

Accounting Demystified
Advanced Calculus Demystified
Advanced Physics Demystified
Advanced Statistics Demystified
Algebra Demystified
Alternative Energy Demystified
Anatomy Demystified
Astronomy Demystified
Audio Demystified
Biology Demystified
Biophysics Demystified
Biotechnology Demystified
Business Calculus Demystified
Business Math Demystified
Business Statistics Demystified
C++ Demystified
Calculus Demystified
Chemistry Demystified
Chinese Demystified
Circuit Analysis Demystified
College Algebra Demystified
Corporate Finance Demystified
Data Structures Demystified
Databases Demystified
Differential Equations Demystified
Digital Electronics Demystified
Earth Science Demystified
Electricity Demystified
Electronics Demystified
Engineering Statistics Demystified
Environmental Science Demystified
Ethics Demystified
Everyday Math Demystified
Financial Planning Demystified
Forensics Demystified

French Demystified

Genetics Demystified

Geometry Demystified

German Demystified

Home Networking Demystified

Investing Demystified

Italian Demystified

Japanese Demystified

Java Demystified

JavaScript Demystified

Lean Six Sigma Demystified

Linear Algebra Demystified

Logic Demystified

Macroeconomics Demystified

Management Accounting Demystified

Math Proofs Demystified

Math Word Problems Demystified

MATLAB[®] Demystified

Medical Billing and Coding Demystified

Medical Terminology Demystified

Meteorology Demystified

Microbiology Demystified

Microeconomics Demystified

Minitab Demystified

Nanotechnology Demystified

Nurse Management Demystified

OOP Demystified

Options Demystified

Organic Chemistry Demystified

Personal Computing Demystified

Pharmacology Demystified

Philosophy Demystified

Physics Demystified

Physiology Demystified

Pre-Algebra Demystified

Precalculus Demystified

Probability Demystified

Project Management Demystified

Psychology Demystified

Quality Management Demystified

Quantum Mechanics Demystified

Real Estate Math Demystified

Relativity Demystified

Robotics Demystified

Signals and Systems Demystified

Six Sigma Demystified

Spanish Demystified

Statics and Dynamics Demystified

Statistical Process Control Demystified

Statistics Demystified

Technical Analysis Demystified

Technical Math Demystified

Trigonometry Demystified



Trigonometry

DeMYSTiFieD®

Stan Gibilisco

Second Edition



New York Chicago San Francisco Lisbon London Madrid Mexico City
Milan New Delhi San Juan Seoul Singapore Sydney Toronto

Copyright © 2012, 2003 by The McGraw-Hill Companies, Inc. All rights reserved. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of the publisher.

ISBN: 978-0-07-178025-4

MHID: 0-07-178025-4

The material in this eBook also appears in the print version of this title: ISBN: 978-0-07-178024-7, MHID: 0-07-178024-6.

All trademarks are trademarks of their respective owners. Rather than put a trademark symbol after every occurrence of a trademarked name, we use names in an editorial fashion only, and to the benefit of the trademark owner, with no intention of infringement of the trademark. Where such designations appear in this book, they have been printed with initial caps.

McGraw-Hill eBooks are available at special quantity discounts to use as premiums and sales promotions, or for use in corporate training programs. To contact a representative please e-mail us at bulksales@mcgraw-hill.com.

McGraw-Hill, the McGraw-Hill Publishing logo, Demystified, and related trade dress are trademarks or registered trademarks of The McGraw-Hill Companies and/or its affiliates in the United States and other countries and may not be used without written permission. All other trademarks are the property of their respective owners. The McGraw-Hill Companies is not associated with any product or vendor mentioned in this book.

Information contained in this work has been obtained by The McGraw-Hill Companies, Inc. (“McGraw-Hill”) from sources believed to be reliable. However, neither McGraw-Hill nor its authors guarantee the accuracy or completeness of any information published herein, and neither McGraw-Hill nor its authors shall be responsible for any errors, omissions, or damages arising out of use of this information. This work is published with the understanding that McGraw-Hill and its authors are supplying information but are not attempting to render engineering or other professional services. If such services are required, the assistance of an appropriate professional should be sought.

TERMS OF USE

This is a copyrighted work and The McGraw-Hill Companies, Inc. (“McGraw-Hill”) and its licensors reserve all rights in and to the work. Use of this work is subject to these terms. Except as permitted under the Copyright Act of 1976 and the right to store and retrieve one copy of the work, you may not decompile, disassemble, reverse engineer, reproduce, modify, create derivative works based upon, transmit, distribute, disseminate, sell, publish or sublicense the work or any part of it without McGraw-Hill’s prior consent. You may use the work for your own noncommercial and personal use; any other use of the work is strictly prohibited. Your right to use the work may be terminated if you fail to comply with these terms.

