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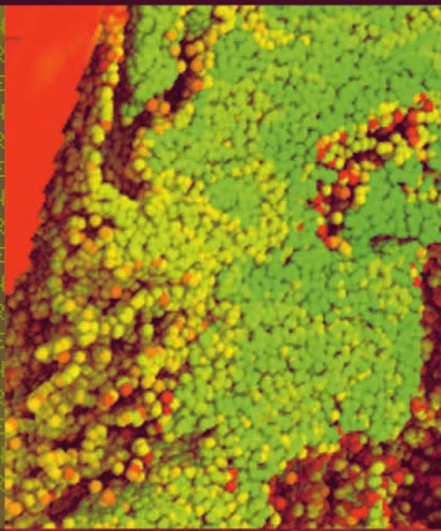

Alwyn C. Scott

THE NONLINEAR UNIVERSE

Chaos, Emergence, Life

Springer

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THE NONLINEAR UNIVERSE

Chaos, Emergence, Life

With 86 Figures

Springer

Alwyn C. Scott†
Tucson, USA

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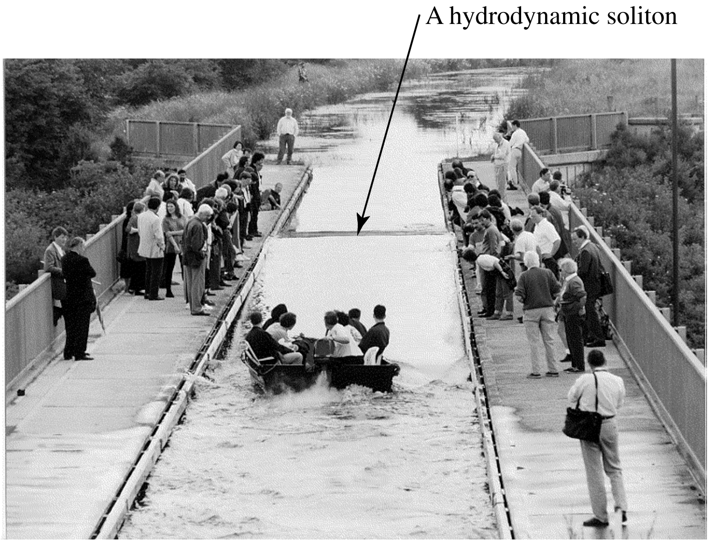

Frontispiece: Recreation of Scott Russell's soliton on Scotland's Union Canal, 12 July 1995

Preface

It has been suggested that the big questions of science are answered – that science has entered a "twilight age" where all the important knowledge is known and only the details need mopping up. And yet, the unprecedented progress in science and technology in the twentieth century has raised questions that weren't conceived of a century ago. This book argues that, far from being nearly complete, the story of science has many more chapters, yet unwritten. With the perspective of the century's advance, it's as if we have climbed a mountain and can see just how much broader the story is.

Instead of asking how an apple falls from a tree, as Isaac Newton did in the 17th century, we can now ask: What is the fundamental nature of an apple (matter)? How does an apple (biological organism) form and grow? Whence came the breeze that blew it loose (meteorology)? What in a physical sense (synaptic firings) was the idea that Newton had, and how did it form?

A new approach to science that can answer such questions has sprung up in the past 30 years. This approach – known as *nonlinear science* – is more than a new field. Put simply, it is the recognition that throughout nature, the whole is greater than the sum of the parts. Unexpected things happen. Minute causes can explode into mighty effects. Metaphorically, a butterfly flaps its wings in the Gobi Desert, causing a tornado in Texas. Nonlinear science provides the tools to study these phenomena. It is a metascience, a tree trunk that supports and governs the organization of almost every other branch of inquiry. Like Copernicus putting the Sun at the center of the Solar System, nonlinear science is a revolution. And just as Newton's work offered a basis for scientific discoveries in the three centuries that followed, so nonlinear science will support research in the 21st century and beyond.

Yet for all of its usefulness, nonlinear science is not widely known. The public still thinks Newton's laws, and others of a similar nature, are sufficient to explain what causes a plane to crash or a cancer to grow. But this *reductionism*, as will be shown, is inadequate to deal with the more intricate questions of the 21st century.

