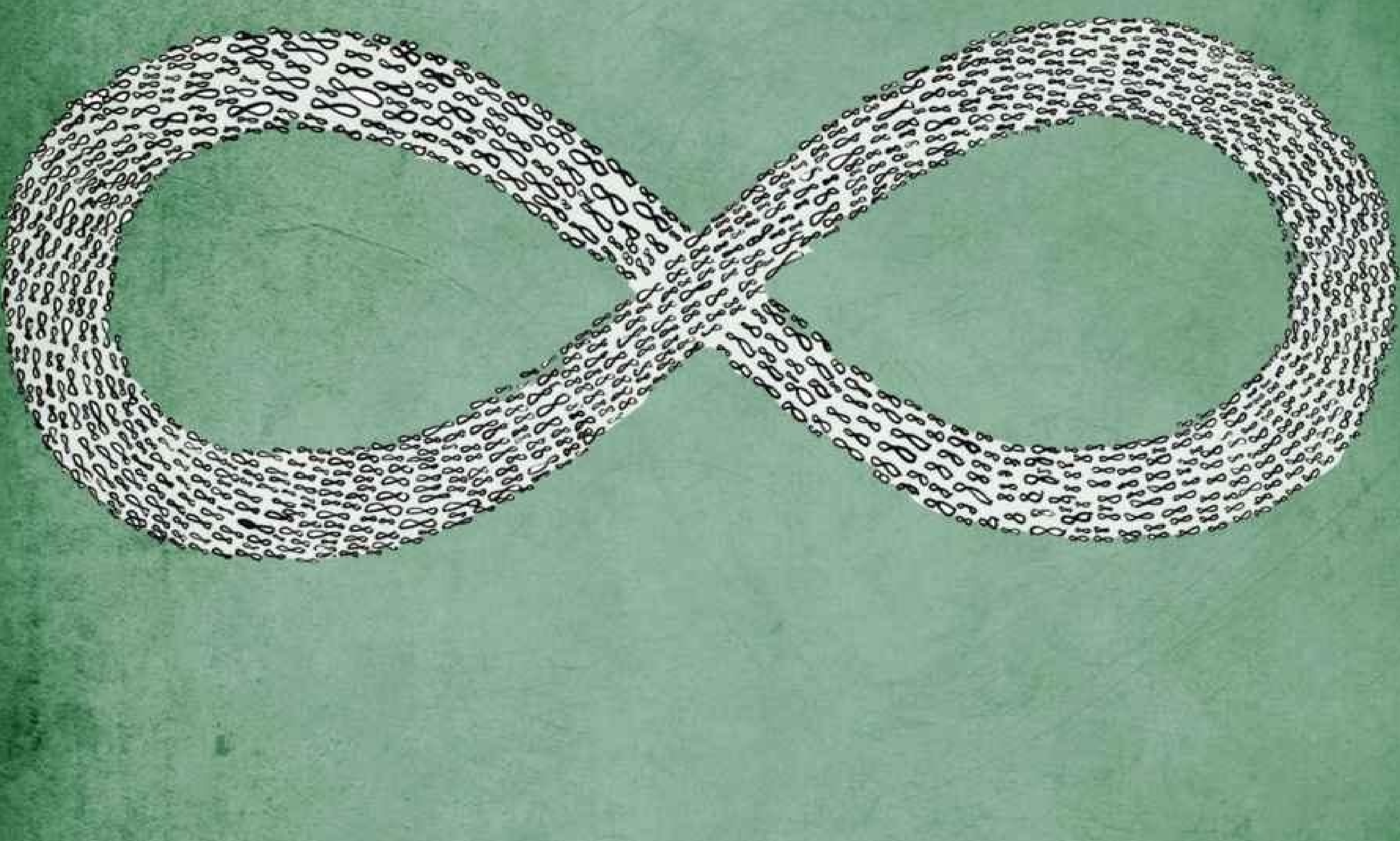
Sašo Dolenc

THE MAN WHO COUNTED INFINITY

and Other Short Stories from Science, History and Philosophy



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INTRODUCTION

The universe is made of stories, not atoms.

Muriel Rukeyser (*The speed of darknes*

Albert Einstein once said: “Everything should be made as simple as possible, but not simpler.” Th is the guiding spirit of the books in this series of “Short stories from science, history and philosophy The objective here is to explain science in a simple, attractive and fun form that is open to all.

The first axiom of this approach was set out as follows: “We believe in the magic of science. W hope to show you that science is not a secret art, accessible only to a dedicated few. It involv learning about nature and society, and aspects of our existence which affect us all, and which w should all therefore have the chance to understand. We shall interpret science for those who might n speak its language fluently, but want to understand its meaning. We don’t teach, we just tell storie about the beginnings of science, the natural phenomena and the underlying principles through whic they occur, and the lives of the people who discovered them.”

The aim of the writings collected in this series is to present some key scientific events, ideas a personalities in the form of short stories that are easy and fun to read. Scientific and philosophic concepts are explained in a way that anyone may understand. Each story may be read separately, but the same time they all band together to form a wide-ranging introduction to the history of science a areas of contemporary scientific research, as well as some of the recurring problems science h encountered in history and the philosophical dilemmas it raises today.

NUMBERS

For a moment, nothing happened. Then, after a second or so, nothing continued to happen.