THE WORK IS PROVIDED “AS IS.” MCGRAW-HILL AND ITS LICENSORS MAKE NO GUARANTEES OR WARRANTIES AS TO THE ACCURACY, ADEQUACY OR COMPLETENESS OF OR RESULTS TO BE OBTAINED FROM USING THE WORK, INCLUDING ANY INFORMATION THAT CAN BE ACCESSED THROUGH THE WORK VIA HYPERLINK OR OTHERWISE, AND EXPRESSLY DISCLAIM ANY WARRANTY, EXPRESS OR IMPLIED,

INCLUDING BUT NOT LIMITED TO IMPLIED WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. McGraw-Hill and its licensors do not warrant or guarantee that the functions contained in the work will meet your requirements or that its operation will be uninterrupted or error free. Neither McGraw-Hill nor its licensors shall be liable to you or anyone else for any inaccuracy, error or omission, regardless of cause, in the work or for any damages resulting therefrom. McGraw-Hill has no responsibility for the content of any information accessed through the work. Under no circumstances shall McGraw-Hill and/or its licensors be liable for any indirect, incidental, special, punitive, consequential or similar damages that result from the use of or inability to use the work, even if any of them has been advised of the possibility of such damages. This limitation of liability shall apply to any claim or cause whatsoever whether such claim or cause arises in contract, tort or otherwise.

About the Author

Stan Gibilisco, an electronics engineer, researcher, and mathematician, has authored multiple titles for the McGraw-Hill *Demystified* and *Know-It-All* series, along with numerous other technical books and dozens of magazine articles. His work has been published in several languages.

Contents

Introduction

Part I Origins and Theory

CHAPTER 1 Angles, Distances, and Triangles

Angles and Distances

Triangles

Trigonometric Triangles

Quiz

CHAPTER 2 Cartesian Coordinates

How It's Assembled

Radial Distance of a Point from the Origin

Distance between Two Points

Finding the Midpoint

Quiz

CHAPTER 3 The Unit-Circle Paradigm

The Unit Circle

Primary Circular Functions

Secondary Circular Functions

Quiz

CHAPTER 4 Mappings, Relations, Functions, and Inverses

What's a Mapping?

Types of Mappings

Examples of Relations

Examples of Functions

Inverses of Circular Functions

Graphs of Inverse Circular Functions

Quiz

CHAPTER 5 Hyperbolic Functions

The "Hyper Six"

Hyperbolic Function Graphs

Hyperbolic Inverses

Hyperbolic Inverse Graphs

Quiz

CHAPTER 6 Polar Coordinates

The Variables

Three Basic Graphs

Coordinate Transformations

The Navigator's Way

Quiz

CHAPTER 7 Three-Space and Vectors

Spatial Coordinates

Vectors in the Cartesian Plane

Vectors in the MPC Plane

Vectors in Three Dimensions

Quiz

Test: Part I

Part II Extensions and Applications

CHAPTER 8 Scientific Notation

Formatting Methods

Rules for Use

Approximation, Error, and Precedence

Significant Figures

Quiz

CHAPTER 9 Surveying, Navigation, and Astronomy

Terrestrial Distance Measurement

Interstellar Distance Measurement

Direction Finding and Radiolocation

Quiz

CHAPTER 10 Electrical Waves and Phase

Alternating Current

Phase Angle

Inductive Reactance

Capacitive Reactance

Quiz

CHAPTER 11 Geometrical Optics

Reflection

Refraction

CHAPTER 12 Trigonometry on a Sphere

The Global Grid

Arcs and Triangles

Global Navigation

Quiz

CHAPTER 13 The Infinite-Series Paradigm

Repeated Addition

Repeated Multiplication

Limits

Expansion of the Sine

Expansion of the Cosine

Expansion of the Tangent

Quiz

Test: Part II

Final Exam

Answers to Quizzes, Tests, and Final Exam

Appendix: Circular and Hyperbolic Identities

Suggested Additional Reading

Index

Introduction

This book can help you learn the principles of trigonometry without taking a formal course. It can also serve as a supplemental text in a classroom, tutored, or home-schooling environment. None of the mathematics goes beyond the high-school-senior level (12th grade). If you need a “refresher,” you can select from several *Demystified* books dedicated to mathematics topics. In particular, I recommend *Geometry Demystified* as a prerequisite for this course. If you want to build a “rock-solid” mathematics foundation before you start this course, you can go through *Pre-Algebra Demystified*, *Algebra Demystified*, *Geometry Demystified*, and *Algebra Know-It-All*. If you want to extend your knowledge beyond the scope of this course, I recommend *Pre-Calculus Know-It-All* and *Calculus Know-It-All*.

