me to solve a problem and not really understand what I had just accomplished.

In the past two decades, graphing technologies, computer software, and Internet applets have changed the way calculus is taught and understood. The factual information is still the same, but students can now view local linearity, watch an applet turn a secant line into a tangent line, and see the number of inscribed rectangles increase to produce increasingly better approximations of areas under graphs. The beautiful geometric principles in the course are more salient, and realistic applications are more available. I hope your experience with this book inspires in you a similar passion for this wonderful subject.

Sincerely,

Greg Hill

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A step-by-step guide to the basics of calculus—*in plain English!*

Greg Hill
National Council of Teachers of Mathematics

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This book is dedicated to my good friends and colleagues John Brunsting and John Diehl, two of my most significant mentors in mathematics—and particularly in the field of calculus.

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The Top 10 Ways to Be Successful in Calculus

1. Develop a solid foundation in algebra, geometry, and trigonometry.
2. Seek a balance between analytic skills and conceptual understanding. Understanding the big ideas makes mastering the mechanics much easier.
3. Memorize the facts that are the foundation of the course, including derivative and integral rules and major theorems.
4. Think about most concepts on a microscopic level. Calculus is the study of change on infinitely small intervals and how those changes accumulate.
5. Remember the chain rule. It shows up in almost every derivative and integral.
6. Use visualization tools to make connections between ideas that are important in the subject. Graphing technologies and Internet applets are excellent sources of such tools.
7. Be patient. Master each concept before moving ahead, because the subject builds upon itself incrementally.
8. Read and study from multiple sources. A single explanation of an idea may not make it clear, and there are numerous paths to understand any concept.
9. Find a friend or mentor to discuss the ideas that arise in the course.
10. Look for the beauty and wonder in this subject! Change is everywhere in the world, and calculus helps us analyze and understand it.

Introduction

FOR DECADES, CALCULUS has struck fear into the hearts of countless high school seniors and first-year college students. The mere thought of mastering the mysteries of the subject has caused far too many people to give up before they even get started. But for those who gain an understanding of it, calculus is a beautiful integration (no pun intended) of all the math topics that lead up to it. Calculus uses arithmetic, algebra, geometry, and trigonometry to develop new and fascinating ideas. The most important thing to do as you work through this book is to really believe that you can learn calculus. Its reputation as an unconquerable mountain is totally undeserved.

Centuries ago, the mathematicians Isaac Newton, Gottfried Leibniz, Leonhard Euler, and others worked to develop the ideas of calculus in an attempt to study and understand the world around them. At its most basic, calculus is the mathematics of change. These mathematicians realized that by thinking in terms of infinitely small increments, they could better understand ideas of limits, rates of change, and even areas and volumes of irregularly shaped regions and objects.

Today, calculus is a vital element in the foundation of many practical fields, such as engineering, biological sciences, medical studies, economics, and even the automobile and film industries. People in these professions are not necessarily sitting down at their desks and working calculus problems. It is more likely that the tools they use, particularly computer programs, have calculus processes at their core. Those tools are used to achieve an end or produce a result, and without the help of calculus, the tasks would be significantly more difficult—and sometimes even impossible.

This book is intended to remove the mystery of learning calculus. You will see how calculus utilizes many of the foundational ideas introduced in earlier courses of study to develop new ideas. You will also discover that, taken in small steps and developed gradually, the big ideas of calculus are very accessible. It is not the evil monster many people make it out to be. The book will give the subject relevance by helping you understand how the calculus concept of measuring and exploring summations of infinitely small change shed light on what is happening at any instant, what has happened in the past, and what may happen in the near future. Currently, global warming is a major concern for environmentalists. Scientists are constantly studying changes in the Earth's temperature. Those changes are examined over short intervals to establish current rates of change, and they are studied over longer intervals to get a picture of the long-term impact of the phenomenon. Although it is cloaked here in environmental science, this is the essence of calculus: the study of change.