Douglas Adams (*The Hitch Hikers Guide to the Galax*

A mathematical melodrama

On the 30th of May 1832, when the day had barely started to dawn in the southern suburbs of Pari a young man called Evariste Galois, still drowsy, but with a gun in his hand, stood facing an artiller officer named Pescheux d'Herbinville. A classic duel was about to take place, one where opponen use weapons to defend their honor and reputations. Today, it is not known precisely what the argumen was really about, but it seems likely that the main cause of dispute between the two men was th beautiful Stéphanie Félice Poterine du Motel.

From the distance of twenty-five feet the rivals aimed the barrels of their guns at each other an only a moment later the young Evariste fell to the ground, wounded. The bullet hit him in the stomac puncturing his intestines, and only an immediate operation might have saved his life. However, n doctor was at hand and those watching the duel as well as Evariste's opponent left the scene as soon a he fell. It was only three hours later that the injured Galois was noticed by a passer-by who manage to get him to a nearby hospital. At that particular moment Galois was still fully conscious, but he wa aware that his time was fast running out. When his younger brother came to see him, he uttered h last words: "Don't cry, Alfred! I'll need all my courage to die at twenty years of age." The nex morning he would not wake up.

Even before this fatal May morning, Galois was well aware that accepting the duel would almo certainly sentence him to death. He was a brilliant mathematician, but hardly as skilled in wieldin weapons as his opponent d'Herbinville, the soldier. And so he spent the night before the duel trying t jot down as many of the mathematical ideas that had occurred to him during his short working life, b which he had not had chance to record. We all know that we are usually much more productive whe haunted by a deadline than when we have plenty of time at our disposal. However, it is somethin quite different to be only twenty years old and, in your mind's eye, see the solutions to sever mathematical problems which you are certain nobody else knows, while a duel that you are bound lose is set to take place the very next morning.

That night, Galois filled several sheets of paper with equations and sent them to a good frien asking him to pass them on to the famous mathematicians Jacobi and Gauss. On these sheets he trie to gather all the ideas that he thought relevant. However, as the surviving notes imply, he found difficult to focus on mathematics alone. The edges of the pieces of paper contain the words "un femme" (a woman) and a cry of desperation: "Je n'ai pas le temps!" (I don't have time!).

Evariste Galois was undoubtedly a great mathematical genius, but his short life was not witho difficulties. Even though he came from a fairly wealthy family – his father was the mayor of a sma town near Paris – and received a good education, the trouble was that his teachers, at least when came to mathematics, could not measure up to his standards. At the early age of fifteen he was alread reading – in the original – through every possible paper published by his contemporaries. In order find a more appropriate study environment, he tried to enroll into the prestigious Ecole Polytechniqu but unfortunately failed to pass the entry exams. When he had already published his first scientif paper later he tried his luck at the Polytechnique again, but was no more successful in convincing th board of admissions than he was the first time.

He also sent his mathematical ideas to be evaluated by the French Academy of Science (*L'Académie des sciences*) which was at that time at the centre of scientific work in France and inde Europe. Galois' work was reviewed by the Baron Cauchy, one of the giants of the French school o mathematics, who praised the achievements of the young mathematician and suggested that he b given a special award from the Academy. Galois developed his ideas further and sent them to th

Academy’s secretary, but he died shortly after, and the young mathematician’s article was nowhere t be ~~found, so he regrettably never received the award even though everybody agreed that he was~~ a mo talented young man.

After Galois’ sudden death rumors started spreading that the unfortunate incident was not really th result of an honorable duel, but a murder, plotted against the young mathematician because he wa thought to be an ardent republican. It is true that he was often present in the first lines of protests an had been arrested on several occasions, but there is no proof to support the theory of premeditate murder. It was actually when serving a prison sentence because of his political beliefs that he mad the acquaintance of the fatal Stéphanie. She was the daughter of the doctor of the clinic to which th youngest prisoners were transferred during a cholera epidemic.

Galois’ friends and his brother tried to save Evariste’s notes after his death, to organize them a much as possible and to pass them on to renowned mathematicians so they could prepare them f publication. However, it took several decades before the world of mathematics came to understand th extraordinary significance of the ideas of this young mathematician.

Anyone who has gone to school in the past couple of decades will remember the unusu mathematical structure called the group. Such knowledge escapes from the head of the average stude right after he hands in his last math test, but we should be aware of the fact that modern physics wou be quite inexplicable without the use of the group theory. Groups are not only essential to describin the elementary particles which make up the universe; they are a fundamental mathematical tool use by experts from all fields of science. Groups as a basic mathematical structure, indeed one of th foundations of modern mathematics, came into existence on that very fatal night in 1832 whe Evariste Galois decided to scribble down equations from his head when he would have done better practice his shooting.

What is randomness?

You might think it must be easy to define randomness, but nothing could be further from the trut Not only is it difficult to create random events or sequences of numbers, verifying whether somethi that we have produced really is random is no easy task either. Many great mathematicians througho history have examined the problem of randomness, but it was only a short while ago, in the era computers and information technology, that the questions concerning randomness revealed themselv in all their complexity and appeal.