How to Use This Book

This book contains abundant multiple-choice questions written in standardized test format. You’ll find an “open-book” quiz at the end of every chapter. You may (and should) refer to the chapter texts when taking these quizzes. Write down your answers, and then give your list of answers to a friend. Have your friend reveal your score, but not which questions you missed. The correct answer choices appear in the back of the book. Stick with a chapter until you get most of the quiz answers correct.

Two major parts constitute this course. Each part ends with a multiple-choice test. Take these tests when you’re done with the respective parts and have taken all the chapter quizzes. Don’t look back at the text when taking the part tests. They’re easier than the chapter-ending quizzes, and they don’t require you to memorize trivial things. A satisfactory score is three-quarters correct. The correct answer choices appear in the back of the book.

The course concludes with a 100-question final exam. Take it when you’ve finished all the parts, all the part tests, and all of the chapter quizzes. A satisfactory score is at least 75 percent correct answers. Again, the correct answer choices are listed in the back of the book.

With the part tests and the final exam, as with the quizzes, have a friend divulge your score without letting you know which questions you missed. That way, you won’t subconsciously memorize the answers. You might want to take each test, and the final exam, two or three times. When you get a score that makes you happy, you can (and should) check to see where your strengths and weaknesses lie.

I’ve posted explanations for the chapter-quiz answers (but not for the part-test or final-exam answers) on the Internet. As we all know, Internet particulars change; but if you conduct a phrase search on “Stan Gibilisco,” you’ll get my Web site as one of the first hits. You’ll find a link to the explanations on that site. As of this writing, it’s www.sciencewriter.net.

Strive to complete one chapter of this book every 10 days or two weeks. Don’t rush, but don’t go too slowly either. Proceed at a steady pace and keep it up. That way, you’ll complete the course in a few months. (As much as we all wish otherwise, nothing can substitute for “good study habits.”) When you’ve finished up, you can use this book as a permanent reference.

I welcome your ideas and suggestions for future editions.

Stan Gibilisco

Part I
Origins and Theory

chapter 1

Angles, Distances, and Triangles

The word *trigonometry* comes from the Sanskrit expression “triangle measurement.” In its simplest form, trigonometry involves the relationships between the sides and angles of triangles where one of the angles measures 90° (a *right angle*). But, as you’ll learn as you go through this book, trigonometry involves a lot more than triangles! Before we get into “real trigonometry,” let’s review the basic principles and jargon for angles, distances, and triangles on *Euclidean* (flat) surfaces.

CHAPTER OBJECTIVES

In this chapter, you will

- Quantify angles in radians and degrees.
- Learn how lines, angles, and distances relate on flat surfaces.
- Classify triangles according to their interior angles.
- Define the trigonometric functions as ratios among lengths of the sides of a right triangle.
- Learn the Pythagorean theorem for right triangles.

Angles and Distances

When two lines intersect, we get four distinct angles at the point of intersection. In most cases, we’ll find that two of the angles are “sharp” and two are “dull.” If all four of the angles happen to be identical, then they all constitute right angles, and we say that the lines run *perpendicular*, *orthogonal*, or *normal* to each other at the intersection point. We can also define an angle using three points connected by two line segments. In that case, the angle appears between the line segments.

Naming Angles

The italic, lowercase Greek letter *theta* has become popular as an “angle name.” It looks like an italic numeral *0* with a horizontal line through it (θ). When writing about two different angles, a second Greek letter can go along with θ . The italic, lowercase letter *phi* works well for this purpose. It looks like an italic lowercase English letter *o* with a forward slash through it (ϕ).

Sometimes, mathematicians use the italic, lowercase Greek letters *alpha*, *beta*, and *gamma* to represent angles. We write these symbols as α , β , and γ , respectively. When things get messy and we have a lot of angles to write about, we can add numeric subscripts to italic Greek letters and denote our angles as symbols such as θ_1 , θ_2 , θ_3 , and so on, or α_1 , α_2 , α_3 , and so on.