As a physical scientist who graduated from the university in the middle of the twentieth century, I have been engaged in studies of nonlinear science over the past four decades. Such studies include applications to computers, living creatures, weather prediction, oceanography, planetary motions, the brain

– in fact, all fields of science from physics and chemistry to the biological and social sciences. Over the years I have become convinced that this new perspective is essential for every scientist working today.

Why? Consider this example. It's commonly assumed that the Earth orbits the Sun in a precisely regular way. Of course, we only know Earth's present position within a certain range of measurement error. Our intuition tells us that this small error stays small over time, implying that Earth's position was about the same in the past. However, nonlinear science shows that our intuition is wrong. Massive computations possible only within the past decade prove that the error in our knowledge of the Earth's distance from the Sun doubles about every four million years. This error is trivial over the few millennia of recorded human history, but over geological eons, it compounds into a giant uncertainty. This means *we don't know* where the Earth was in relation to the Sun half a billion years ago, when multicelled life began. Most people are unaware of this startling fact. Do scientists studying climate and evolution take it into account? Mostly no. Because of the parochial nature of the many branches of science, nonlinear science is often ignored even by those working at the cutting edges of their fields.

My aim in writing this book is to show general readers and academics how nonlinear science is applied to research, both in physical and biological sciences. Some of the outstanding questions have practical applications and are being studied. Can we observe gravitational waves and understand their dynamical nature? Is there – as Einstein believed – a nonlinear theory that incorporates quantum theory and explains elementary particles? Is Einstein's geometrical theory of gravity related to quantum theory, as string theorists believe? How did living organisms manage to emerge from the lifeless molecules of the hot chemical soup during Earth's Hadean eon, four billion years ago? To what new vistas are Internet developments in the storage, transmission and manipulation of information leading us? How does the human brain work? Where is Life headed?

Other problems are more philosophical, pushing the envelope of what is possible to know. Are predictions of all future events from present knowledge possible "in principle" – as reductionist science continues to believe – or are important aspects of dynamics *chaotic*, precluding this possibility? What is the relationship between quantum theory and chaos? Where does the above-noted "butterfly effect" leave the concept of causality? Indeed, what do we *mean* by causality? What is the fundamental nature of *emergence*, when qualitatively new entities come into being? Is it possible for there to be something "new under the sun"? Does emergence lead to new *things* or mere *epiphenomena*, which can be explained in other ways? How is the phenomenon of biological evolution to be viewed? Are chaos and emergence related? Can we comprehend Life?

Far from esoteric, all of these questions can be addressed by nonlinear science. In describing such intricate phenomena as planetary motion and the state of the weather, nonlinear science can't always give a precise answer, but it can tell us whether a precise answer exists. To paraphrase the old saying, nonlinear science gives us the insight to accept the things we cannot calculate, the ability to calculate the things we can and the wisdom to know the difference.

Written for general readers who would understand science and for university undergraduates who would become researchers in or teachers of science, the book begins with descriptions of the three fundamental facets of nonlinear science: chaos; the emergence of independent entities in energy-conserving systems; and the quite different emergence of independent entities in dissipative (nonconservative) systems. Like the legs of a milk stool, these three facets are interrelated, with more general systems, like rungs, linking them. Chapters 6 and 7 then serve as backbones, presenting applications to the physical and biological sciences.

The book concludes with one of the most pressing questions in modern science: the debate over the Newtonian notion that all effects can be reduced to simple causes (the whole is equal to the sum of its parts). Most scientists believe this, because they hold that everything we experience is based on physical matter. Although I am committed to physicalism, I show that there are many phenomena – including Life, Mind and Spirit – that cannot be described merely by the actions of atoms, molecules, genes, synapses, memes, or whatever. Thus from the mountain top of nonlinear science, we see that reductionism is invalid, and that we can study the many fascinating questions it fails to address.

Tucson, Arizona,
December 2006

Alwyn Scott

Alwyn C. Scott (1931–2007)

My earliest memories of my father are of his intense desire to understand the world. It permeated everything he did, read, wrote, and said. Life for Alwyn Scott was a thing to be passionately explored, and life's meaning a thing to be pursued with every waking moment. Time with my father was spent building plastic models of molecules, carrying out amateur and sometimes messy chemistry experiments with household products, and investigating the capillary attraction of the candle on our kitchen table. The greatest lesson he taught me was to approach everything with curiosity and wonder.