Ironically, calculus itself continues to change in certain respects. The capabilities of computers and calculators have opened new pathways to understanding the ideas of calculus. Graphing calculators and computer algebra systems reduce some of the challenges that students faced in past decades. Graphing calculators can rapidly produce graphs, solve equations, and even numerically evaluate derivatives and integrals. Computer algebra systems can significantly reduce the manipulations necessary for solving complicated equations, taking numerous derivatives, or finding antiderivatives. Geometry software programs make possible the dynamic visualization of many calculus concepts. Used properly as tools of study, these newer technologies can reduce the mechanical obstacles and sharpen the focus on the big ideas of the course.

CHAPTER 1

Prerequisite Skills

With any new endeavor, you usually need to have certain basic skills to move ahead successfully. Could you imagine becoming a triathlete if you didn't know how to swim or ride a bike? The same is true for calculus. All the math courses you've taken create a solid foundation for calculus. Don't believe all the hype about how hard calculus is. If you've taken algebra, geometry, and trigonometry, you've got everything you need to get started.

Important Algebra Skills

Some people say that by the end of middle school, students have learned all of the math they need to know to get by in life. That may be true for many people, but for those who want to build a career using any kind of math skill, a mastery of basic algebra is indispensable in all successive math courses.

Algebra introduces abstract thinking into the world of numbers and equations via the use of variables to represent unknown quantities. Algebra students learn how to use the Cartesian coordinate system to view graphs and data. Many students first learn to use a graphing calculator when they study algebra, and this skill is a huge part of calculus. Many other things you learned in algebra, such as pattern finding, variable expressions and functions, powers of variables, and properties of exponents, are used regularly in calculus. You'll recognize polynomials and factoring here, as well as domain and range. And you've already learned perhaps the most important tool for working in calculus: solving equations and inequalities.

This chapter is a quick review of these concepts from algebra class. You won't need to go over everything you learned in Algebra I and II, but you'll concentrate on the algebra building blocks you'll need. Remember, as you're working through the problems in this book, you can refer to the key algebra formulas in Appendix A.

Solving Equations

When you solve an equation, you find the value or values of the variables that make that equation true. You first simplify the expressions on either side of the equation. As you do this, keep the equation balanced by always doing the same mathematical step to both sides of the equation. Look at the example that follows. It's a pretty simple first-degree equation. Do you remember those?

EXAMPLE 1-1

Solve $3(x-1)-7=\dfrac{x}{2}$.

Simplify the left-hand side by distributing the 3. $\quad 3x-3-7=\dfrac{x}{2}$

Combine the constants. $\quad 3x-10=\dfrac{x}{2}$

Multiply both sides of the equation by 2.	$6x-20=x$
Subtract $6x$ from both sides of the equation.	$-20=-5x$
Divide both sides of the equation by –5.	$4=x$

Solving Inequalities

In calculus, you'll often need to determine where a function is zero, positive, or negative. This may happen at specific numbers or over a whole set of numbers, so you must pay attention to the domain of the problem. Changing the previous example into an inequality problem should remind you of several important ideas about solving inequalities and about solving over a certain domain.

ALERT

> A small detail in solving inequalities makes a big difference between getting the right answer and getting the wrong answer. If you multiply or divide both sides of an inequality by a negative number, you must turn the inequality arrow around.

Suppose the problem were to solve $3(x-1)-7<\dfrac{x}{2}$.

The initial steps would have been similar up to the point where $-20<-5x$. When you now divide both sides of the equation by –5, the inequality symbol turns around to give $4>x$. If no domain for the solution or solutions is stated or can be determined from the context of the problem, you should assume that the domain is all real numbers. In this case, every number less than 4 would solve the original problem. But if the problem had been stated as "Solve $3(x-1)-7<\dfrac{x}{2}$ over the whole numbers," the only acceptable solutions would have been the whole numbers less than 4, which are 0, 1, 2, and 3.