TIP *If you don’t like Greek symbols, you can represent angle variables with more familiar characters such as x , y , and z . As long as you know the context and stay consistent in a given situation, you can call an angle anything you want.*

Radian Measure

Imagine two *rays* (or “half-lines”) pointing out from the center of a circle so that each ray intersects the circle at a specific point. Suppose that the distance between these points, as measured along the curve of the circle (not along a straight line), is the same as the circle’s radius. In that case, the angle between the rays measures one *radian* (1 rad).

A circle has 2π rad going exactly once around, where π (the lowercase Greek letter pi, not in italics) stands for the ratio of a circle’s circumference to its diameter. This number is the same for all perfect circles in a flat plane. The number π is *irrational*, meaning that we can’t express it as a ratio between whole numbers. It equals approximately 3.14159.

Mathematicians commonly use the radian as their standard unit of angular measure. Sometimes, they’ll omit the “rad” after an angle when they know that they’re working with radians (and when they’re sure that you know it too). Based on that convention, we can sum up the nature of the radian as follows:

- An angle of $\pi/2$ represents 1/4 of a circle
- An angle of π represents 1/2 of a circle
- An angle of $3\pi/2$ represents 3/4 of a circle
- An angle of 2π represents a full circle

We can categorize angles into several types:

- An *acute angle* has a measure of more than 0 but less than $\pi/2$
- A *right angle* has a measure of exactly $\pi/2$
- An *obtuse angle* has a measure of more than $\pi/2$ but less than π
- A *straight angle* has a measure of exactly π
- A *reflex angle* has a measure of more than π but less than 2π

Degree Measure

The *angular degree* ($^\circ$), also called the *degree of arc*, is the unit of angular measure familiar to lay people. One degree (1°) represents 1/360 of a full circle. We can summarize degree measure by noting a few facts:

- An angle of 90° represents 1/4 of a circle
- An angle of 180° represents 1/2 of a circle
- An angle of 270° represents 3/4 of a circle
- An angle of 360° represents a full circle

When we use degrees, the general angle types break down as follows:

- An acute angle has a measure of more than 0 but less than 90°
- A right angle has a measure of exactly 90°
- An obtuse angle has a measure of more than 90° but less than 180°
- A straight angle has a measure of exactly 180°
- A reflex angle has a measure of more than 180° but less than 360°

TIP Whenever you see a quantitative (numerical) reference to an angle and no unit goes with it, and especially if you see the symbol π in the expression, you can assume that the author wants to express the angle in radians. However, you should always check the context to make certain. If you're writing a paper and you want to express an angle in radians, you can write "rad" after the quantity for the angle value. That way, you can make sure that your readers won't get confused. If you write "2 rad," for example, your readers will know that you mean two radians, not two degrees. Similarly, if you want your reader to know for sure that you mean to express an angle in degrees, you should put a little degree symbol after the quantity.

Minutes and Seconds of Arc

Once in awhile, you'll read about fractions of a degree called *minutes of arc* or *arc minutes*. One arc minute (symbolized as 1 arc min or 1') equals 1/60 of a degree. You might also read about *seconds of arc*, also known as *arc seconds*. One arc second (symbolized as 1 arc sec or 1") equals 1/60 of an arc minute, or 1/3600 of a degree.

TIP Arc minutes and arc seconds differ from the minutes and seconds that astronomers sometimes use for defining the positions of objects in the sky. You'll learn about those minutes and seconds in [Chap. 7](#).



PROBLEM 1-1

Imagine that the measure of a certain angle θ equals $\pi/6$. What fraction of a complete circular rotation does this angle represent? What's the measure of θ in degrees?



SOLUTION

A full circular rotation represents an angle of 2π . The value $\pi/6$ equals 1/12 of 2π . Therefore, the angle θ represents 1/12 of a full circle. In degree measure, that's 1/12 of 360° , or 30° .



Still Struggling

If you've always measured and defined angles in degrees, the radian can seem strange at first. "Why," you ask, "would anyone want to divide a circle into an *irrational* number of angular parts?" Mathematicians prefer the radian-measure system because it works out more simply (believe it or not) than the degree-measure scheme in some disciplines. The radian is actually a *more natural* angular quantity than the degree! We can define the radian in geometric terms without talking about any numbers whatsoever, just as we can define the diagonal of a square as the distance from one corner to the opposite corner. The radian is a purely geometric unit. The degree was contrived by humans, probably dating back to ancient times when people knew that a year contained roughly 360 days. When we think of a year as a "circle in time," it's easy to extend the idea to all geometric circles. If our distant ancestors had seriously

considered the true length of the year, which comes out to a fractional number of days (and, one might argue, an irrational number that doesn't even stay the same as the centuries pass), maybe they'd have chosen some other standard angle to represent the degree, such as 1/100 of a circle or 1/1000 of a circle. Even so, the radian is the most natural unit possible!