The paradox of the definition of randomness

It would be easiest to define randomness as a series of events taking place without any meaning independent of any possible rule. Random is what has neither cause nor meaning. Nevertheless, it very important not to confuse our subjective unawareness of rules with the objective nonexistence such laws. We can quite easily come to the conclusion that a certain sequence of numbers is rando when we cannot recognize any rule that might govern it, while it is likely that we just cannot make o the pattern. A good example of this is the number pi which represents the ratio of a circle circumference to its diameter. The definition of pi is very simple, but if we were to see nothing but th long list of decimals with the beginning 3.1415… hidden, it would be extremely difficult to figure o that it is, in fact, a sequence which is far from random.

One could even say that the definition of randomness is, in a way, a paradox. On the one hand, v say that a truly random sequence cannot conceal any rule that would enable us to recreate th sequence, while on the other hand, requiring the absence of all patterns within a sequence leads to very restricting definition which is almost impossible to apply in practice. For something to b random, it must meet very well defined conditions. Randomness is thus defined by the comple absence of form which is, on the other hand, a very strictly defined form in itself, only with a negativ sign.

The shortest possible instruction

In the mid-1960s, the mathematicians Andrej Kolmogorov, Gregory Chaitin and Ray Solomono all independently invented a way to effectively define randomness in the era of computers and th digital recording of information. They linked the definition of randomness to the concept algorithmic complexity. This sounds very complicated, but is actually based on a simple idea.

Andrej Kolmogorov, a renowned Russian mathematician, defines the complexity of a thing as th length of the shortest recipe (algorithm) required to make it. Cakes are usually more complex tha bread, because the instructions for making them are normally more extensive. Similarly, orange juic is less complex than beer, for example, as we can reduce the recipe for making the juice to no mo than two words: “Squeeze oranges”, while the instructions for making beer are much longer.

Following the same pattern, the complexity of a number is defined as the length of the most simp computer program that can write it out. The sequence 01010101010101010101 can be reduced to “1 times 01”. We notice immediately that the sequence can be memorized in a shorter and clearer wa When observing a sequence like 01000101000011101001, however, it is not possible to recognize th rule right away, so we have to memorize it as a whole.

If your phone number happened to be 01 1111-111, you would be able to memorize it straight away just as you would the number 01 2345-678. The rules hidden behind both numbers are simple and v need no more than a single piece of information to recall them. When it comes to more compl numbers, though, we have to commit several pieces of information to our memories. Sometimes a pa of ~~the number contains the date of our birth or a similar sequence that we can easily rec~~all, s memorizing such numbers presents a smaller difficulty than memorizing numbers that possess n apparent pattern. These are, in our eyes at least, completely random.

No recipe for randomness

We can use the concept of algorithmic complexity to define what is random as being a thing fo which there exists no shorter recipe than a detailed description of the thing itself. If we limit ourselv to number sequences, then the sequence which is random cannot be reduced to any shorter form algorithm than the entire actual list of numbers. As there is no rule to sum up the list of numbers, w can only memorize it in its entirety, like a phone number which does not contain a single set o numbers we can recognize.

A binary number, used in the language of computers that can only read ones and zeros, is rando when its complexity is equal to the number of its digits. A program that a computer reads in binar code as well cannot be shorter than the number itself. In simpler terms, there is no shorter recipe write down the number than writing it with all the digits it contains. There can be no other short algorithm or program.

All programs for creating random numbers built into our computers are actually only creating th so-called “pseudo-random numbers”. The algorithms that create them are shorter than the numbe they produce, so they do not really meet the strict criterion of randomness based on algorithm complexity. In an article in 1951, John Von Neumann, the famous mathematician, physicist and fathe of computer science, summarized this problem in a single sentence: “Anyone who conside arithmetical methods of producing random digits is, of course, in a state of sin.”

The “Monte Carlo” method

However, modern science could hardly progress without generators of (pseudo-) random number Today, random numbers are most frequently used to enable simulations or to help with more comple calculations. The method of using random numbers to solve mathematical problems is very similar public opinion polls. If we pose a certain question to a smaller sample of population and if the samp only contains randomly chosen representatives of a population that are not, for example, mostly seni citizens or students, their answers can give us a pretty good idea about the population’s opinio concerning a certain subject.

In science, we can deal with the problem of random numbers in a similar way. Suppose we had t calculate the surface of an irregular shape in the form of a heart. Using a procedure th mathematicians named the Monte Carlo method we can evaluate the surface of the heart b circumscribing it with a rectangle, the surface of which is not difficult to calculate. Now, all there left to do is to evaluate what part of the rectangle is covered by the heart and the problem is solve But what is the easiest way to assess the ratio of the surface of the entire rectangle to the area covere by the heart? Try randomly placing dots on the rectangle and counting whether you have hit the hea or not. If the dots are really distributed in a random manner, the ratio of the number of dots inside th entire rectangle to the number of dots on the heart will gradually approach the ratio of the rectangle surface to the heart’s surface.

Compressing data

Naturally, verifying whether a long sequence of numbers is random is no easy feat. Howeve mathematicians have developed several methods for testing different random number generators an checking if the random sequences are good enough to be used for a given task.