This hunger for understanding was apparent in Alwyn Scott from an early age. As a boy, he was fascinated by water waves and built his own Ham radio. As a doctoral candidate at MIT, he studied shock waves and nerve impulses, which eventually led him to dive into the vast study of consciousness. His contributions to nonlinear science made him an undisputed pioneer of the field. Yet with admirable humility, he always expressed an acute awareness of the complexity that confronted him in his search for answers to the questions of the universe. During our many after dinner discussions on science and philosophy, he was fond of quoting Socrates, saying "The only thing I know is that I know nothing." For these words to come from someone with such a reservoir of knowledge, not only about science, but art, politics, history, philosophy, and culture, was humbling and inspiring. To have such a man as my mentor has been invaluable.

I know that I am not alone in having been touched by Alwyn Scott's overwhelming intellectual drive. In his seventy-five years, he profoundly affected both the scientific and philosophical world views of those he worked with. It was impossible to walk into a room where Alwyn Scott was making a point without stopping and taking notice; his was clearly a mind to be reckoned with.

Completed shortly before his diagnosis of lung cancer in 2006, *The Nonlinear Universe: Chaos, Emergence, Life* embodies the same spirit of voracious curiosity that drove my father's life. In this, his final work, he once again strives to understand the dazzling complexity of the world around us, a world so many of us take for granted. Those of us who mourn the loss of such a great thinker and dear human being can find solace in the fact that his life's work and his power to inspire will live on in his writing.

New York City
August 2007

Lela Scott MacNeil

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1 Introduction

I just want to know what Truth is!

Thomas Kuhn

In addition to providing us with the above epigraph [516],[1] Thomas Kuhn wrote a book on the history and philosophy of science entitled *The Structure of Scientific Revolutions* [515], which has sold over a million copies and remains in print after four and a half decades. Upon first reading this book in the early 1960s, I did not realize that the program of research that I was then embarking upon – theoretical and experimental studies of nonlinear wave motion – would soon become part of a Kuhnian revolution.

In *Structure* (as he called his book), Kuhn famously proposed that the history of science comprises two qualitatively different types of activity. First are the eras of "normal science", during which the widely-accepted models of collective understanding (he called them paradigms) are agreed upon, and the primary activity is "puzzle solving" by adepts of those paradigms [515].[2] Occasionally, these eras of normal science are "punctuated" by "revolutionary periods" of varying magnitude and theoretical importance, during which the previously accepted paradigms change rapidly, leading the scientific community to see the world in new ways. According to Kuhn, such rapid changes in scientific perspective occur in a "Gestalt-like manner", qualitatively similar to the psychological changes of perception where one sees a familiar image (like the Necker cube) in a new way, and also with the rapid development of a new species in the "punctuated evolution" of NeoDarwinism [625]. After a collective switch of perceptions, the "lexicon" that scientists use to describe reality changes – new words are coined for new concepts, and old words are assigned new meanings, often with implications for the wider culture.

An example of this phenomenon that was studied in detail by Kuhn in the 1950s is the Copernican Revolution [514], during which the European percep-

[1] See the reference section for these citations.

[2] The third edition of Kuhn's classic book is particularly valuable as it includes a final chapter responding to critics of the first edition.

tion of planetary motions changed from the geocentric (Ptolemaic) picture to the heliocentric view proposed in 1543 by Copernicus in his *De Revolutionibus Orbium Caelestium*. Although the heliocentric idea had been suggested by Aristarchus of Samos in the third century BCE and seems evident to us now – familiar as we are with the amazing photographs that space probes have sent back from remote corners of the solar system – heliocentrism was not obvious to astronomers of the sixteenth century. In addition to the fact that Earth does not *seem* to move, Ptolemy's formulation of Aristotle's cosmology predicted all observations of celestial motions recorded over many centuries by Chinese, Greek, Islamic and European astronomers, and a geocentric Universe is in accord with the Christian myths that had recently been so vividly described by Dante Alighieri in his classic *La Divina Commedia*.

The basic structure of the Ptolemaic Universe consisted of two primary spheres, the inner one being the stationary surface of the earth and the outer (stellar) sphere carrying the stars around us, once every day. Within the stellar sphere were seven lesser spheres – the *orbium* of Copernicus's title – inhabited by the seven moving bodies: Saturn, Jupiter, Mars, Sun, Mercury, Venus, and Moon, all of which were driven in their motions by the daily rotation of the outer (stellar) sphere. Beyond the stellar sphere there was nothing, as the Ptolemaic Universe was finite.