Angle Notation

Imagine that P , Q , and R represent three distinct points. Let L represent the line segment connecting P and Q , and let M represent the line segment connecting R and Q . We can denote the angle between L and M , as measured at point Q in the plane defined by the three points, by writing $\angle PQR$ or $\angle RQP$ as shown in Fig. 1-1.

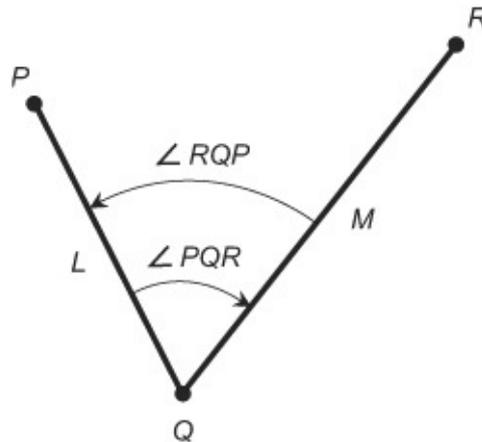


FIGURE 1-1. Angle notation.

If we want to specify the *rotational sense* of the angle, then $\angle RQP$ indicates the angle as we turn counterclockwise from M to L , and $\angle PQR$ indicates the angle as we turn clockwise from L to M . We consider counterclockwise-going angles as having positive values, and clockwise-going angles as having negative values.

In the situation of Fig. 1-1, $\angle RQP$ is positive while $\angle PQR$ is negative. If we make an approximate guess as to the measures of the angles in Fig. 1-1, we might say that $\angle RQP = +60^\circ$ while $\angle PQR = -60^\circ$.

TIP *Rotational sense doesn't make any difference in basic geometry. However, it does matter when we work in coordinate geometry involving graphs. We'll get into coordinate geometry, also known as analytic geometry, later in this book. For now, let's not worry about the rotational sense in which we express or measure an angle. We can consider all angles as having positive measures.*

Angle Bisection Principle

Let's look at an angle called $\angle PQR$ that measures less than 180° , and that we can define with three points P , Q , and R as shown in Fig. 1-2. In this situation, there's *exactly one* (in other words, *one and only one*) ray M that *bisects* $\angle PQR$ (divides $\angle PQR$ in half). If S represents a point on M other than point Q , then $\angle PQS = \angle SQR$.

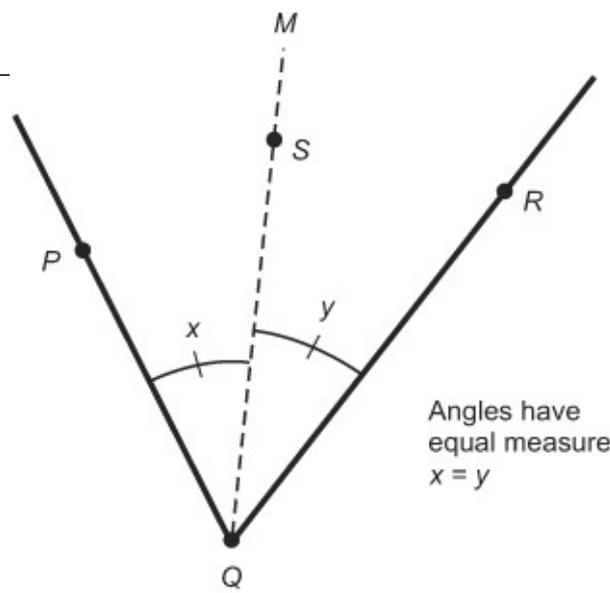


FIGURE 1-2. The angle bisection principle.

Perpendicular Principle

Consider a line L that passes through two distinct points P and Q as shown in Fig. 1-3. Suppose that R represents a point that doesn't lie on L . In that case, there's exactly one line M through point R , intersecting line L at some point S , such that M runs perpendicular to L (M and L intersect at a right angle) at point S .

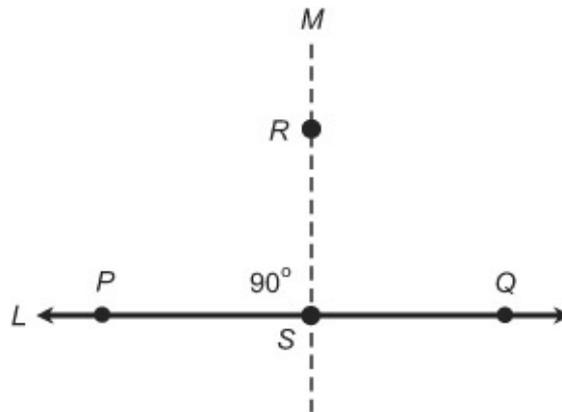


FIGURE 1-3 The perpendicular principle.