Graphing on the Cartesian Coordinate System

Don't let the fancy term intimidate you. The Cartesian coordinate system, named for the French mathematician René Descartes, is just the xy-plane in which all graphing occurs. The graphs of two basic types of functions, linear and quadratic, are often introduced in algebra, and they resurface frequently in further math. Calculus, a very visual subject, requires working with quite a few graphs of lines, parabolas, and other functions. Most often, you will have the equations and will need to produce a quick sketch, but at times, you may need to write the equation for a line given certain information.

RULE

The slope-intercept form of a linear equation has the form $y = mx + b$, where m is the slope and b is the y-intercept. When the power on the x is 1, the graph will always produce a line, which is why the equation is called *linear*.

One way to graph any function is to pick a few x values, plug them into the equation to calculate the corresponding y values, and then plot the points and connect them. Because many graphs used in calculus are just tools employed to analyze a problem or work toward a solution, a quick sketch is often sufficient. Generating individual points can be too time-consuming. Let's look at a better approach.

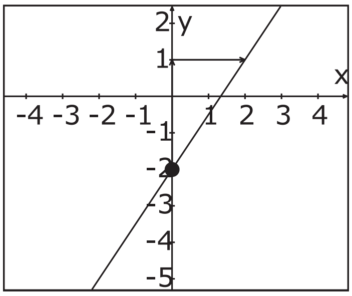
The slope of a line (the m in $y = mx + b$) is the ratio of how much the y values change to how much the x values change. If you know the slope, you have an easy way to move from point to point on the graph. The b value is the y-intercept because when $x = 0$, then $y = b$. This is where the graph will cross the y-axis. Therefore, a much more efficient way to produce the graph of a line is to plot a point at the y-intercept and then use the slope to find other points on the line before connecting them. All linear equations, with the exception of vertical lines, can be rearranged into slope-intercept form, so this quick sketch method can almost always be used.

EXAMPLE 1-2

Graph $y = \frac{3}{2}x - 2$.

Plot a point at $(0,-2)$ because that is the y-intercept. From that point, move up 3 units and right 2 units to find another point on the line. You can also move down 3 units and left 2 units because $\frac{3}{2}=\frac{-3}{-2}$. Connect the points with a straight line.

Figure 1-1



Graphing quadratic equations is somewhat more involved, but remember that in calculus, a rough sketch, with knowledge of a few key characteristics of the function, is frequently all the information you need. It's often enough to know the location of the vertex, the y-intercept, and the x-intercepts of the graph, if they exist. Where the graph lies in relation to the x-axis will provide important information—specifically, where the function values are positive and where they are negative.

RULE

The standard form of a quadratic equation is $y=a\cdot x^2+b\cdot x+c$, where *a*, *b*, and *c* are all real numbers. Quadratic equations produce U-shaped graphs called parabolas. The *x*-coordinate of the vertex is found by $x=\frac{-b}{2a}$. The *y*-coordinate is found by substituting the calculated *x* value into the equation. From $y=a\cdot x^2+b\cdot x+c$, the *y*-intercept is *c* because when $x=0,\ y=c$.

EXAMPLE 1-3

Produce a sketch of the quadratic function $y = 2x^2 - 4x - 3$.

By comparison to standard form $y = a \cdot x^2 + b \cdot x + c$, $a = 2$, $b = -4$, and $c = -3$.

Using $x = \dfrac{-b}{2a}$ reveals that the x-coordinate of the vertex is $x = \dfrac{-(-4)}{2 \cdot 2}$, or $x = 1$.

The y-coordinate is $y = 2 \cdot 1^2 - 4 \cdot 1 - 3$, or $y = -5$.

Because a is a positive number, the parabola is an upward-oriented U-shape.

The y-intercept is c, which is -3.

Even without the x-intercepts, you should now be able to produce a quick sketch of the graph.