Because a daily rotation of the stars about a fixed Earth is the simplest way to think about some calculations, the geocentric formulation is still used to teach celestial navigation, and although we modern scientists snicker at the absurd notions of astrology – which claims that motions of the heavenly bodies can influence terrestrial events – this concept is not unreasonable in the context of Ptolemaic astronomy. Aristotle supposed that the motion of the stars causes Saturn's motion, which in turn causes Jupiter's motion, and so on, suggesting that some terrestrial phenomena could be partially influenced by the motions of the stars and planets, and in accord with this view, it is well established that the ocean tides are governed by motions of the Sun and Moon. That we now find such difficulty seeing the Universe through Ptolemaic eyes shows not how dimwitted our ancestors were but how markedly our collective view of reality has changed. And as the polemics over Kuhn's work have shown, it is difficult for some modern thinkers to accept that Ptolemaic truth seemed as valid as ours in its day [515].

Why then did Copernicus propose a heliocentric formulation of astronomy? He had long been concerned with a "problem of the planets" – when Mercury, Venus, Mars, Jupiter and Saturn appear brightest, their motions through the heavens cease and reverse directions for a while before resuming their more regular westward paths. Although the Ptolemaic astronomers could describe and predict these retrograde motions through a well-defined system of deferents, epicycles, ecliptics and equants, there is an ad hoc character of their explanations that can be avoided by assuming Earth to be a planet lying between Venus and Mars, rotating on its polar axis each day, and revolving around the Sun. Although the Ptolemaics had a credible explanation for this retrograde motion, Copernicus did not believe that they had the *correct* explanation.

Within a century of Copernicus's death, Johannes Kepler used the careful observations of his colleague Tycho Brahe – the best pre-telescopic data then available – to show that planets, including Earth, can be more simply and accurately assumed to follow elliptical orbits about the Sun. Furthermore, he observed that a line joining a planet to the Sun sweeps out equal areas in equal times, a result that became known as Kepler's law. Galileo Galilei's application of the newly-invented telescope to celestial observations then revealed the moons of Jupiter and the phases of Venus, adding further data in support of the heliocentric theory.

In addition to the religious implications of Earth no longer being located at the center of the Universe, the Copernican revolution also altered the scientific concept of motion. Aristotle's fundamental picture was that moving objects are impelled to move toward or away from the center of the Universe, whereas the corresponding Galilean picture was of isolated massive bodies ideally moving in an infinite Universe with constant speed along straight lines. Galileo assumed that such uniform motion would continue until and unless a mass is acted upon by gravity, impact, friction, or other mechanical forces. Thus the stage was set for Isaac Newton – born just a century after the death of Copernicus and within a year of Galileo's death – to propose a self-consistent dynamical model of the Universe in his *Principia Mathematica* (entitled to complement René Descartes' *Principia Philosophiae*). Being from Britain, Newton was truly standing "on the shoulders of giants" [407], as the work leading to his *Principia* comprised the efforts of an impressive international group, involving essential contributions from Greece (Aristotle and Ptolemy), Poland (Copernicus), Denmark (Brahe), Germany (Kepler), Italy (Galileo), and France (Descartes) – all member states of modern Europe.

My central claim in this book is that the concepts of nonlinear science comprise a Kuhnian revolution which will have profound implications for scientific research in the present century. As we shall see, research in nonlinear science underwent significant changes over the last three decades of the twentieth century, particularly during the 1970s. Before this decade, important ideas lay undiscovered or were not widely noted, and communications among researchers doing mathematically related work in different fields of science varied from poor to nonexistent. Nowadays, these conditions have changed dramatically. Several international conferences on nonlinear science are held every year, mixing participants from a variety of professional backgrounds to a degree that was not imagined in the 1960s. Nonlinear science centers have spread across the globe, bringing together diversely educated young researchers to collaborate on interdisciplinary activities, combining their skills in unexpected ways. Dozens of nonlinear science journals have been launched, and hosts of textbooks and monographs are now available for introductory and advanced courses in nonlinear science.

In addition to its many interesting and important applications in the physical sciences and technology, we shall see that nonlinear science offers new perspectives on biology and answers a deep question that arose in the context of Newton's mechanistic model of the Universe: What is the nature of Life?