In our daily lives, we encounter the process of evaluating the degree of randomness whe compressing files on our computers which happens almost every day. As we all know, we can us special software devised to compress data and greatly reduce the size of a file on our computer. Thes file compressing programs look for repetitions within data and create new dictionaries to write dow the information in shorter form. Suppose we repeatedly used the word “problem” in our document. good file compressor will notice that and substitute all the occurrences of the word with a single sig like *. The sentence: “The problem lies in the problematical approaches to solving problems”, will b written in a coded and compressed form: “The * lies in the *atical approaches to solving *s”. And th dictionary will add the explanation that the asterisk (*) means “problem”.

The following rule applies: the more that the size of the file is reduced by compressing informatio the less random data is being compressed. When compressing a written document it is thus easy evaluate the size of our vocabulary. The more we repeat words in our text, the easier it will be for th file compressor to reduce the size of the file containing our document.

This is why file compressing programs are a relatively good judge of the randomness of a certa sequence. The more the data can be compressed, the less random it is. A truly random sequence cann be compressed by using file compressing programs because the entire computer algorithm is also th shortest possible algorithm describing this sequence. A file of a truly random sequence cannot b compressed.

When our sense of probability deceives us

The influence of numerous columns published in popular newspaper supplements where so-calle “experts” shower us with all kinds of advice is not to be underestimated. These columns do not on form people’s opinions and change habits of entire nations; every once in a while they can als provoke large-scale polemics involving wider audiences as well as more professional circles. In th area of health and nutrition such fervent reactions are to be expected, but it is a completely differe thing when they are brought about by a mathematical question.

Two goats and a car

Parade magazine, which is distributed as a supplement of more than four hundred America newspapers every Sunday and reaches around seventy million readers, has long featured a colum called “Ask Marilyn”. It is edited by Marilyn vos Savant who became famous in the 1980s when sh was listed in the Guinness Book of World Records for the highest IQ on the planet. In her column, sh has been answering various questions and solving problems posed by readers for more than twen years.

Among all the questions she has ever dealt with, a special place belongs to a seemingly very simp problem which was posed to her on September 9, 1990 by a Mr. Craig F. Whitaker:

> Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goa You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. H says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?[1]

The question became known as “the Monty Hall problem”, after the host of the popular America television show *Let’s Make a Deal* in which the host Monty Hall challenged his guests to accept refuse different offers he made. In her column, Marilyn answered her reader that it would definitely b wiser to switch doors because the probability to win the car would increase by two times. This was h answer:

> Yes; you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good w to visualize what happened. Suppose there are a million doors, and you pick door #1. Then the host, who knows what's behi the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pret fast, wouldn't you?[2]

Could the most intelligent woman in the world be wrong?

Of course, all of this would have gone unnoticed if it had not provoked a storm of critics aimed Mrs. vos Savant. *Parade magazine* received more than ten thousand letters sent by outraged reade among which a great many were teachers of mathematics. Nearly one thousand letters were signe with names followed by a PhD and numerous letters were written on sheets of paper bearing the titl of renowned American universities (many of these letters are available on Marilyn’s websit www.marilynvossavant.com). All the critics agreed that Marilyn was misleading readers with he answer, because the probability of winning could not be changed by choosing the other door. On professor of mathematics was very blunt: “You blew it, and you blew it big! … As a profession mathematician I am very concerned with the general public’s lack of mathematical skills. Please hel by confessing your error and in the future being more careful.” Another was so upset that he ended u calling Marilyn herself a goat.

The argument quickly spread across the limits of the Sunday magazine and even landed on th cover of The New York Times, and some well-known names from the world of mathematics joined th debate. This is how a reporter described the atmosphere created by vos Savant's explanation: "H ~~suggestion to switch the doors has been the subject of debate in the corridors of the CIA as well~~ as o Air Force bases by the Persian Gulf. It has been analyzed by mathematicians from the MIT ar computer programmers of the Los Alamos laboratories in New Mexico." Apart from the offensiv letters which denounced her response, Marilyn did receive some letters of approval. Among other one came from a professor at the prestigious MIT: "You are indeed correct. My colleagues at wor and I had a ball with this problem, and I dare say that most of them, including me at first, thought yo were wrong!"

Marilyn was not discouraged by the critics - after all, she was objectively proven to be smarter tha all her critics in terms of her IQ – and in one of her following columns she gave teachers across th country the task to play this simple game with their students in class (without real goats and car obviously) and send her their results. She later published these results and they were complete consistent with her advice that in the case in question, it was actually wiser to choose the other door.

Who's right?

The debate brought about by the Monty Hall problem falls into the category of wh mathematicians call conditional probability. Simply put, this is a branch of mathematics dealing wi the question of adapting the probability of an event taking place when new data enters the equatio The essence of the problem which provoked such a widespread and emotionally charged reaction fro readers was that most overlooked a crucial piece of information. It is very important to take in consideration the fact that the game-show host already knows which door hides the car.

In the second part of the challenge when the host opens the door which hides the goat, he alread knows that there is no car behind this door. If the host did not possess this knowledge and were t open the doors completely at random- as the contestant must– the probability of guessing correct really would not be increased by changing the decision. In this case, the more than one thousar readers who had concluded that Marilyn was not familiar with the basics of mathematics and se angry letters to the magazine would have been right. Our mathematical intuition suggests that th probability that the car is behind one or the other door when two of the doors are still closed remai equal. This would of course be true, if there were not for the host who knows more than we do.