Perpendicular Bisector Principle

When we have a line segment connecting two points P and R , we can always find exactly one line M that runs perpendicular to line segment PR and that intersects PR at a point Q , such that the distance from P to Q (which we can write as PQ) equals the distance from Q to R (which we denote as QR). In other words, every line segment has exactly one *perpendicular bisector*. Figure 1-4 illustrates this principle.

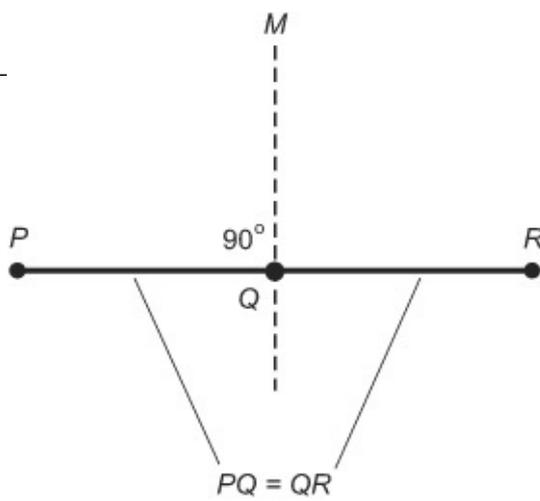


FIGURE 1-4. The perpendicular bisector principle. Illustration for [Problem 1-2](#).

Distance Addition and Subtraction

Imagine that P , Q , and R represent points on a line, arranged so that Q lies somewhere between P and R . The following equations hold true concerning the distances between pairs of points as we measure them along the line as shown in [Fig. 1-5](#):

$$PQ + QR = PR$$

$$PR - PQ = QR$$

$$PR - QR = PQ$$

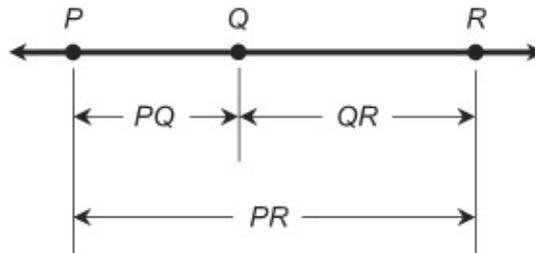


FIGURE 1-5. Distance addition and subtraction.

Angular Addition and Subtraction

Suppose that P , Q , R , and S represent points that all lie in the same *Euclidean plane*. In other words, all four points lie on a single, perfectly flat surface. Let Q represent the convergence point of three angles $\angle PQR$, $\angle PQS$, and $\angle SQR$, with line segment QS between line segments QP and QR as shown in [Fig. 1-6](#). In that case, we'll always find that the following equations hold true:

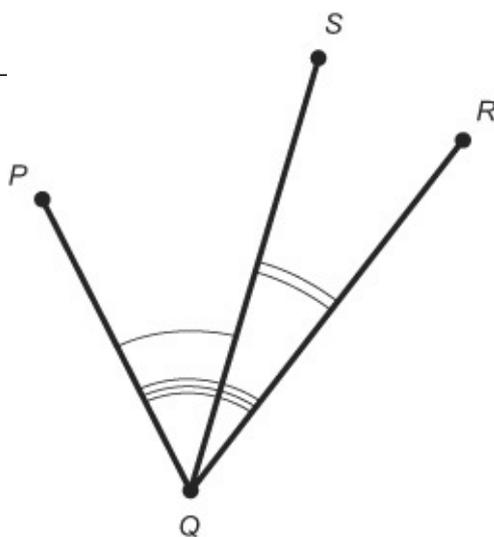


FIGURE 1-6. Angular addition and subtraction. Illustration for [Problem 1-3](#).

$$\angle PQS + \angle SQR = \angle PQR$$

$$\angle PQR - \angle PQS = \angle SQR$$

$$\angle PQR - \angle SQR = \angle PQS$$



PROBLEM 1-2

Go back to [Fig. 1-4](#) and examine it again. Imagine some point S , other than point Q , that lies on line M (the perpendicular bisector of line segment PR). What can you say about the lengths of line segments PS and SR ?