1.1 What Is Nonlinear Science?

When asked this question at a cocktail party, I often paraphrase Aristotle, saying that nonlinear science is the study of those dynamic phenomena for which the whole differs from the sum of its parts [31] – or just claim it is the science of Life. In other words, particular effects cannot be assigned to particular causal components (as is so for linear systems) because all components interact with each other. If she does not disappear to refresh her drink, I proceed by pointing to dynamic phenomena in virtually every area of modern research that are currently being investigated under the aegis of nonlinear science, including the following:

- **Chaos.** Sensitive dependence on initial conditions or the *butterfly effect*, strange attractors, Julia and Mandelbrot sets, problematic aspects of weather prediction, executive toys, electronic circuits.
- **Turbulence.** Wakes of ships, aircraft and bullets; waterfalls; clear air turbulence; breaking waves; fibrillation dynamics of heart muscle.
- **Emergent Structures.** Chemical molecules, planets, tornadoes, rogue waves, tsunamis, lynch mobs, optical solitons, black holes, flocks of birds and schools of fish, cities, Jupiter's Great Red Spot, nerve impulses.
- **Filamentation.** Rivers, bolts of lightning, woodland paths, optical filaments, rain dripping down window panes.
- **Threshold Phenomena.** An electric wall switch, the trigger of a pistol, electronic flip-flop circuits, tipping points, the all-or-nothing behavior of a neuron.
- **Spontaneous Pattern Formation.** Fairy rings of mushrooms, the Gulf Stream, ecological domains, biological morphogenesis.
- **Phase Changes.** Freezing and boiling of liquids, the onset of superconductivity in low temperature metals, superfluidity in liquid helium, magnetization in ferromagnetic materials, polarization in ferroelectric materials.
- **Harmonic Generation.** Digital tuning of radio receivers, conversion of laser light from red to blue, symphonic music and overdriven amplifiers for rock bands.
- **Synchronization.** Coupling of pendulum clocks, mutual entrainment of electric power generators connected to a common grid, circadian rhythms, hibernation of bears, coordinated flashing of Asian fireflies.

- **Shock Waves.** Sonic booms of jet airplanes, the sound of a cannon, bow waves of a boat, sudden pile-ups in smoothly-flowing automobile traffic.
- **Hierarchical Systems.** Stock markets, the World Wide Web, economies, cities, living organisms, human cultures.
- **Psychological Phenomena.** Gestalt perceptions, anger, depression, startle reflex, love, hate, ideation.
- **Social Phenomena.** Lynch mobs, war hysteria, emergence of cultural patterns, development of natural languages.

All of these phenomena and more comprise the subject matter of nonlinear science, which is in some sense a metascience with roots reaching into widely diverse areas of modern research.[3]

In the United States, the first use of the term "nonlinear science" may have been in a 1977 letter written by Joseph Ford to his colleagues, which defined our subject and is included here as the epigraph to Chap. 5 [877].[4] This letter was historically important as it introduced Ford's *Nonlinear Science Abstracts*, an ambitious project that soon evolved into *Physica D: Nonlinear Phenomena* – the first journal devoted to nonlinear science. Since the middle of the twentieth century, of course, the adjective "nonlinear" has been employed to modify such nouns as: analysis, dynamics, mechanics, oscillations, problems, research, systems, theory, and waves – particularly in the Soviet Union [92, 746] – but Ford defined a broad and cohesive field of interrelated activities; thus it is his sense of the term "nonlinear science" that is used in this book.

A yet deeper characterization of nonlinear science recognizes that the definition of nonlinearity involves assumptions about the nature of causality. Interestingly, the concept of causality was carefully discussed by Aristotle some twenty-three centuries ago in his *Physics*, where it is asserted that [29]:

> We have to consider in how many senses *because* may answer the question *why*.

As a "rough classification of the causal determinants of things", Aristotle went on to suggest four types of cause [136].

- **Material Cause.** Material cause stems from the presence of some physical substance that is needed for a particular outcome. Following Aristotle's suggestion that bronze is an essential factor in the making of a bronze

[3]A fairly complete listing and description of such applications can be found in the recently published *Encyclopedia of Nonlinear Science*, which aims to make the facets of the field available to students at the undergraduate level [878].