This problem is most easily clarified, if we take a look at the action from behind the scenes whe we can see which door hides the car and which hides the goats. If the contestant guesses where the c is in his first try, the host will open any of the other two doors in which case it is unwise for th contestant to change his decision. However, this only applies if the contestant chooses the door whic hides the car in his first try for which the probability is 1:3 or approximately 33 %. But if in his fir try the contestant chooses the door which hides a goat, the host will have to open the only remainir door which hides the other goat. In this case the contestant would be wise to switch the doors becau that would certainly win him the car.

If the contestant chooses the goat in his first try it is always to his advantage to switch doors whi this does not apply if he chooses the car. The probability that he will choose the goat in his first try 2:3 while the probability that he will choose the car is only 1:3. If the contestant follows the tactic switching doors, the probability of winning the car is 2:3; if he refuses to switch doors the probabili decreases by half and is only 1:3.

If the contestant chooses to switch doors when the host offers him the chance, he will win the c every time when he chooses the goat in his first try. This choice will win him the car in twice as mar cases as it would if he decides not to switch. The probability of guessing correctly increases from 3 percent to 66 percent.

If you are having difficulties understanding this right now, do not worry. Many mathematiciar needed quite some time to clarify the issue.

~~Let us conclude with a similar puzzle that you can try to solve yourselves: a family wi~~th tw children has at least one boy. What is the probability that it also has a girl if we presume that average an equal number of boys and girls are born? Of course, the correct answer is not 50%.

The man who counted infinity

“There is a concept that is the corrupter and destroyer of all others. I speak not of Evil, who limited empire is that of ethics; I speak of the infinite.” Even though one might expect these words have been written down by a mathematician or a scientist, this is not so. Their author is an Argentin writer, Jorge Luis Borges, who succeeded in revealing the very essence of a problem that had intrigue many thinkers before him. Infinity is a concept that has appeared time after time in all kinds different philosophical, mathematical and physical discussions, but has always been clouded b difficulties and contradictions. People simply cannot grasp infinity directly as they do other concept We have to imagine infinity in an indirect way. Usually, we describe infinity as an endless or limitles sequence, but we quickly come upon problems, as with infinity we enter a dimension where o intuitions may not be trusted completely.

There is no single infinity

Throughout history, a great number of thinkers and scientists have pondered the concept of infini and reached many interesting and important conclusions, but the greatest development understanding this difficult idea occurred in the second half of the 19th century, when th mathematician Georg Cantor (1845-1918) approached the subject from a completely new angle.

With a couple of simple definitions Cantor succeeded in setting new boundaries to this comple area of thought, which allowed him to examine the concept of infinity thoroughly and systematicall To the great amazement of the scientific and philosophical community he demonstrated that there no single infinity, but, in fact, an infinite number of different infinities. He came to this conclusio after careful and accurate consideration and was also able to prove formally all the stages leading his conclusion His ideas subsequently became incorporated into the very core of modern mathematic

One of the first challenges he had to overcome was defining what infinity actually was. Of cours the simplest way to define infinity was to describe it as having no limits: infinity is what is larger tha anything finite. This is where Cantor made an important step forward: for he defined something infinite, if some of its parts are as big as the whole. This might sound contradictory as we are a intuitively used to finite dimensions whose parts are always smaller than the whole, but as we hav already mentioned, when it comes to questions of infinity, intuition does not offer the best answers.

How to compare infinities?

Not only did Cantor invent new ways of dealing with infinity, he also came up with the well-know set theory that children have been learning in elementary school for decades. When thinking about h new theory, he soon had to face the question of how to compare the size of two sets. He proceede from the completely intuitive supposition that two sets were of the same size if each element from th first set could be matched-up with exactly one element from the second set. Two groups of childre are of the same size if a child from one group can hold hands and pair up with a child from the oth group.

Cantor generalized the method of comparison by “holding hands” to infinite sets. Two infinite se are of equal size if we can match-up each element from one set with an element from the other se According to this definition, the set of even numbers is just as big as the set of all natural numbe since we can pair up each natural number with an even number simply by multiplying it by two. O holds hands with two, two holds hands with four, three holds hands with six and so on all the way infinity. As we have matched up each element from the first set with an element from the second se both of the sets, according to our definition, are of the same size, even though we could also say th the ~~entire list of natural numbers is twice as large as the list of all even numbers, because~~ natur numbers are made up of both even and odd numbers.

It was with the help of this strange notion that two sets were of the same size even though one twice as large as the other that Cantor defined the infinite set. An infinite set contains a subset of th same size, when size is defined according to the principle of holding hands. Natural, even and od numbers thus represent infinite sets.

How to count points on a line?

The question that immediately arises is whether all infinite sets are of the same size. For exampl is a set of all points on a line as big as the set of natural numbers? In other words: is it possible count all the dots on a line? One of Cantor's important achievements is the proof that it is impossib to count all the dots on a line. Let us have a look at his proof, also called the diagonal argument.

Suppose we are counting points on a line which is one unit long. Each point is matched up with number between zero and one. As points fill out all the line because there can be no space betwee them, otherwise we would not have a line, but a sequence of dots, all of the points on a line can on be matched up with numbers if we use numbers with an infinite decimal representation, called re numbers. Every point on a line which is one unit long can be matched up with a real number betwe zero and one.