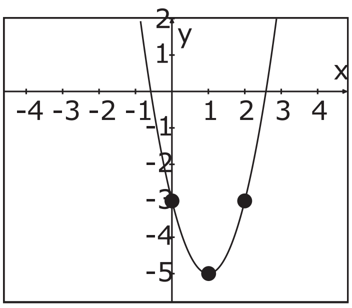


Figure 1-2

Note the point plotted at $(2, -3)$. Remember that all parabolas are symmetric, so you can easily find an additional point by using symmetry to the y-intercept.

Factoring Polynomial Expressions

The factoring you learned in algebra is used over and over again in successive math courses, and this is certainly true for calculus. The most common factoring you'll do in calculus is factoring the greatest common factor (GCF) out of an expression. You'll also use factoring to break a quadratic polynomial into the product of two simple binomials in order to solve an equation. Taking out the GCF means you essentially reverse the distributive property. Factoring a quadratic expression into a product of two binomials is a bit more complicated. To find the GCF of a polynomial, first find the GCF of all coefficients. For the variables in the expression, choose the lowest power of each variable that appears in all terms. To factor out the greatest common factor, divide each term of the original polynomial by the GCF.

EXAMPLE 1-4

Factor the greatest common factor out of $12x^3y^2 - 18x^4y^3z + 30x^4y^5$.

By observation, you should be able to tell that the greatest common factor of 12, 18, and 30 is 6. This is the first factor in your GCF.

The lowest power of x that is in each term is x^3.

The lowest power of y in all terms is y^2.

Because z does not appear in all terms, it is not part of the greatest common factor.

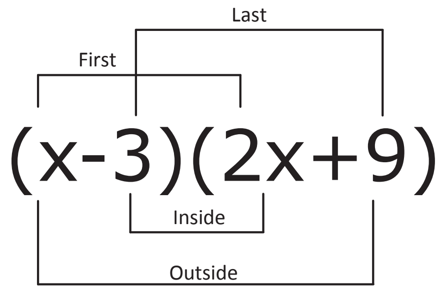
The GCF is $6x^3y^2$, and you must divide it out of the original polynomial to find the remaining factor.

$$\frac{12x^3y^2 - 18x^4y^3z + 30x^4y^5}{6x^3y^2} = 2 - 3xyz + 5xy^3$$

The final factored form is
$12x^3y^2 - 18x^4y^3z + 30x^4y^5 = 6x^3y^2(2 - 3xyz + 5xy^3)$.

In calculus, factoring a quadratic expression is used almost exclusively to find the solutions to an equation and to determine where the quadratic is positive, zero, or negative. Remember that a quadratic is usually the result of multiplying together two binomials. For example, $(x-3)(2x+9)=2x^2+3x-27$. Factoring takes the expanded quadratic and breaks it back down into its two factors. Do you remember an acronym called FOIL from algebra class? It stands for **F**irst, **O**utside, **I**nside, **L**ast.

Figure 1-3



This is one way in which two binomials can be multiplied. The $2x^2$ above is the result of multiplying the first term in one of these binomials by the first term in the other: x times $2x$. The "outside" terms are x and 9, and the "inside" terms are –3 and $2x$. If you multiply each pair, you get $9x$ and $-6x$, which sum to $3x$. The last term in each binomial is –3 and 9. Their product is –27. When you factor a binomial, concentrate on getting the product of the first terms and the product of the last terms correct, and then check the middle term by summing the "outside" and "inside" products.

EXAMPLE 1-5

Factor x^2-5x+6.

Get the first terms in place to produce x^2.	$(x \quad\quad)(x \quad\quad)$
Try the last terms to produce 6.	$(x \; - \; 1)(x \; - \; 6)$

This doesn't work, because the middle term would be
$-6x+(-1x)=-7x$.

Try other numbers that multiply to 6. $(x-2)(x-3)$

Note that the outside product plus the inside product is $-3x+(-2x)=-5x$.

Thus x^2-5x+6 factors into $(x-2)(x-3)$.