[4]Joseph Ford (1927–1995) was both fun to be around and an inspiration to many in the early years of research in nonlinear science. Ever striving to understand the philosophical implications of chaos, Joe was often at odds with the physics community, but without his research and his encouragement of others, the revolution described in this chapter would have had an even more difficult birth.

statue, many other examples come to mind: atoms of iron are necessary to produce hemoglobin, obesity in the United States is materially caused by our overproduction of corn, water is essential for Life. At a particular level of description, a material cause may be considered as a time or space average over dynamic variables at lower levels of description, entering as a slowly varying *parameter* at a higher level of interest.
- **Formal Cause.** For some particular outcome to occur, the requisite materials must be arranged in an appropriate form. The blueprints of a house are necessary for its construction, the DNA sequence of a gene is required for synthesis of the corresponding protein, and a pianist needs the score to play a concerto. At a particular level of description, formal causes might arise from the more slowly varying values of dynamic variables at higher levels, which then enter as *boundary conditions* at the level of interest.
- **Efficient Cause.** For something to happen, there must be an "*agent* that produces the effect and starts the material on its way". Thus, a golf ball moves through the air along a certain trajectory because it was struck at a particular instant of time by the head of a properly swung club. Similarly, a radio wave is launched in response to the alternating current that is forced to flow through an antenna. Following Galileo, this is the limited sense in which physical scientists now use the term causality [136]; thus an efficient cause is usually represented by a *stimulation–response* relationship, which can be formulated as a differential equation with a *dependent variable* that responds to a *forcing term*.
- **Final Cause.** Events may come about because they are desired by some intentional organism. Thus a house is built – involving the assembly of materials, reading of plans, sawing of wood, and pounding of nails – because someone wishes to have shelter from the elements, and economic transactions are motivated by future expectations [448]. Purposive answers to the question "why?" seem problematic in the biological sciences, and they emerge as central issues in the social sciences because such phenomena don't conform to a general belief in reductionism [873,876]. As we will see in Chap. 8, final causes offer additional means for closed causal loops of dynamic activity which must be included in realistic models, leading to a class of physical systems that cannot be simulated [817].

In more modern (if not more precise) terms, Aristotle's material and formal causes are sometimes grouped together as *distal causes*, his efficient cause is called a *proximal cause*, and his final cause is either disparaged as a *teleological cause* or disregarded altogether. The present-day disdain of many scientists for final causes is a serious oversight, as an event often transpires because some living organism wills it so. The ignoring of such phenomena is rooted in Newtonian reductionism, which we will consider in the closing chapter of this book.

While these classifications may seem tidy, reality is more intricate, as Aristotle was aware [29]. Thus causes may be difficult to sort out in particular cases, with several of them often "coalescing as joint factors in the production of a single effect". Such interactions among the components of complex causes are a characteristic property of nonlinear phenomena, where distinctions among Aristotle's "joint factors" are not always easy to make. There is, for example, a subtle difference between formal and efficient causes that appears in the metaphor for Norbert Wiener's *cybernetics*: the steering mechanism of a ship [1036]. If the wheel is connected directly to the rudder (via cables), then the forces exerted by the helmsman's arms are the efficient cause of the ship's executing a change of direction. For larger vessels, however, control is established through a servomechanism in which changing the position of the wheel merely resets a pointer that indicates the desired position of the rudder. The forces that move the rudder are generated by a feedback control system (or servomechanism) that minimizes the difference between the actual and desired positions of the rudder. In this case, one might say that the position of the pointer is a formal cause of the ship's turning, with the servomotor of the control system being the efficient cause. And of course the overall direction of the ship is determined as a final cause by the intentions of the captain and his navigator.

Another example of the difference between formal and efficient causes is provided by the conditions needed to fire the neurons in our brains. If the synaptic weights and threshold are supposed to be constants, they can be viewed as formal causes of a firing event. On a longer time scale associated with learning, however, these parameters change; thus they can be considered collectively as a *weight vector* that is governed by a learning process and might be classified as efficient causes of neuron ignition [874]. Although the switchings of real neurons are far more intricate than this simple picture suggests, the point remains valid – neural activity is a nonlinear dynamic process, melding many causal factors into the overall outcome.