Now imagine we wrote down all of the real numbers one under the other in an endlessly long list o which every line contains one real number and the lines are marked with natural numbers. Cant proved that we can always find at least one real number missing from every such list of numbers. Ho come? We simply take away the first decimal from the first real number on the list and change it, the take the second decimal from the second number and change it … If we continued to do so until w came to the end of the list, we would create a new real number different from any number alread written on the list.

Coming back to our analogy with the two groups of children that we have compared in size b matching the children up in pairs, we have now proven that pairing up natural numbers with points o a line or real numbers never works. We can always find one real number or point on the line left th has no pair.

Infinities come in infinite numbers

The diagonal argument can also be used to prove that the set of all subsets of a given set is alway larger than the set itself. It is the easiest to do so with natural numbers. We write down the subsets o natural numbers as lists of on and off numbers in the set of natural numbers. Odd numbers, f example, are written as {1,0,1,0,1,0 …}, and prime numbers as {1,1,1,0,1,0,1 …}. Next we arran these sets into a long list and apply the same argument as before to show that we can always constru another subset of natural numbers that has not been on the list before, simply by taking one eleme from each subset and changing its value. The set of all subsets of an infinite set is larger than the s itself. Infinite sets come in infinite numbers.

However, a problem already occurs with the smallest infinite sets. We know that natural numbe are the smallest infinite set. It still remains unclear though which is the next largest infinite set. Is the set of real numbers or points on a line? Or is there another infinity in between, bigger than natur numbers and smaller than real numbers? Cantor presumed that such a set does not exist, but he w unable to prove it. Many years later, mathematicians solved this question not by finding the answe but by showing that the question had no answer at all.

Many books about Cantor's efforts to solve problems concerning infinity also mention his illne which caused him to spend the last years of his life in a mental hospital. Today, he would have bee diagnosed with bipolar disorder or manic depression, but at the time most people suffering from th illness were simply labeled insane. Many writers have implied it was actually his work on infinity th drove Cantor over the edge of sanity. This might sound like a reasonable theory, but it is unlikely th his illness and his research were directly related.

The hermit of the Pyrenees

In August of 1991, Alexander Grothendieck, generally regarded as one of the most importa mathematicians of the 20th century and whose influence is often compared to that of Albert Einstei suddenly left his home in the south of France and headed for the Pyrenees. Since then, he has be living as a hermit high in the mountains somewhere between France and Spain, completely cut o from civilization. In the mid-nineties, a few mathematicians managed to reach him at his home in th wilderness, but for the last couple of years he has remained unseen. His mail is still piling up at th University of Montpellier, but he explicitly prohibited even the handful of his friends who, at th beginning, knew where in the mountains he lived, to bring it to him. Today, even his closest relativ are not completely certain if he is still alive.

Even before his departure into the deep wilderness, Grothendieck lived a very secluded, ascetic li in an old house with no electricity in a village near Montpellier in France. After a successf mathematical career in the 1950s and '60s, when he was also one of the principal members of th infamous Bourbaki group, he became increasingly interested in ecological and anti-war politic movements in the 1970s. He became so involved with the struggle for social justice that he traveled Vietnam in protest, participated in numerous demonstrations and even went took his ideals so far as refuse a national research scholarship, in order to avoid even tacitly supporting a system of nation politics he utterly opposed.

Merely to begin understanding Grothendieck's complete devotion first to mathematics and the politics and ecology, one must look back to his childhood. His father Sasha was a convinced anarchi and participated in several rebellions in imperial Russia at the beginning of the 20th century. In 192 he moved from Russia to Berlin where he moved in radical circles and met his future partner, Hank who came from a wealthy bourgeois family, but associated with members of avant-garde movement They had little Alexander on March 28, 1928. At the time, the young family also supported Hanka daughter Maidi from her first marriage.

In 1933, when Nazis came to power, Alexander's father Sasha fled Berlin for Paris and was soo followed by Alexander's mother, but she did not take her son and daughter with her. She place Alexander into foster care with a family that lived near Hamburg, and left her daughter in institution for handicapped children, even though she was a perfectly healthy child. Alexander live with his foster family from his fifth to his eleventh year. He rarely received letters from his moth and never even heard from his father nor from his other relatives who lived in nearby Hambur Naturally, this period of separation from his parents left a deep mark on young Alexander.

In 1939, the political pressure became too great and the foster family could no longer take care all the children. The problem with Alexander was that he looked distinctly Jewish, an appearance wi ominous implications not only for him but also the rest of the family. So his relatives found Hank with the help of the French consulate, sat little Alexander on a train and sent him from Hamburg Paris. Both of his parents had spent some of the years away from their son in Spain, where they foug against Franco. On his return to France his father was arrested as a "dangerous foreigner" by th French authorities of the time and sent to an internment camp. He died a few years later in Auschwit

Hanka in Alexander spent the war in different internment camps, but as soon as the war was over h enrolled at university to study mathematics. He was not impressed with his teachers, so he most studied on his own. Before his twentieth birthday, much like young Einstein, he independently mad several important mathematical discoveries, unaware that they had already been made and publishe by other mathematicians.

When he moved to Paris he started to spend time with the most prominent French mathematicia of ~~the time and joined the Bourbaki circle of which he quickly became a driving force.~~ He w becoming more and more famous for his highly abstract approach to solving mathematical problem His friends later claimed that he was unable to think about concrete things, because his mind on functioned on a universal level.