The Geometry of Calculus

There is a fairly common problem in most calculus courses that goes something like this: "A piece of steel 20 feet long and 6 feet wide is to be shaped into a long trough. It can be bent into an isosceles triangle, an isosceles trapezoid, or a semicircle. Which shape will produce the trough of greatest volume?" You don't need to solve this problem right now, but let's look at how geometry is involved in problems such as this.

In all possible cases, the trough will be in the shape of a prism. The volume of a prism is found by multiplying the area of its base times its length. The length is simply 20 feet, but to find the area of the base, you will need to know area formulas for all three possible shapes: triangle, semicircle, and trapezoid. You'll also need to know the angle at which you might bend the sides to form the triangle or the trapezoid. This could even introduce some trigonometry into the problem. There are several places where geometry concepts surface in calculus, and a quick survey of those concepts will prepare you to apply them when the need arises.

Area and Volume Formulas

The most common geometry formulas you will need to know involve area. Throughout calculus, shapes such as rectangles, triangles, trapezoids, and circles make frequent appearances. You'll also need to recall some volume formulas, especially the formulas for cylinders, washer-shaped objects, and spheres. You probably won't encounter more obscure geometry formulas very often, but you can look them up as needed. Common area and volume formulas are listed in Appendix A.

RULE

The most famous formula from geometry should always be at your fingertips for any calculus application. The Pythagorean Theorem expresses the relationship between the sides of a right triangle. It tells us that the sum of the squares of the two legs equals the square of the hypotenuse, the side across from the right angle: $(\text{Leg})^2 + (\text{Leg})^2 = (\text{Hypotenuse})^2$.

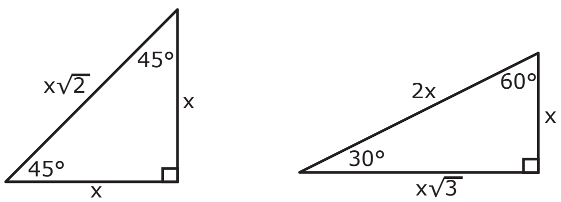
Tangents and Secants

In geometry, tangents and secants are associated primarily with circles. A *secant* is any line or segment that cuts across a circle, intersecting it at two points. A *tangent* line intersects a circle at only one point. In calculus, the context of these two terms is expanded and applied to graphs of functions. A line can be tangent to the graph of a function at a given point, but then intersect that function again at another point. Often, the context of the situation will be dealing with what is happening over a very local domain.

Special Right Triangles

Two special right triangles that you learned about in geometry should be reviewed. An isosceles right triangle has both legs congruent, which results in the angle measures being $45°, 45°$, and $90°$. What is more important is that the sides have a ratio of x to x to $x\sqrt{2}$. The other special right triangle has angles that measure $30°$, $60°$, and $90°$. Its sides are in a ratio of $2x$ to x to $x\sqrt{3}$. Figure 1-4 shows the relationships visually. It is very helpful to be familiar with these two triangles when you work with trigonometric values of common angle or radian measures.

Figure 1-4



A Bit of Trigonometry

What is the exact area between the graph of $y = \sin(x)$ and the x-axis on the domain $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$? This is the kind of question that calculus can help you answer, but there are some basic trigonometry skills you will need while working on the task. Obviously, you will need to know what the graph of $y = \sin(x)$ looks like. You will also need to know some trigonometric values at the endpoints of the given interval. In general, most studies of trigonometry go into far more depth than is required for basic calculus. Of course, a calculus problem could take you to the outer limits of a trigonometry course, but for the most part, the trigonometric relationships that emerge throughout a calculus course are the simpler ones. What, then, should you remember?

Right Triangle Trigonometry

The basic definitions of the six trigonometric functions are expressed using a right triangle. The six functions are:

- Sine
- Cosine
- Tangent
- Cotangent
- Secant
- Cosecant

Each function has its own definition. For instance, the sine function for an angle is always the ratio of the length of the leg opposite that angle to the length of the hypotenuse. To describe all six would get rather wordy. Instead, study the triangle and the definitions in Figure 1-5 to review the basic trigonometric ratios.

Figure 1-5