Finally, when a particular protein molecule is constructed within a living cell, sufficient quantities of appropriate amino acids must be available to the messenger RNA as material causes. The DNA code, determining which amino acids are to be arranged in what order, is a formal cause, and the chemical (electrostatic and valence) forces acting among the constituent atoms are efficient causes. Thus in the realms of the chemical and biological sciences, it is not surprising to find several different types of causes involved in a single nonlinear event – parameter values, boundary conditions, forcing functions, and intentions combining to influence the outcome of a particular dynamics. Can these ideas be extended to social phenomena?

Just as supercooled water, resting quietly in its fluid state, may experience the onset of a phase change during which it suddenly turns to ice, collective social phenomena can unexpectedly arise, sweeping away previous assumptions and introducing new perspectives. Examples of such "social phase changes" abound – the revolutions in eighteenth-century France and twentieth-century Russia, lynch mobs, the outbreak of war, England's collective heartbreak over the untimely death of Princess Diana, and the Copernican revolution [514], among many others. In the 1970s, I claim, something similar happened in the organization and practice of nonlinear science.

Although research in nonlinear dynamics goes back at least to Isaac Newton's successful treatment of the two-body problem of planetary motion, such activities were until recently scattered among various professional areas, with little awareness of the common mathematical and physical principles involved. Beginning around 1970, this situation changed. Those interested in nonlinear problems became increasingly aware that dynamic concepts first observed and understood in one field (population biology, for example, or flame-front propagation or nonlinear optics or planetary motion) could be useful in others (such as chemical dynamics or neuroscience or plasma stability or weather prediction). Thus research activities began to be driven more by an interest in generic types of nonlinear phenomena than by specific applications, and the concept of nonlinear science began to emerge.[5] Apart from particular applications, we shall see, there are three broad classes of nonlinear problems.

- **Low-Dimensional Chaos.** As discussed in the following chapter, an important discovery of nonlinear science is that one cannot – not even "in principle" – predict the behaviors of certain very simple dynamical systems. Due to a phenomenon now popularly known as the butterfly effect, systems with as few as three dependent variables can exhibit "sensitive dependence on initial conditions". Errors in such systems grow exponentially with time, which renders predictions of future behaviors mathematically impossible beyond a certain characteristic (Lyapunov) time.
- **Solitons.** In energy-conserving nonlinear fields, it is often observed that energy draws itself together into localized "lumps", becoming particle-like entities (new "things") that remain organized in the subsequent course of the dynamics. For an example see the frontispiece, where a hydrodynamic soliton has been generated on a Scottish canal by suddenly stopping a motorboat, whereupon a soliton emerges from the bow wave. Similar examples arise in optics, acoustics, electromagnetics, and theories of elementary particles, among other nonlinear dynamical systems.
- **Reaction-Diffusion Waves.** Since the middle of the nineteenth century, it has been known that localized waves of activity travel along nerve fibers, carrying signals along motor nerves to our muscles and from one neuron to another within our brains. As nerve fibers do not conserve energy, their dynamics are characterized by an interplay between the release of stored electrostatic energy and its consumption through dissipative processes

[5]This development has recently been noted by Rowena Ball in her introduction to *Nonlinear Dynamics: From Lasers to Butterflies* [50], where she points out that studies in nonlinear science are now often driven by new ideas generated within the field rather than merely responding to national needs like research on cancer, plasma confinement, or weapons technology.

(circulating ionic currents). To grasp this phenomenon, think of a candle where chemical energy is stored in the unburned wax and released by a moving flame at the same rate that it is dissipated by radiation of heat and light. Thus a candle models the nonlinear processes on nerve fibers, with the flame corresponding to a nerve impulse – exemplifying a second general type of emergence, distinctly different from that of energy conserving systems.

These three types of nonlinear phenomena – low-dimensional chaos, solitons, and reaction-diffusion fronts – are of central concern in this book. "Chaos" is a familiar word of Greek origin, describing, perhaps correctly, the original character of the Universe, but it is now also used in a new sense to imply "low-dimensional chaos" in nonlinear science. The term "soliton", on the other hand, was coined in 1965 by Norman Zabusky and Martin Kruskal to indicate the particle-like properties of the solitary-wave solution of energy-conserving wave systems [1074].[6] Describing processes in which energy (or some other essential quantity) is released by the ongoing dynamics, the adjective "reaction-diffusion" is widely but not universally used; thus such phenomena are also referred to as self-excited waves or self-organizing waves. Following a coinage by Rem Khokhlov in 1974, they are also called autowaves in the Russian literature [678].