After a long and productive collaboration with the Bourbaki, he left the group in protest, becau most of the members refused to accept his suggestion to replace the old set theory for formalizi mathematics with the new more general category theory which he had helped create. The set theo was constrained by a number of paradoxes and so it became too narrowly oriented to be appropria for describing the entire diverse scope of modern mathematics. The mathematician Pierre Cartier, o of the more important members of the Bourbaki group summed up the essence of the problem: “S theory is too restrictive: an element is either a member of a set or not, there is nothing in between.”

The decision of the Bourbaki to refuse Grothendieck’s suggestion to move away from the set theor to the category theory was, as it soon turned out, a big mistake. Category theory became a ve important area in mathematics in the years to follow, and Grothendieck received many awards for h achievements, among others the Fields Medal, also known as the “Nobel Prize of Mathematics”.

Is marriage a mathematical operation?

One autumn Sunday in 1934 the director of the prestigious Parisian school *École Norma Supérieure* (ENS) called up a young philosopher called Claude Lévi-Strauss and asked him if he mig be interested in applying for the position of Professor of Sociology at the University of Sao Paolo Brazil. Lévi-Strauss, who had not yet reached his thirties, accepted the offer to leave for this farawa place mainly because he wished to distance himself mentally from the European intellectual scene. H thought it to be too involved in dealing with abstract problems and was more interested in immersi himself in anthropological field work.

A World on the Wane

Lévi-Strauss later remembered that the director of the ENS was, at the time he offered him the jo in Sao Paolo, certain that natives of the country's interior were already to be found in the suburbs o the Brazilian metropolis, and that the philosopher, interested in the relatively new discipline o gathering information about peoples of different cultures and their customs in the field, could stu their culture during weekends.

Naturally, there were no natives near the university where Lévi-Strauss spent the following coup of years lecturing. On several occasions, however, he did set off for more remote places in th rainforest where he could learn about the customs and way of life of the Indian tribes. Even though h spent a lot of time in the field and is today considered one of the principal names in twentieth-centu anthropology and philosophy, he was neither a classical field anthropologist nor a classic philosopher who almost never leaves his desk.

In fact, he was interested in solid data describing the customs of specific cultures and wanted to u the information he gathered to reveal the universal structure, characteristic not only of a certain trib but of all human societies. He later described his expeditions to the Amazon region in detail in h book entitled *A World on the Wane*, which combines his autobiographical notes with an analysis of th lives of Indian tribes.

Are you the guy who makes jeans?

On his return from Brazil he quickly realized that the situation in Europe was even worse than wh he had left for South America. The air was thick with the anticipation and anxiety as the Secon World War drew near, and because he was of Jewish origins, he only thought it wise to leave Franc once again. He decided to accept an invitation from the renowned private institution, *The New Scho for Social Research* in New York, which offered him tenure.

Immediately after his arrival to the US, his acquaintances kindly suggested that he change his nam as soon as possible. If he refused to do so, anyone who met him would think that he sold jeans. Eve though it all seemed to be a joke at first, he quickly came to realize that the similarity of his name that of the famous jeans manufacturer was more of a nuisance than an entertaining coincidence. order to avoid unwanted misunderstandings he decided to sign his name Claude L. Strauss during h stay in the US. However, this change was not effective enough to save him from receiving orders f the legendary blue jeans to his address every once in a while.

The French in Manhattan

In New York, Lévi-Strauss made the acquaintance of the Russian linguist Roman Jakobson who als lectured at The New School. They quickly discovered they had very similar approaches to scienc

Lévi-Strauss tried to introduce the method that Jakobson had been developing in the field ~~linguistics to anthropology. In order to become familiar with each other's past research they~~ als frequented each other's lectures.

Many other French intellectuals spent the wartime years in New York. During the week, Lév Strauss spent his time with Jakobson, immersed in problems concerning anthropology and linguistic while at weekends he joined the painter Max Ernst and the writer André Breton to look for Nativ American art in the local markets.

The Elementary Structures of Kinship

Jakobson was very impressed with Lévi-Strauss's doctoral thesis so he encouraged him to publish in book form as well. In his thesis, Lévi-Strauss examined the structures of family relations different cultures around the world. Each culture has its own specific rules determining who can mar whom, and determines which newly created family bonds are strictly forbidden.

The Elementary Structures of Kinship, as the book was called when it was finally released after th Second World War, is still considered to be one of the most important works in the field o anthropology. It also had a great influence on other fields of science.

When Lévi-Strauss was preparing his manuscript for publishing in New York, however, he realize that he was still missing the essential part. He had plenty of data which he had gathered in the fiel but he wanted to arrange it into a coherent whole, and in this he hit an obstacle. While using the san approaches that Jakobson had been applying to linguistics, he failed to find the inner logic within h field data on the acceptable and prohibited ways of creating new family relations.

As he knew that finding a structure within data was also a mathematical problem, he turned to h mathematician colleagues, but they were not of much help. One of the representatives of the old generation even advised him to stop looking for principles because he would not find an "Mathematics only knows four operations and marriage isn't one of them."