SOLUTION

You can “streamline” the solutions to problems like this by making your own drawings. With the aid of your own sketch, you should see that for any point S that lies on line M (other than point Q), the distance PS exceeds the distance PQ (that is, $PS > PQ$), and the distance SR exceeds the distance QR (that is, $SR > QR$).



PROBLEM 1-3

Look again at [Fig. 1-6](#). Suppose that you move point S either straight toward yourself (out of the page) or straight away from yourself (back behind the page). In either case, point S no longer lies in the same plane as points P , Q , and R do. What can you say about the measures of $\angle PQR$, $\angle PQS$, and $\angle SQR$?



SOLUTION

You can use your “three-dimensional mind’s eye” to envision these situations. Either way you should be able to see that the sum of the measures of $\angle PQS$ and $\angle SQR$ exceeds the measure of $\angle PQR$ because the measures of $\angle PQS$ and $\angle SQR$ both increase if point S departs at a right angle from the plane containing points P , Q , and R .

Triangles

In technical terms, a *triangle* consists of three line segments, joined pairwise at their end points, and including those end points. In order to determine a triangle, the three points must not be *collinear* (they must not all lie on the same straight line). For now, let's assume that the surface or *universe* for all our triangles is Euclidean ("flat"), and not "curved" like the surface of a sphere, cone, or cylinder.

TIP *In a Euclidean universe, we can always determine the shortest distance between two points by finding the straight line segment connecting the points, and then measuring the length of the line segment. Conversely, if the shortest distance between two points always constitutes a straight line, then we know that we're working in a Euclidean universe.*

Vertices, Names, and Sides

Figure 1-7 shows three points called A , B , and C , connected by line segments to form a triangle. We call each point a *vertex* of the triangle, so the figure has three *vertices* (or *vertexes*). We call the whole figure "triangle ABC ."

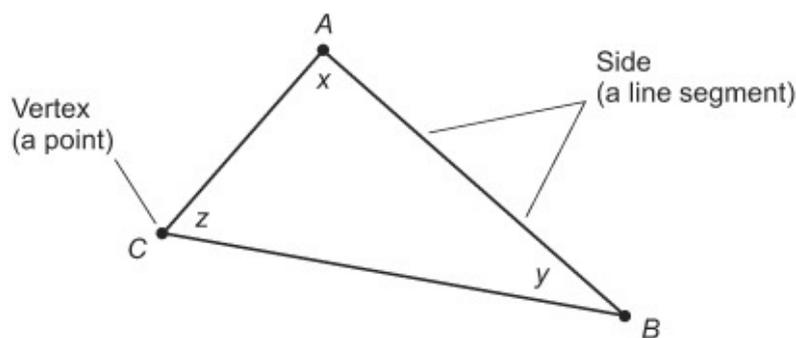


FIGURE 1-7. Vertices, sides, and angles of a triangle.

When naming triangles, geometers write an uppercase Greek letter delta (Δ) in place of the word "triangle." According to that notation, Fig. 1-7 portrays ΔABC . Alternatively, we can call it ΔBCA , ΔCAB , ΔCBA , ΔBAC , or ΔACB .

We can name the sides of the triangle in Fig. 1-7 according to their end points. In this case, our triangle has three sides: line segment AB (or BA), line segment BC (or CB), and line segment CA (or AC).

Interior Angles

Each vertex of a triangle corresponds to a specific *interior angle*, which always measures more than 0° (or 0 if we express it in radians) but less than 180° (or π if we express it in radians). In Fig. 1-7, we denote the interior angles using the lowercase italic English letters x , y , and z . Alternatively, we can use italic lowercase Greek letters to symbolize the interior angles. Subscripts can help us distinguish the angles from one another; for example, θ_a , θ_b , and θ_c could represent the interior angles at vertices A , B , and C , respectively.

Directly Similar Triangles

Two triangles are *directly similar* if and only if they have the same proportions in the same rotational

sense (that is, as we go around them both in the same direction). Two triangles *are not* directly similar if we must flip one of them over, in addition to changing its size and rotating it, in order to place it exactly over the other one.

Directly Congruent Triangles

Two triangles are *directly congruent* if and only if they're directly similar and the corresponding sides have identical lengths. If we take one of the triangles and rotate it clockwise or counterclockwise to the correct extent, we can “paste” it precisely over the other one. Rotation and motion are allowed, but flipping-over, also called *mirroring*, is forbidden.