1.2 An Explosion of Activity

Although the roots of these three components of modern nonlinear science go back at least to the nineteenth century, the frequency with which they appeared in scientific publications began to grow explosively 1970, as Fig. 1.1 shows. More precisely, the curves indicate an exponential rise (or Gestalt switching to use Kuhn's metaphor) from 1970 to 1990, with a doubling time of about three years. This was followed by an apparent leveling off (or saturation) around the beginning of the present century at a rate of more than 3000 papers per year, or about eight per day – evidently a lower estimate because some nonlinear-science papers don't use the terms "chaos", "soliton" or "reaction-diffusion" in their titles or abstracts. From the perspectives of other manifestations, these curves look much like the heat emitted from a freshly lit bonfire, the onset of applause in a theater, or the initial growth of a biological population.

How are we to understand these data? What causes the early rise of the curves? Why do they saturate? How do they get started? More generally, can

[6] This new term took a couple of decades to work its way into English dictionaries, and it was not uncommon for manuscripts being published in the 1970s to have "soliton" replaced with "solution" by overly zealous copy-editors of physics journals. The word entered the public mind from a *Star Trek* episode of the early 1990s, and to my great relief it is now in the official Scrabble dictionary.

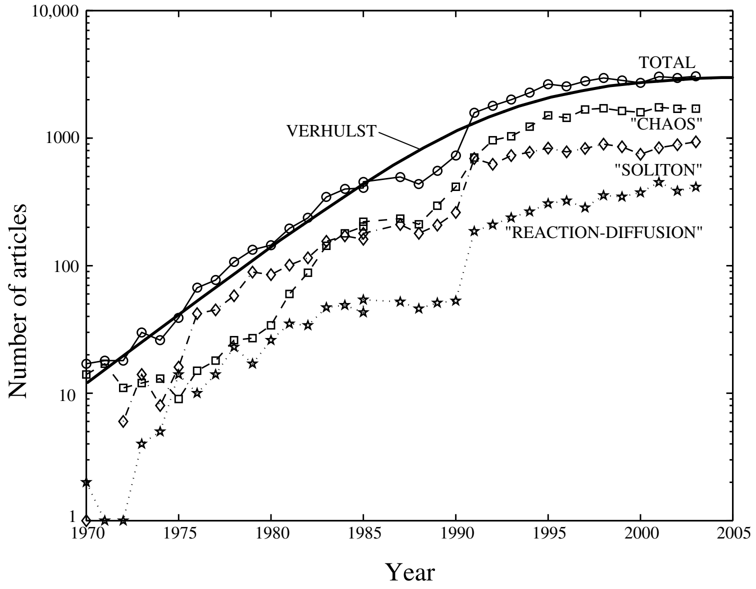

Fig. 1.1. The annual number of articles in scientific publications that have used the term CHAOS, SOLITON, and REACTION-DIFFUSION in their titles, abstracts, or key words, and the TOTAL of these three plots. (Data from the Science Citation Index Expanded.) The VERHULST curve is calculated from (1.1) to approximate the TOTAL curve with the parameters given in Table 1.1. (Note that the values of the CHAOS plot are misleadingly high before about 1975, as authors then used the term in its classical meaning)

the tools of nonlinear science help us understand a social phenomenon: the way that modern nonlinear science emerged and grew?

Before introducing a brief mathematical representation of the data in Fig. 1.1, let's consider qualitative descriptions of the above examples. To get the dynamics started, some *threshold* must be overcome: a lighted match for the bonfire, one person's burst of enthusiasm in the theater, and the presence of at least one male and one female of a biological species. Once triggered, these processes begin to grow as a result of *positive feedback* around *closed causal loops*. Thus the initial burning of the bonfire releases more heat which burns more fuel which releases even more heat, and so on, in an endless loop of causality. Similarly, the initial applause in a theater induces more enthusiasm, which elicits more applause, which induces yet more enthusiasm, etc. In the example of population growth, the early reproduction rate is proportional to the population size, which again leads to a positive feedback loop and growth at an exponentially increasing rate. Other examples, albeit with very different time scales, are the explosive increase of neutrons inside an atomic bomb and the growth of dandelions on a poorly tended lawn.