A mathematician comes to the rescue

After a while, he finally found some good fortune. In New York, he met a young Frenc mathematician, André Weil, who also gave lectures at one of the American universities. Weil was mathematician of a new breed who did not have much in common with his elderly colleagues. He w one of the founders of a group of young French mathematicians who published their scientif discussions and university textbooks under the collective pseudonym Nicolas Bourbaki.

Their basic principle was to set mathematics on a new footing, mostly with the aid of set theory. I fact, The Bourbaki group was responsible for the famous new mathematics reform, moving the focu from arithmetic to sets and operations involving sets.

In his research, Lévi-Strauss gathered a large quantity of data on family relations in differe cultures of the world. He found out, for example, that similarities exist even between family structur of groups living as far apart as the Indian tribes of Brazil and the Aborigines of Australia. However, l could not succeed in revealing the general structure or system of all the acceptable newly forme family bonds.

"When in doubt, look for groups!"

After Lévi-Strauss explained his problem to him, André Weil immediately suspected that structure that could organize the mass of anthropological data into a coherent whole could be mathematical group. This completely abstract algebraic structure, which was not as well known in th mathematical community then as it is today, turned out to be one that can also be found governing th most unlikely situations, such as marriages between Australian natives.

Weil was well known for his motto: “When in doubt, look for the group!” And this abstrac alg~~ebraic approach was indeed useful when applied to anthropology. The essential principle tha~~t We applied to Lévi-Strauss’s data was that he ignored the very elements among which he was supposed t seek out regularities. He did not focus on the types of marriages, but on the relations betwee weddings. The structure was not hidden in the marriages as such, but in the differences between typ of marriages. This approach was in complete accordance with the ideas of the Bourbaki group th relations and structures are the central elements of mathematics.

Weil found out that the structure of relations between the marriages of the members of differe generations and tribes was defined by what he called a permutation group. He described his findings a discussion which was published as an appendix to Lévi-Strauss’s influential work. More than concrete analysis of family relations, this book is important because of its influence, as it introduc the notion of structure as a system of differences, causing a veritable intellectual revolution.

The interdisciplinary cooperation of Lévi-Strauss, Jakobson and Weil in New York in 1943 becam a thing of legend. Three leading experts on anthropology, linguistics and mathematics, each brillia in their own field, so different at first glance that it is hard to imagine how they could talk abo anything other than the weather, came together to achieve an amazing scientific breakthroug According to science historians, this was when the structuralist movement, which influenced all soci sciences, mathematics and philosophy in the second half of the 20th century, came to life.

A mathematical intrigue at the Swedish court

Monetary prizes for solving complex scientific and technical problems are not awarded as often they used to be. Today, tenders where a committee of experts offers prizes for the best creations se to its address by a certain date are a common way of finding solutions in the fields of architecture ar other artistic/technical disciplines, but not so much in science. Every once in a while an organizatio or an individual still announces that a prize will be given to anyone who can solve some seeming insoluble problem, but these awards are not really significant in terms of maintaining or increasing th financing of scientific work. In the past, however, this used to be quite different.

A little more than a hundred years ago, the Swedish mathematician Gösta Mittag-Leffler persuade King Oscar II of Sweden to organize a mathematics competition in honor of his sixtieth birthday. A the king was a student of mathematics himself, he took to the idea quickly, especially because Mittag Leffler had succeeded in convincing two highly distinguished European mathematicians to take part the jury that would set the tasks and evaluate the solutions. The invitation was accepted by Ka Weierstrass of Berlin and Charles Hermite of Paris.

Is the solar system stable?

Four tasks were given, all relating to problems that were also principal subjects of mathematic research at the time. Even though the king's birthday was not until 1889, preparations for the awa already started in 1884, so that scholars from across Europe would have enough time to examine th problems thoroughly and come up with their solutions. In the middle of 1885 *Nature* magazine printe an announcement inviting scientists to participate in a mathematical competition in honor of Kin Oscar II's sixtieth birthday. The deadline for submitting the solutions to the Swedish court and th president of the jury Gösta Mittag-Leffler was June 1, 1888. Naturally, a precondition of entry wa that all submissions be made anonymously, to guarantee unbiased evaluation of the work. That is wh all candidates were required to enclose with their solutions a sealed envelope containing their names.

The first of the four questions which were given in formal mathematical language pertained to th problem of the movement of three bodies. In other words, the jury was interested in the question of th stability of our solar system. Today, more than a hundred years after the event took place, wh happened a couple of months after the announcement of the winner is more interesting than the initi question and its answers. When the winning solution was already being prepared for publication th editor of the magazine noticed that the jury had rewarded a work which was not completely flawles One of the mistakes actually turned out to be of key importance.

The winner was no surprise

The winner of the competition was none other than the eminent French mathematician Hen Poincaré who, despite his young age at the time, already enjoyed a very high reputation. In the peri between the announcement of the competition and the deadline for submissions he was even elected member of the French Academy of Sciences, a great honor for a man of only thirty-two.

The competition was not about winning the money. The awarded sum came nowhere close to that today's Nobel Prize. The winner received 2500 Swedish kronor which was approximately one third a professor's yearly salary. The award improved the young mathematician's scientific care significantly and enabled him to secure a good position at a renowned university, but it certainly d not make him rich. As we shall see shortly, the prizewinning Poincaré actually lost more than h gained from the award.