Still Struggling

You should remember these fundamental facts when you work with triangles that look a lot alike:

- If two or more triangles are directly congruent, then the corresponding sides have equal lengths as you proceed around them all in the same direction. The converse also holds true. If two or more triangles have corresponding sides with equal lengths as you proceed around them all in the same direction, then all the triangles are directly congruent.
- If two or more triangles are directly congruent, then the corresponding interior angles have equal measures as you proceed around them all in the same direction. However, the converse does not always hold true. Two or more triangles can have corresponding interior angles with equal measures when you go around them all in the same direction, and nevertheless fail to be directly congruent (although they'll always be directly similar).

Acute Triangle

We have an *acute triangle* if and only if each of the three interior angles is acute. In such a triangle, none of the angles measure as much as a right angle (90° or $\pi/2$); they're all smaller than that. [Figure 1-8](#) shows some examples.

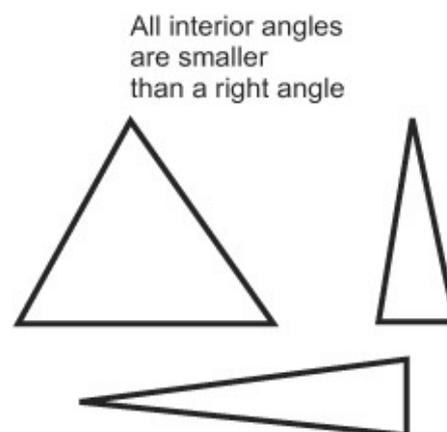


FIGURE 1-8 In an acute triangle, all angles measure less than a right angle.

Obtuse Triangle

We have an *obtuse triangle* if and only if one of the three interior angles is obtuse, measuring more than a right angle (90° or $\pi/2$) but less than a straight angle (180° or π). In a triangle of this type, the two nonobtuse angles are both acute. [Figure 1-9](#) shows some examples.

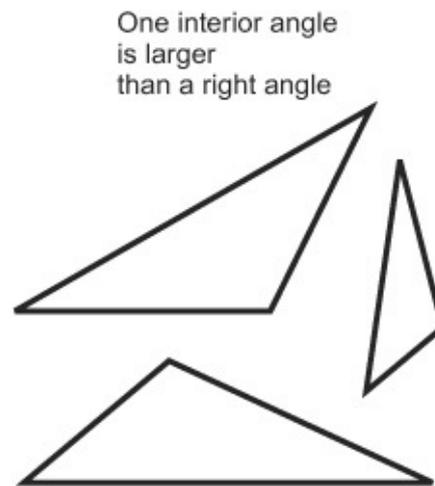


FIGURE 1-9. In an obtuse triangle, one angle measures more than a right angle.

Isosceles Triangle

Imagine a triangle with sides that have lengths s , t , and u . Let x represent the angle opposite the side of length s , let y represent the angle opposite the side of length t , and let z represent the angle opposite the side of length u . Now suppose that *at least one* of the following equations holds true:

$$s = t$$

$$t = u$$

$$s = u$$

$$x = y$$

$$y = z$$

$$x = z$$

[Figure 1-10](#) shows an example of such a situation, where $s = t$. Whenever we find a triangle that has two sides of identical length, we call it an *isosceles triangle*. For sides and angles in the orientation of [Fig. 1-10](#):

$$s = t \leftrightarrow x = y$$

- [download online Darkness Hunts \(Dark Angels, Book 4\)](#)
- [read online Clinical Strategies for Becoming a Master Psychotherapist \(Practical Resources for the Mental Health Professional\) pdf](#)
- [**click The American President: From Teddy Roosevelt to Bill Clinton**](#)
- [click Paris to the Pyrenees: A Skeptic Pilgrim Walks the Way of Saint James pdf, azw \(kindle\), epub, doc, mobi](#)
- [read The Cover-Up at Omaha Beach: D-Day, The US Rangers, and the Untold Story of Maisy Battery](#)

- <http://hasanetmekci.com/ebooks/Jude-s-Herbal-Home-Remedies--Natural-Health--Beauty---Home-Care-Secrets--Living-with-Nature-Series-.pdf>
- <http://nautickim.es/books/The-Case-of-the-Gilded-Fly--Gervase-Fen--Book-1-.pdf>
- <http://serazard.com/lib/The-American-President--From-Teddy-Roosevelt-to-Bill-Clinton.pdf>
- <http://yachtwebsitedemo.com/books/Paris-to-the-Pyrenees--A-Skeptic-Pilgrim-Walks-the-Way-of-Saint-James.pdf>
- <http://aneventshop.com/ebooks/C--sar.pdf>